Angular Correlation Of Scattered Annihilation Radiation

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SUMMARY

There exists an angular correlation between the two scattered «annihilation quanta» because of their initial cross polarization, when this polarization effect is calculated a very important azimuthal asymmetry leads to the conclusion that the two «annihilation quanta» are linearly polarized at right angles to each other. The main concern of the present work is to reach this conclusion experimentaly, and the results found are consistent with the theory.

ÖZET

Başlangıç polarizasyonundan dolayı iki saçılmış anhilasyon kuantası arasında bir açısal bağıntı vardır.Bu polarizasyon tesiri hesaplanacak olursa çok önemli bir kutupsal asimetri değeri, iki anhilasyon kuantasının birbirine dik lineer polarizasyona sahip oldukları sonucunu verir. Bu çalışmanın ana gayesi bahsi geçen sonuca deneysel olarak ulaşmaktadır, bulunan değerler teori ile uyum halindedirler.

INTRODUCTION

In 1945 J. A. Wheeler[1] proposed an experiment to verify that the two annihilation quanta are polarized at right angles to each other. The theoretical investigations were published by Pryce and Ward[2] (1974) and by Snyder, Pasternack and Hornbostel[3] (1948). Bleuler and Bradt[4] (1948)

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and Hanna[5] (1948) performed coincidence experiments by using two and window Ceiger Müller tubes, and because of inefficieny of these counters they could not obtain the theoretical results. In 1949 C.S. Wu and I. Shaknov[6] did the same experiment by using two anthracene crystals with photomultupliers and obtained a very satisfactory result. Later on the experiment was repeated by Vlassov[7] (1950) and by Hereford[8] (1951) and they also confirmed that the theory is correct. Recently Butt et al (1976) studied the case in detail. The purpose of the present paper is to confirm the experimentel methods accuracy.

INITAL CROSS POLARIZATION OF THE TWO ANNIHILATION QUANTA

First we show that the tw annihilation quanta are polarized at right angles[9].

Singlet Positronium (a hydrogenlike atom, whose nucleus is a positron instead of a proton) is an S_0 state hence the total angular momentum is zero and parity is odd; therefore the total wave function of the annihilation photons must also have these properties, so that both must have same circular polarization. Define ψ_{RR} and ψ_{LL} as two right circularly polarized waves in $+$ z and $-$ z directions, and two left circularly polarized waves in the same directions respectively. Then the odd function is

$$\psi_{RR} - \psi_{LL}$$

We can express creation operators for circularly polarized photons as linearly polarized photons as follows :

$$a^{R*}_{+} = \frac{1}{\sqrt{2}}(a^{*}_{+1} - ia^{*}_{+2}) \qquad a^{L*}_{+} = \frac{1}{\sqrt{2}}(a^{*}_{+1} + ia^{*}_{+2})$$

$$a^{R*}_{-} = \frac{1}{\sqrt{2}}(a^{*}_{-1} - ia^{*}_{-2}) \qquad a^{L*}_{-} = \frac{1}{\sqrt{2}}(a^{*}_{-1} - ia^{*}_{-2})$$

Where the plus and minus sign below a's refer to the positive and negative z directions while the subscripts 1, and 2 the x and ydirections respectively (i.e. a_{+1} creates a photon in $+$z direction linecarly polarized in x direction.)

Then the states ψ_{RR} and ψ_{LL} can be expressed by means of the operators a :

$$\psi_{RR} = a_{+}^{R^*} \; a_{-}^{R^*} \psi_{00}$$

$$\psi_{LL} = a_{+}^{L^*} \; a_{-}^{L^*} \psi_{00}$$

Here ψ_{00} denotes the vacuum. When the values of operators a are substituted

$$\psi_{RR} - \psi_{LL} = -i\,(a^*_{+1}\, a^*_{-2} + a^*_{+2}\, a^*_{-1})\ \psi_{00}$$

is obtained. Since $a_{+1} a_{-2}\, \psi_{00}$ defines a linearly polarized wave in positive z direction with its polarization vector in x direction and a linearly polarized wave in negative z direction with its polarization vector in y direction, and since $a_{+2}\, a_{-1}\, \psi_{00}$ defines a linearly polarized wave in positive z direction with its vector of polarization in y direction, and a linearly polarized wave in negative z direction with its polarization vector in x direction, the two annihilation photons are polarized at right angles to each other.

