# Discharge Calculation in Interfering Walls By Modified Total Drawdown-Discharge Equations 

# Geliştriilmiş Toplam Düşüm-Debi Denklemieri Yardım ile Giriṣim Yapan Kuyuların Debilerinin Hesaplanması 

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Bu araştırmada, girişim yapan kuyularda, rarklı düşi m, rırkn yamęap ve farkiı çalışnıa süresi nlması halinde kuyuların debilerinin hesabı için geliştirilmiş toplam düșüm-debi clenklemlerinden faydalanılmiştır.

Ayrıca basınçsız akiferdeki kuyular için tanı toplam düsüm debi denklemi kullanılmıstır.

In this research it was attempted to calculate discharges of interfering wells in the case of different drawdowns, diameters and operating periods by means of modified total drawdown discharge equations.

Also exact total drawdown-discharge equation was used for the wells which are in unconfined aquifers.

## I.VTRODUCTION

Muskat (1) used total drawdown - discharge equations to calculate discharges of interfering wells for steady flows in the case of equal drawns, diameters and operating periods, and furthermore gave some special solutions. However, Hantush (2) used similar equations for unsteady flows and presented some special equations with the same conditions.

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## DERIVATION OF EQUATIONS

Muskat employed the following equations for diseharge calculations in the interfering wells for the case of steady flows as,

$$
\begin{align*}
s_{\mathrm{T}}=\mathrm{H}-h_{\mathrm{T}} & =\sum_{\mathrm{i}=1}^{n} \frac{\mathrm{Q}_{\mathrm{i}}}{2 \cdot \pi k \cdot b} \cdot \ln \left(R_{\mathrm{i}}^{\prime} r_{i}\right)  \tag{1}\\
\mathrm{H}^{2}-h^{2} & =\sum_{i=1}^{n} \frac{Q_{\mathrm{i}}}{\pi \cdot k} \cdot \ln \left(\mathrm{R}_{\mathrm{i}} / r_{\mathrm{i}}\right) \tag{2}
\end{align*}
$$

Eqs. (1) and (2) are valid for confined and unconfined aquifers respectively. Eq. (1) was originally derived on the basis of superposition principle as,

$$
\begin{equation*}
s_{\mathrm{T}}=\sum_{\mathrm{i}=1}^{\mathrm{n}} s_{\mathrm{i}} \tag{3}
\end{equation*}
$$

but Eq. (2) is an approximate equation and is valid only for small values of drawdowns $\left(s_{\mathrm{r}}<2 \mathrm{H}\right)$, where
$\boldsymbol{s}_{\boldsymbol{T}}$ : Total drawdown in a well, $L$.
$h_{\Gamma}$ : Height of water in a well which corresponds to total drawdown in the same well, $h_{\mathrm{T}}=H-s_{\mathrm{T}}, L$.
$s_{\mathrm{i}}$ : Individual drawdown in the $i-t h$ well, $L$.
$H$ : Height of piezometric pressure from the base of a confined aquifer or thickness of an unconfined aquifer $L$.
$b$ : 'Thickness of a confined aquifer, $L$.
$k$ : Coefficient of permeability, $L T$.
$Q_{i}$ : Discharge of the $i-t h$ well, $L^{3} / T$.
$r_{1}$ : Radius of the $i-$ th well, $L$.
$R_{\mathrm{i}}$ : Radius of influence in the $i-t h$ well, $L$.
In the solution of Eqs. (1) and (2) Muskat accepted that drawdowns, diameters and operating periods are the same in all the interfering ivells. In this research, it was attempted to find solutions for different drawdowns, diameters as well as operating periods for each individual interfering wells. Also, exact total drawdown equation was
used, Eq. (3), for wells in unconfined aquifers instead of approximate equation, Eq. (2). First of all Eq. (3) is modified and written in a new form as,

$$
\begin{equation*}
S_{T_{i}}=\sum_{i=1}^{n} s_{i j} \quad(i=1,2, \ldots n) \tag{4}
\end{equation*}
$$

where
$s_{\mathrm{Ti}}:$ Total drawdown in the $i-t h$ well, $L$.
$s_{i j}$ : Influence (drawdown) in the $i-t h$ well which is caused by $j$-th well, $L$.

Eq. (4) can be written explicitly as,

$$
\left.\begin{gather*}
s_{\mathrm{T}_{1}}=s_{11}+s_{12}+\ldots+s_{1 \mathrm{n}}  \tag{5}\\
s_{\mathrm{T}_{2}}=s_{21}+s_{22}+\ldots+s_{2 \mathrm{n}} \\
\cdot \\
s_{\mathrm{T}_{\mathrm{n}}}=s_{\mathrm{n} 1}+s_{\mathrm{n} 2}+\ldots+s_{\mathrm{nn}}
\end{gather*} \right\rvert\,
$$

However, for confined aquifers (in the case of steady flow) mutual drawdown effects can be expressed as,

$$
\begin{equation*}
s_{\mathrm{i}}=\frac{\ln \left(\mathrm{R}_{\mathrm{i}} / r_{\mathrm{ij}}\right)}{2 \cdot \pi \cdot k \cdot b} \cdot \mathrm{Q}_{\mathrm{i}} \tag{6}
\end{equation*}
$$

and for unconfined aquifers this expression turns out to be,

$$
\begin{equation*}
s_{i \mathrm{i}}=\mathrm{H}-\sqrt{\mathrm{H}^{2}-\frac{\ln \left(\mathrm{R}_{\mathrm{V}} / r_{i j}\right)}{\pi \cdot k} \cdot Q_{\mathrm{i}}} \tag{7}
\end{equation*}
$$

By defining a new variable as,

$$
\begin{equation*}
\alpha_{i j}=\frac{\ln \left(R_{i} / r_{1 j}\right)}{2 \cdot \pi \cdot k \cdot b} \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{11}=\frac{\ln \left(\mathrm{R}_{1} / r_{11}\right)}{\pi \cdot k} \tag{9}
\end{equation*}
$$

Eqs. (6) and (7) can be written implicitly as,

$$
\begin{equation*}
s_{i j}=\alpha_{i j} \cdot Q_{i} \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
s_{\mathrm{ij}}=\mathrm{H}-\sqrt{\mathrm{H}^{2}-\beta_{\mathrm{ij}} \cdot Q_{\mathrm{i}}} \tag{11}
\end{equation*}
$$

where
$r_{\text {IJ }} \quad: \quad$ The distance between the $i-t h$ and $j-t h$ well, $L$.
$R_{i} \quad:$ Radius of influence in the $j-t h$ well, $L$.
$Q_{j} \quad: \quad$ Discharge of the $j$-th well, $L^{3} / T$.
$\left.\begin{array}{l}\alpha_{i j} \\ \beta_{j}\end{array}\right\}$ : Dummy variables.
If Eq. (10) is substituted into Eq. (5), then for confined aquifers one can find,

$$
\begin{align*}
& s_{T_{1}}=\alpha_{11} \cdot Q_{1}+\alpha_{12} \cdot Q_{2}+\ldots+\alpha_{10} \cdot Q_{n} \\
& s_{T_{2}}=\alpha_{21} \cdot Q_{1}+\alpha_{22} \cdot Q_{2}+\ldots+\alpha_{20} \cdot Q_{n}  \tag{12}\\
& \cdot \\
& s_{T_{0}}=\alpha_{n 1} \cdot Q_{1}+\alpha_{n 2} \cdot Q_{2}+\ldots+\alpha_{n n} \cdot Q_{n}
\end{align*}
$$

or shortly,

$$
\begin{equation*}
s_{T 1}=\sum_{j=1}^{n} \alpha_{i 1} Q_{i} \quad(i=1,2, \ldots, n) \tag{12a}
\end{equation*}
$$