THE ASYMMETRY RATIO

To verify that the polarization directions of the two annihilation quanta are at right angles to each other, we use the polarization dependence of the Compton scattering cross - section. Consider the scattering of the two quanta, assuming that their polarization vectors are perpendicular to each other.

The two quanta are scattered and then counted in coincidence. The differential cross - section for the scattering of polarized photons by free electrons is given by Klein and Nishina formula[17]:

$$\frac{d\sigma e}{d\Omega} = \frac{r_o^2}{2}\left(\frac{k'_i}{k_i}\right)^2 (\gamma_i - 2\sin^2\theta_i\ \cos^2\phi_i)$$

$$\left(i = 1{,}2\ ;\ k'_i = \frac{k_i}{2-\cos\theta_i}\ ;\quad \gamma_i = \frac{k_i}{k_o} + \frac{k_o}{k_i}\right)$$

If the polarization of the first photon is in its plane of scattering : $\varphi_1 = 0$ then $\varphi_2 = \frac{\pi}{2} - \psi$. Using this the differential cross - sections for the scattering of each quanta will be :

$$P_1(\theta_1) \backsim \left(\frac{k'_1}{k_2}\right)^2 (\gamma_1 - 2 \sin^2 \theta_1)$$

$$P_2(\theta_2) \backsim \left(\frac{k'_2}{k_2}\right)^2 (\gamma_2 - 2 \sin^2 \theta_2 \sin^2 \psi)$$

And the coincidence cross section $P'(\theta_1, \theta_2)$ is the product of $P_1(\theta_1)$ and $P_2(\theta_2)$:

$$P'(\theta_1, \theta_2) \backsim \left(\frac{k_1' k_2'}{k_1 h_2}\right)^2 (\gamma_1 - 2 \sin^2 \theta_1)(\gamma_2 - 2 \sin^2 \theta_2 \sin^2 \psi)$$

For $\varphi_1 = \frac{\pi}{2}$, $\varphi_2 = \pi - \psi$ this gives $\cos^2 \varphi_0 = \cos^2 \psi$. Then two scattering cross sections are :

$$P_3 \backsim \left(\frac{k'_1}{k_1}\right)^2 \gamma_1$$

$$P_4(\theta_2) \backsim \left(\frac{h'_2}{k_2}\right)^2 (\gamma_2 - 2 \sin^2 0_2 \cos \psi)$$

For this case the coincidence cross - section $P''(\theta_2)$ becomes :

$$P'(\theta_3) \backsim \left(\frac{k_1' k'_2}{k_1 k_2}\right)^2 \gamma_1 (\gamma_2 - 2 \sin^2 0_2 \cos^2 \psi).$$

Since both polarization forms occur equally often, the total differential cross - section and hence the total coincidence probability will be :

$$P(\theta_1, \theta_7) = A \left(\frac{k'_1 k'_2}{k_2 k_2}\right)^2 (\gamma_1 \gamma_2 - \gamma_1 \sin^2 \theta_2 - \gamma_2 \sin^2 \theta_2 + 2 \sin^2 \theta_2 \sin^2 \psi)$$

(Wehere A is a normalization constant to be obtained by integrating over all scattering angles of $0 \leq \theta_1 \leq \pi, 0 \leq \theta_2 \leq \pi$)

putting $k_1 = k_2 = k$, $k_1' = k_2' = k'$, and $\theta_1 = \theta_2 = \theta$ this becomes :

$$P(\theta) = A \left(\frac{k'}{k}\right)^4 (\gamma^2 - 2 \gamma \sin^2 \theta + 2 \sin^4 \theta \sin^2 \psi)$$

when $\psi = \frac{\pi}{2}$ $$P_1(\theta) = A \left(\frac{k'}{k}\right)^4 (\gamma^2 - 2 \gamma \sin^2 \theta + 2 \sin^4 \theta)$$

and $\psi=0 \qquad P_{11}(\theta) = A\left(\frac{h'}{h}\right)^4 (\gamma^2-2\gamma\sin^2\theta)$