However, for unconfined aquifers, first, it is useful to define

$$
\begin{equation*}
\delta_{i j}=\sqrt{H^{2}-\beta_{i l} \cdot Q_{j}} \tag{13}
\end{equation*}
$$

and accordingly Eq. (11) becomes,

$$
\begin{equation*}
s_{i j}=\mathrm{H}-\delta_{i j} \tag{14}
\end{equation*}
$$

On the other hand, if Eq. (14) is substituted into Eq. (5) it leads to,

$$
\left.\begin{array}{l}
s_{\mathrm{T}_{1}}=n \cdot \mathrm{H}-\left(\delta_{11}+\delta_{12}+\ldots+\delta_{10}\right) \\
s_{\mathrm{T}_{2}}=n \cdot \mathrm{H}-\left(\delta_{21}+\delta_{22}+\ldots+\delta_{2 \mathrm{a}}\right) \\
\cdot \\
s_{\mathrm{T}_{\mathrm{a}}}=n \cdot \mathrm{H}-\left(\delta_{\mathrm{n} 1}+\delta_{\mathrm{a} 2}+\ldots+\delta_{\mathrm{na}}\right)
\end{array}\right\}
$$

or
or

$$
\left.\begin{align*}
& n \cdot \mathrm{H}-s_{\tau_{1}}=\delta_{11}+\delta_{12}+\ldots+\delta_{1 n}  \tag{16}\\
& n \cdot \mathrm{H}-s_{\tau_{2}}=\delta_{21}+\delta_{22}+\ldots+\delta_{2 \mathrm{n}} \\
& \cdot \cdot \cdot \\
& n \cdot \mathrm{H}-s_{\tau_{\mathrm{n}}}=\delta_{\mathrm{n} 1}+\delta_{\mathrm{n} 2}+\ldots+\delta_{\mathrm{on}}
\end{align*} \right\rvert\,
$$

Furthermore, it can be rewritten shortly as,

$$
\begin{equation*}
n \cdot \mathrm{H}-s_{\mathrm{Ti}}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \delta_{\mathrm{ij}} \quad(i=1,2, \ldots, n) \tag{16a}
\end{equation*}
$$

Since, $\delta_{1 J}$ is an irrational function, it is necessary to use computer for numerical solutions of Eq. (16).

However, modified approximate equations can be used also for confined aquifers in the case of different drawdowns, diameters, and operation periods provided that drawdowns are small.

Modified approximate general equation can be written as follows,

$$
\begin{equation*}
\mathrm{H}^{2}-h^{2} \mathrm{Ti}_{\mathrm{i}}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \frac{\ln \left(\mathrm{R}_{\mathrm{i}} / r_{\mathrm{ij}}\right)}{\pi \cdot k} \cdot \mathrm{Q}_{\mathrm{i}} \tag{17}
\end{equation*}
$$

under the light of Eq. (9), Eq. (17) can be rewritten as,

$$
\begin{equation*}
\mathrm{H}^{2}-h^{2}{ }_{T i}=\sum_{j=1}^{n} \beta_{1 \mathrm{l}} \cdot Q_{\mathrm{i}} \quad(i=1,2, \ldots, n) \tag{18}
\end{equation*}
$$

or explicitly as,

$$
\left.\begin{align*}
& \mathrm{H}^{2}-h^{2}{ }_{\mathrm{T}}=\beta_{11} \cdot Q_{1}+\beta_{12} \cdot Q_{2}+\ldots+\beta_{1 \mathrm{a}} \cdot Q_{\mathrm{n}}  \tag{19}\\
& \mathrm{H}^{2}-h^{2} \mathrm{~T}_{2}=\beta_{21} \cdot Q_{1}+\beta_{22} \cdot Q_{2}+\ldots+\beta_{2 \mathrm{a}} \cdot Q_{\mathrm{n}} \\
& \cdot \cdot \\
& \mathrm{H}^{2}-h^{2} \mathrm{~T}_{2}=\beta_{\mathrm{n} 1} \cdot Q_{1}+\beta_{\mathrm{n} 2} \cdot Q_{2}+\ldots+\beta_{\mathrm{na}} \cdot Q_{\mathrm{n}}
\end{align*} \right\rvert\,
$$

Also, Eq. (19) takes the following form of equation system,

$$
\begin{gather*}
M_{1}=\beta_{11} \cdot Q_{1}+\beta_{12} \cdot Q_{2}+\ldots+\beta_{1 \mathrm{n}} \cdot Q_{\mathrm{o}}  \tag{20}\\
M_{2}=\beta_{21} \cdot Q_{1}+\beta_{22} \cdot Q_{2}+\ldots+\beta_{2 \mathrm{n}} Q_{\mathrm{n}} \\
M_{\mathrm{n}}=\beta_{\mathrm{n} 1} \cdot Q_{1}+\beta_{\mathrm{n} 2} \cdot Q_{2}+\ldots+\beta_{\mathrm{an}} \cdot Q_{\mathrm{n}}
\end{gather*}
$$

which can be written briefly as,

$$
M_{i}=\sum_{j=1}^{n} \beta_{11} \cdot Q_{1} \quad(i=1,2, \ldots, n)
$$

where in all the above equations

$$
\begin{equation*}
\mathrm{M}_{\mathrm{i}}=\mathrm{H}^{2}-h_{\mathrm{T}}^{2} \tag{21a}
\end{equation*}
$$

Eq. (20) is similar to Eq. (12) and can be solved easily.
On the other hand, for unsteady flow in confined aquifers it can be written from Theis (3) equation as,

$$
\begin{equation*}
s_{i \mathrm{i}}=\frac{\mathrm{W}\left(u_{i j}\right)}{4 . \pi \cdot k \cdot b} \cdot Q_{i} \tag{2}
\end{equation*}
$$

with the definition of the following new variables,

$$
\begin{equation*}
\bar{\alpha}_{i_{i}}=\frac{w\left(u_{1 \mid}\right)}{4 \cdot \pi \cdot k \cdot b} \tag{23}
\end{equation*}
$$

Eq. (22) can be conciesly written as,

$$
\begin{equation*}
s_{i \mathrm{ij}}=\bar{\alpha}_{\mathrm{ij}}, Q_{\mathrm{i}} \tag{24}
\end{equation*}
$$

And for unsteady flow in confined aquifers from modified Theis equation one can write,

$$
H^{2}-h^{2}=\frac{Q}{2 \cdot \pi \cdot \dot{k}} \cdot W(u)
$$

ft can be written that,

$$
s=\mathrm{H}-\sqrt{\mathrm{H}^{2}-\frac{\mathrm{W}\left(u_{\| j}\right)}{2 \cdot \pi \cdot k} \cdot \mathrm{Q}}
$$

due to $h=H-s$, or

$$
\begin{equation*}
s_{\mathrm{i}}=\mathrm{H}-\sqrt{\mathrm{H}^{2}-\frac{W\left(u_{i j}\right)}{2 \cdot \pi \cdot k} \cdot Q_{\mathrm{i}}} \tag{25}
\end{equation*}
$$

with the definition of the following new variable

$$
\begin{equation*}
\bar{\beta}_{\mathrm{ij}}=\frac{W\left(u_{\mathrm{ij}}\right)}{2 \cdot \pi \cdot k} \tag{26}
\end{equation*}
$$

Eq. (25) can be written briefly as,

$$
\begin{equation*}
s_{i \mathrm{i}}=\mathrm{H}-\sqrt{\mathrm{H}^{2}-\bar{\beta}_{i \mathrm{i}} \cdot Q_{\mathrm{i}}} \tag{27}
\end{equation*}
$$

And also if it is defined that,

$$
\begin{equation*}
\bar{\delta}_{i f}=\sqrt{H^{2}-\bar{\beta}_{i j} \cdot Q_{i}} \tag{28}
\end{equation*}
$$

Eq. (27) becomes as,

$$
\begin{equation*}
s_{i j}=H-\bar{\delta}_{1 \mathrm{i}} \tag{29}
\end{equation*}
$$

In this case Eqs. (12), (16) and (20) can be used for unsteady flows by replacing $\bar{\alpha}_{i j}, \bar{\beta}_{1 j}, \bar{\delta}_{i j}$ instead of $\alpha_{i j}, \beta_{i j}, \delta_{i j}$. The meanings of some parameters in the above derivations are as follows,
$W\left(u_{\mathrm{ij}}\right) \quad: \quad$ Well function.
$u_{i j} \quad: S \cdot r_{i j} / 4 \cdot T \cdot t_{0}$.
$r_{i j} \quad:$ Distance between the $i-t h$ and $j$-th wells, $L$.
$S \quad:$ Storage coefficient.
$T$ : Transmissibility. $L^{2 /} / T$.
$\bar{\alpha}_{i j} \overline{\bar{\delta}}_{i j} \bar{\beta}_{i j} ;$ : Dummy variables.
$t_{0} \quad: \quad$ operation time, $T$.