$\frac{P_1(0)}{P_{11}(0)} = \rho$ is called as the asymmetr ratio.

$$\rho=1+\frac{2\sin^4\theta}{\gamma^2-2\gamma\sin^2\theta}$$

Which is of course defined as the ratio of the number of coincidence counts per unit time when the central axes of the two detectors are perpenducular to each other and at angle θ to the central axes of the two scatterers, to the number of coincidence counts per unit time when the two decettors are parallel.

When this is verified experimentally it will imply that the two annihilation quanta are linearly polarized with the unit polarization vectors being perpendicular to each other. In an attempt to do the experiment; however, an ideal geometry can not be designed. Since the scatterers and the detectors will have finite dimensions, instead of being points in space, θ_1, θ_2 and ψ vary in finite ranges. Therefore the formula obtained for the asymmetry ratio should be generalized to the experimental designs. This can be done[3] by integrating over finite ranges in θ_1, θ_2 and ψ_1, ψ_2, since we have to set $\psi = \psi_2 - \psi_1$, with ψ_1 and ψ_2 the azimuths of an element of the first and the second counters respectively.

Let $d\Omega = \sin\theta \, d\theta \, d\psi$

$x = 2 - \cos\theta$

$\gamma = x + 1/x$

$$I=\int\gamma\left(\frac{k'}{h}\right)^2 \sin\theta\, d\theta=\ln x = \frac{1}{2x^2}\Bigg|_{\theta_{min}}^{\theta_{max}}$$

$$I'=\int\left(\frac{k'}{k}\right)^2 \sin^3\theta\, d\theta=-x=4\ln x+\frac{3}{x}\Bigg|_{\theta_{min}}^{\theta_{max}}$$

Then

$$\Delta P=\iint d\psi_1\, d\psi_2\iint P(\theta_1,\theta_2)\sin\theta_1\sin\theta_2\, d\theta_1\, d\theta_2$$

$$=A\iint[I^2-2II'+2I'^2\sin^2(\psi_2-\psi_1)]\, d\psi_1\, d\psi_2$$

The azimuth of the first counter ψ_1 and that of the second counter ψ_2 will have the limits $-\alpha \leq \psi_{1,2} \leq \alpha$ when the two counters are prallel, then

$$\Delta P_{11} = A \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} [I^2 - 2II' + 2I'^2 \sin^2 (\psi_2 - \psi_1)]\, d\psi_1\, d\psi_2$$

$$= A \left\{ (I^2 - 2II') \,.\, 4\alpha^2 + 2I'^2 \left(2\alpha^2 - \frac{1}{2} \sin^2 2\alpha \right) \right\}$$

When the two counters are perpendicular to each other. $(\psi_2 - \psi_1)$ becomes $\left(\psi_2 - \psi_1 + \frac{\pi}{2}\right)$ and hence $\sin^2 \left(\psi_2 - \psi_1 + \frac{\pi}{2}\right) = \cos^2 (\psi_2 - \psi_1)$ and $-\alpha \leq \psi_1 \leq \alpha$, $\left(\frac{\pi}{2} - \alpha\right) \leq \psi_2 \leq \left(\frac{\pi}{2} + \alpha\right)$ then

$$\Delta P_1 = A \int_{-\alpha}^{\alpha} \int_{\pi/2-\alpha}^{\pi/2+\alpha} [I^2 - 2II' + 2I'^2 \cos^2 (\psi_2 - \psi_1)\, d\psi_1\, d\psi_2$$

$$= A \left\{ (I^2 - 2II') \,.\, 4\alpha^2 + 2I'^2 \left(2\alpha^2 + \frac{1}{2} \sin^2 2\alpha \right) \right\}$$