## APPLICATIONS

## Problem I:

4 wells in a corfined squifer which are randomly scattered in the field in the case of steady flow.


## Solution :

First, Eq. (12) is written

$$
\begin{align*}
& s_{T_{1}}=\alpha_{11} \cdot Q_{1}+\alpha_{12} \cdot Q_{2}+\alpha_{13} \cdot Q_{3}+\alpha_{14} \cdot Q_{1} \\
& s_{T_{2}}=\alpha_{21} \cdot Q_{1}+\alpha_{22} \cdot Q_{2}+\alpha_{23} \cdot Q_{3}+\alpha_{24} \cdot Q_{4} \\
& s_{T 3}=\alpha_{31} \cdot Q_{1}+\alpha_{32} \cdot Q_{2}+\alpha_{33} \cdot Q_{3}+\alpha_{34} \cdot Q_{4} \\
& s_{T_{4}}=\alpha_{41} \cdot Q_{1}+\alpha_{42} \cdot Q_{2}+\alpha_{43} \cdot Q_{3}+\alpha_{44} \cdot Q_{4}
\end{align*}
$$

Then according to Eq. (8) the values of $\alpha_{i j}$ are calculated as follows,

$$
\begin{aligned}
& \alpha_{11}=\ln \left(R_{1} / r_{11}\right) / 2 \cdot \pi \cdot k \cdot b \\
& \alpha_{12}=\ln \left(R_{2} / r_{12}\right) / 2 \cdot \pi \cdot k \cdot b \\
& \alpha_{43}=\ln \left(R_{9} / r_{13}\right) / 2 \cdot \pi \cdot k \cdot b \\
& \alpha_{14}=\ln \left(R_{4} / r_{44}\right) / 2 \cdot \pi \cdot k \cdot b
\end{aligned}
$$

The values $s_{\mathrm{T} 1}, s_{\mathrm{T} 2}, s_{\mathrm{T} 3}, s_{\mathrm{T} 4}$ are given in the beginning as data. For a special case, if it is assumed that four wells are on the corners of a square, and drawdowns, diameters and operation periods are equal to each other in the well group as shown in the following sketch, the calculations can be achieved as follows


Due to the symmetric well distribution one can write the following points :

1) $r_{12}=r_{29}=r_{34}=r_{41}, r_{13}=r_{24}$
2) $r_{11}=r_{22}=r_{39}=r_{44}=r$
3) $s_{\mathrm{T}_{1}}=s_{\mathrm{T}_{2}}=s_{\mathrm{T}_{9}}=s_{\mathrm{T}_{4}}=s_{\mathrm{T}}$
4) $t_{1}=t_{2}=t_{3}=t_{4}=t$ or

$$
R_{1}=R_{2}=R_{3}=R_{4}=R
$$

From the above mentioned knowledge the following results are found,

$$
\alpha_{11}=\alpha_{22}=\alpha_{33}=\alpha_{14}=\frac{1 \text { ? }}{2 \cdot \pi \cdot k \cdot b} \cdot \ln (\mathrm{R} / r)
$$

and

$$
\left.\begin{array}{l}
\left.\alpha_{12}=\alpha_{23}=\alpha_{34}=\alpha_{11}\right) \\
\left.\alpha_{21}=\alpha_{32}=\alpha_{43}=\alpha_{14}\right\}
\end{array}\right)=\frac{1}{2 \cdot \pi \cdot k \cdot \frac{z}{t} \cdot \ln (\mathrm{R} / \mathrm{L})}
$$

and

$$
\alpha_{15}=\alpha_{31}=\alpha_{24}=\alpha_{42}=\frac{1}{2 \cdot \pi \cdot k \cdot b} \cdot \ln (\mathrm{R} / \sqrt{2} \cdot \mathrm{~L})
$$

After these procedures all $\alpha_{i s}$ values are substituted in Eqs. (I-1) together with $s_{T 1}=s_{T 2}=s_{T 3}=s_{T 4}=s_{\mathrm{T}}$, and because of the symmetry, it can be written $Q_{1}=Q_{2}=Q_{3}=Q_{4}=Q$. Hence, it is found that

$$
s_{T}=\frac{Q}{2 \cdot \pi \cdot k \cdot b}[\ln (\mathrm{R} / r)+\ln (\mathrm{R} / \mathrm{L})+\ln (\mathrm{R} / \sqrt{2} \cdot \mathrm{~L})+\ln (\mathrm{R} / \mathrm{L})]
$$

where in Eq. (I-1) each of the four equations become the same. Hence, it is found that,

$$
\begin{equation*}
\mathrm{Q}=\frac{2 \cdot \pi \cdot k \cdot b \cdot s_{\mathrm{T}}}{\ln \left(\mathrm{R}^{4} / \sqrt{2} \cdot \mathrm{~L}^{3} \cdot r\right)} \tag{I-2}
\end{equation*}
$$

In fact, this is the same equation that was found by Muskat (1) in a confined aquifer, in the of case four wells which are on the corners of a square, for steady flow.

## Problem II :

3 wells in a confined aquifer which are randomly scattered in the field in the case of steady flow


## Solution :

First, Eq. (16) is written

$$
\begin{align*}
\text { 3. } \mathrm{H}-s_{\mathrm{T} 1} & =\delta_{11}+\delta_{12}+\delta_{19} \\
3 . \mathrm{H}-s_{\mathrm{T}_{2}} & =\delta_{21}+\delta_{22}+\delta_{29}  \tag{II-1}\\
\text { 3. } \mathrm{H}-s_{\mathrm{T}_{3}} & =\delta_{31}+\delta_{32}+\delta_{33}
\end{align*}
$$

where from Eq. (13)

$$
\begin{equation*}
\delta_{\mathrm{t}_{i}}=\sqrt{\mathrm{H}^{2}-\beta_{\mathrm{i}} \cdot Q_{i}} \tag{II-2}
\end{equation*}
$$

and from Eq. (9)

$$
\begin{equation*}
\beta_{i \mathrm{i}}=(1 / \pi, k) \cdot \ln \left(\mathrm{R}_{\mathrm{j}} / \tau_{\mathrm{i}}\right) \tag{II-3}
\end{equation*}
$$

From Eq. (II - 3) it can be seen that :

$$
\begin{aligned}
& \beta_{11}=(1 / \pi \cdot k) \cdot \ln \left(\mathrm{R}_{1} / r_{11}\right) \\
& \beta_{12}=(1 / \pi \cdot k) \cdot \ln \left(\mathrm{R}_{2} / r_{12}\right) \\
& \beta_{13}=(1 / \pi \cdot k) \cdot \ln \left(\mathrm{R}_{5} / r_{13}\right) \\
& \cdot \\
& \beta_{39}=(1 / \pi \cdot k) \cdot \ln \left(\mathrm{R}_{9} / r_{33}\right)
\end{aligned}
$$

Due to the fact that $\delta_{1 j}$ is irrational function of $Q_{j}$, it is necessary to use computer for numerical solutions. If it is desired to use approximate equation for small drawdowns, Eq. (19) can then be written as,

$$
\begin{align*}
& H^{2}-h^{2} \mathrm{~T}_{1}=\beta_{11} \cdot \mathrm{Q}_{1}+\beta_{12} \cdot \mathrm{Q}_{2}+\beta_{13} \cdot Q_{1} \\
& \mathrm{H}^{2}-h^{2} \mathrm{~T}_{2}=\beta_{21} \cdot \mathrm{Q}_{1}+\beta_{22} \cdot Q_{2}+\beta_{23} \cdot Q_{3}  \tag{II-4}\\
& \mathrm{H}^{2}-h^{2} \mathrm{~T}_{3}=\beta_{31} \cdot Q_{1}+\beta_{32} \cdot Q_{2}+\beta_{33} \cdot Q_{3}
\end{align*}
$$

The values of $\beta_{i j}$ are the same as before.
For a special case, where three wells are on the corners of an equi lateral triangle and drawdowns, diameters and operation periods are equal then it is possible to write the following points.