Thus the asymmetry ratio becomes

$$\rho_2 = \frac{\Delta P_1}{\Delta P_{11}}$$

$$= (z + \rho_1)/(2 + z\, \rho_1)$$

where $z = \left(2\alpha^2 - \frac{1}{2} \sin^2 2\alpha \right) \Big/ \left(2\alpha^2 + \frac{1}{2} \sin^2 2\alpha \right)$ and

$$\rho_1 = 1 + \frac{1}{\frac{1}{2} \left(\frac{I}{I'} \right)^2 - \frac{I}{I'}}$$

For a given symmetrical geometry the values of ρ_2 are tabulated in table 1. below.

d (cm)	a (cm)	b (cm)	c (cm)	α	θ min	θ max	ρ_2
8	2.5	1.5	3	32°00′	58°00′	122°00′	1.51
10	2.5	1.5	3	26°40′	63°20′	106°40′	1.64
12	2.5	1.5	3	22°40′	67°20′	112°40′	1.82
15	2.5	1.5	3	18°30′	71°30′	108°30′	2.09

Table 1. Values of the asymmetry ratio for different distances from the midpoint of the scatterers to the crystal of the detector. (a is the radius of the crystal, b is the radius of the scatterer and c is the length of the scatterrer.)

THE APPARATUS

Teh apparatus used during the experiment consists of :

1. The lead shielding and the scatterers. The Cu^{64} positron source is packed in an aliminium sheet and it is put in the center of the lead shield. The two aliminium scatterers, which have cylindrical shapes, are placed at each end of the pipe drilled through the lead shield.

2. The two NaI (TI) crystals with photumultipliers are attached to the lead shield so that their central axis are perpendicular to the central axis of the two aliminium scatterers, and one of these detectors can be moved around one of the scatterers through an angle of 360° while the other one is stationary. ($1\frac{3''}{4}$ Diameter x $1\frac{1''}{2}$ thick thallium activated sodium iodide crytal and RCA 6342 A 2″ Photo tube multiplier, The Harshaw Chemical Company 6801 Cochran RD. - Solon Ohio 44139.)

3. Conventional Coincidence System.

When the equipment is set up properly, Cu^{64} positron source is put in its place and the central axis of the two detectors are made parallel, while they are perpendicular to the central axis of the scatterers; and coincidence counts N_1 are made. Then the movable detector is rotated by an angle of 90° and coincidence counts N_2 are made. At the end the accidentals rates N_a are observed. The experiment is repeated for different distances d betwen the scatterers and the crystals of the detectors and the data is obtained to calculate the asymmetry ratios. The data and the results are tabulated in table 2.

The experimental asymmetry ratio is calculated from the formula

$$2 = \frac{N_2 - N_3}{N_1 - N_3}$$

let s be the statistical error in ρ_2

m_1 be the statistical error in N_1

m_2 be the statistical error in N_2

m_3 be the statistical error in N_3

then $s^2 = \left(\frac{\partial f_2}{\partial N_1}\right)^2 m_1^2 + \left(\frac{\partial \rho_2}{\partial N_2}\right)^2 m_2^2 + \left(\frac{\partial f_2}{\partial N_3}\right)^2 m_3^2$

$$= \left(\frac{1}{N_1 - N_3}\right)^2 (m_2^2 + \rho_2^2\; m_1^2 + (\rho_2 - 1)^2\; m_3^3).$$

And he avarage asymmetry ratio ρ_2 is

$$\bar{\rho}_2 = \frac{\sum_i \frac{(\rho_2)\, i}{s_i^2}}{\sum_i \frac{1}{s_i^2}}$$

(the subscript i refers to the experiment number.) the erro in ρ_2 is given by

$$\frac{1}{s^2} = \sum_i \frac{1}{s_i^2}\,.$$

IMPLICATIONS OF THE EXPERIMENT

This experiment is relevant to the concept of measurement in quantum mechanics. Measurement is defined by Messiah as a filtering process acting on the wave function :

d (cm)	N_1 (counts/sec)	N_2 (counts/sec)	N_3 (counts/sec)
8	.135	.189	.065
8	.391	.521	.180
10	.660	.710	.500
10	.477	.646	.305
10	.524	.579	.305
10	.498	.574	.265
10	.398	.520	.200
10	.398	.490	.165
12	.417	.523	.283
12	.432	.497	.290
15	.153	.220	.110
15	.107	.137	.075