1) $r_{12}=r_{23}=r_{31}=L$
2) $r_{11}=r_{22}=r_{33}=r$
3) $h_{T_{1}}=h_{T_{2}}=h_{T_{3}}=h_{T}$
4) $t_{1}=t_{2}=t_{3}=t$ or $\mathrm{R}_{1}=\mathrm{R}_{2}=\mathrm{R}_{\mathrm{g}}=\mathrm{R}$

From the above mentioned knowledge the following results are found

$$
\left.\begin{array}{l}
\beta_{11}=\beta_{22}=\beta_{39}=\frac{1}{\pi \cdot k} \cdot \ln (\mathrm{R} / r) \\
\beta_{12}=\beta_{23}=\beta_{31} \\
\beta_{31}=\beta_{32}=\beta_{13}
\end{array}\right\}=\frac{1}{\pi \cdot \dot{k}} \cdot \ln (\mathrm{R} / \mathrm{L})
$$

After these calculations all $\beta_{\mathrm{i} j}$ values are substituted into Eq. (II-4) together with

$$
h_{\mathrm{T}_{1}}=h_{\mathrm{T}_{2}}=h_{\mathrm{T}_{3}}=h_{\mathrm{T}}
$$

Also, it is a fact that $Q_{1}=Q_{2}=Q_{3}=Q$ which are due to symmetry. These considerations leads us to,

$$
H^{2}-h^{2}=\frac{Q}{\pi \cdot \vec{k}} \cdot[\ln (R / r)+\ln (R / L)+\ln (R / L)]
$$

where, in Eq. (II - 4) system, each of the three equations become the same. Hence, it is found,

$$
\begin{equation*}
\mathrm{Q}=\frac{\pi \cdot k \cdot\left(\mathrm{H}^{2}-h^{2} \tau\right)}{\ln \left(R^{3} / L^{2} \cdot \tau\right)} \tag{II-5}
\end{equation*}
$$

This is the same equation that was earlier found by Musisat in an unconfined aquifer, for three wells which are located on thie corners of an equilateral triangle, in the case of steady flow.

However, for this special case exact equations, Eqs. (II-1) and (II-2) can be solved without going to computer. First of all it is better to write $\delta_{11}$ values

$$
\begin{aligned}
& \delta_{11}=\sqrt{H^{2}-\beta_{11} \cdot Q_{1}} \\
& \delta_{22}=\sqrt{H^{2}-\beta_{n} Q_{2}} \\
& \delta_{31}=\sqrt{H^{2}-\beta_{33} \cdot Q_{3}}
\end{aligned}
$$

For this special case one can write,

$$
\beta_{11}=\beta_{22}=\beta_{33}=\frac{1}{\pi \cdot k} \cdot \ln (\mathrm{R} / r)=\beta_{r}
$$

and also because $Q_{1}=Q_{2}=Q_{3}=Q$, due to symmetry,

$$
\delta_{11}=\delta_{22}=\delta_{30}=\sqrt{H^{2}-\beta_{r} \cdot Q}=\delta_{r}
$$

Furthermore,

$$
\left.\begin{array}{l}
\beta_{12}=\beta_{23}=\beta_{31} \\
\beta_{21}=\beta_{32}=\beta_{13}
\end{array}\right\}=\frac{1}{\pi \cdot k} \cdot \ln (R / L)=\beta_{\mathrm{L}}
$$

and therefore

$$
\left.\begin{array}{l}
\delta_{12}=\delta_{23}=\delta_{31} \\
\delta_{21}=\delta_{32}=\delta_{13}
\end{array}\right\}=\sqrt{\mathrm{H}^{2}-\beta_{L} \cdot Q}=\delta_{\mathrm{L}}
$$

And also it is given that, $s_{T 1}=s_{T 2}=s_{T 3}=s_{\mathrm{T}}$, hence it can be written that,
or explicitly

$$
\begin{equation*}
\text { 3. } \mathrm{H}-s_{\mathrm{T}}=\delta_{\mathrm{r}}+2 \cdot \delta_{\mathrm{L}} \tag{II-6}
\end{equation*}
$$

$$
\begin{equation*}
\text { 3. } \mathrm{H}-s_{T}=\sqrt{H^{2}-\beta_{\mathrm{r}} \cdot Q}+2 \cdot \sqrt{\mathrm{H}^{2}-\beta_{\mathrm{l}} \cdot Q} \tag{II-7}
\end{equation*}
$$

where, in Eq. (II - 1) system, each of the three equations become the same. After solution Eq. (II-7) it is found that,

$$
\begin{equation*}
Q=\left(-B_{3} \pm \sqrt{B_{3}^{\prime}-4 A_{3} \cdot C_{3}}\right)^{\prime} 2 \cdot A_{3} \tag{11-8}
\end{equation*}
$$

where

$$
\begin{gathered}
A_{3}=\beta_{r} \cdot \beta_{L}-E_{2}^{2} \\
B_{3}=-\left(H^{2} \cdot \beta_{r}+H^{2} \cdot \beta_{L}+2 \cdot E_{1} \cdot E_{2}\right) \\
C_{3}=H^{4}-E_{1}^{2}
\end{gathered}
$$

and

$$
\begin{gathered}
\mathrm{E}_{1}=\left[\left(3 \cdot \mathrm{H}-s_{\mathrm{T}}\right)^{2}-5 \cdot \mathrm{H}^{2}\right] / 4 \\
\mathrm{E}_{2}=\left(\beta_{\mathrm{r}}-4 \cdot \beta_{\mathrm{L}}\right)^{\prime} 4
\end{gathered}
$$

The problem of two interfering wells with equal drawdowns can be solved in the same way,

$$
\left.\begin{array}{l}
2 . \mathrm{H}-s_{T_{1}}=\delta_{11}+\delta_{12}  \tag{II-9}\\
2 . \mathrm{H}-s_{T_{2}}=\delta_{21}+\delta_{22}
\end{array}\right\}
$$

where

$$
\begin{aligned}
& \delta_{11}=\delta_{22}=\delta_{\mathrm{r}} \\
& \delta_{12}=\delta_{21}=\delta_{\mathrm{L}} \\
& s_{\mathrm{T}_{1}}=s_{\mathrm{T}_{2}}=s_{\mathrm{T}}
\end{aligned}
$$

Hence, it is found

$$
\begin{equation*}
2 . \mathrm{H}-s_{\mathrm{T}}=\delta_{\mathrm{r}}+\delta_{\mathrm{L}} \tag{II-10}
\end{equation*}
$$

since

$$
\begin{aligned}
\delta_{r} & =\sqrt{\mathrm{H}^{2}-\beta_{r} \cdot \mathrm{Q}} \\
\delta_{\mathrm{L}} & =\sqrt{\mathrm{H}^{2}-\beta_{\mathrm{L}} \cdot \mathrm{Q}}
\end{aligned}
$$

it can be written

$$
\begin{equation*}
\text { 2. } \mathrm{H}-S_{T}=\sqrt{\mathrm{H}^{2}-\beta_{\mathrm{r}} \cdot Q}+\sqrt{\mathrm{H}^{2}-\beta_{L} \cdot Q} \tag{II-11}
\end{equation*}
$$

After solution Eq. (II-11) system it is found that

$$
Q=\left(-B \pm \sqrt{B_{2}^{2}-4 \cdot A_{2}} \cdot C_{2}\right) / 2 \cdot A_{2}
$$

where

$$
\begin{gathered}
A_{2}=\beta_{r} \cdot \beta_{\mathrm{L}}-\mathrm{F}_{2}^{2} \\
B_{2}=-\left(\mathrm{H}^{2} \cdot \beta_{\mathrm{r}}+\mathrm{H}^{2} \cdot \beta_{\mathrm{L}}+2 \cdot \mathrm{~F}_{1} \cdot \mathrm{~F}_{2}\right) \\
\mathrm{C}_{2}=\mathrm{H}^{4}-\mathrm{F}_{1}^{2}
\end{gathered}
$$

and

$$
\begin{gathered}
F_{1}=\left[\left(2 \cdot H-s_{T}\right)^{2}-2 \cdot \mathrm{H}^{2}\right] / 2 \\
F_{2}=\left(\beta_{\mathrm{r}}+\beta_{\mathrm{L}}\right) / 2
\end{gathered}
$$

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[^0]:    1 Water Supply Department of the Holy Clty of Makkah.