Tablo 2. The data a

ρ_2 experimental	ρ	percentage error in ρ_2	ρ_2 thocrotical
1.90 + .43	1.74 + .27	9.78	1.51
1.63 + .34			
1.36 + .81			
1.98 + .30			
1.27 + .17			
1.35 + .16	1.43 + .08	5.59	1.64
1.62 + .21			
1.39 + .16			
1.80 + .62	1.59 + .25	15.7	1.82
1.46 + .27			
2.56 + .90	2.22 + .65	29.27	2.09
1.94 + .77			

and the results.

«Consider[13] therefore an ideal measurement of the quantity A, and suppose at the start that the value found a_i is a non - degenerate eigen value. According to our hypothesis, we know with certainty that $A = a_i$ once the measurement is completed, hence that the wave function of the system is the eigenfunction Ψ_i (to within a constant) corresponding to the eigenvalue a_i. The arbitrary constant has no physical significance since the statistical distrubution of the results of any subsequent measurement is independent of the choice of that constant. The wave function of the system after measurement is thus known without ambiguity. The measuring device works in some sense like a 'perfect filter'. The wave function before measurement is a function $\Psi = \Sigma_n c_n \Psi_n$. There is a probability $|c_i|^2$ that the result of measurement is a_i. Assuming that the measurement gives a_i, the net effect of the measuring process is to 'pass' (without distortion) only the term $c_i \Psi_i$ of the expression Ψ in a series of eigenfunctions of A.»

«.... When the measurement is not ideal, the 'pasing' of these terms is accompanied by some distortion. That distortion is in principle exactly calculable and depends upon the measuring device used.

In our case we start with the initial wave function

$$\psi = \psi_{RR} - \psi_{LL} = -i \left(a^*_{+1} \; a^*_{-2} + a^*_{+2} \; a^*_{-2} \right) \psi_{00}$$

and after the measurement it is filtered to

$$\psi' = -i \left(c_1 \; a^*_{+1} \; a^*_{-2} + c_2 \; a^*_{+2} \; a^*_{-1} \right) \psi_{00}$$

Thus the measurment on the first photon influences the state of the second photon. The following quotation form Dicke and Wittke is related to this fact.

«As[14] the polarization measurmenet on the first photon is made long after the photons were created, it is very difficult to see how this measurement can be thought of as affecting the polarization of the other photon. The other obvious alternative is, however, equally disturbing. It is celar that a situation is encountered here which is inexplicable in terms of a classical model. With any classical model, a desoription of the system is complete when the polarizations of the photons are each separately described. It is found instead that the photons are correlated in their behavior. The two photons constitute a single dynamical system. Any information obtained about the system is information about both photons. Any interaction on a single photon is an interaction on the system and effects the state of the whole system· The above paradox is very similar to one first discussed by Einstein, Podolsky, and Rosen[15], but the paradoxial behaviour is made to take a particularly acute form in the above example».

In our case experiment shows that even when the system breaks into two noninteracting parts· (as it is pointed out in the quatation above) these parts cannot be treated in isolation. If we do not assume that the wave function describes the behaviour of a single event, but is only applicable to ensembles then each photon has a definite polarization before any measurements, and we will again expect to see the same results. The possibility eliminated by this experiment is that the initial correlation between the polarizations disappears when they do not interact with each other, or with the measuring apparatus.

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An Approach To Telephone Traffic

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Telephone calls made by a telophone system is called the telephone traffic. A traffic is measured in Erlang (previously traffic units). Also it can be measured in traffic units like C. M. (call minute), A. R. C. H., C. C. S. (cent call seconds). Measurement units vary form country to country (1).

A traffic is measured in erlangs which take into account the average duration of calls as well as their number, thus, if the average number of calls carried by a system in an hour is (C), and the average duration or holding time per call is (T) measured in hours then the telephone traffic is (TC) erlangs (2).

Calling rate is the average call price that is made by a subscriber in a certain period. Sometimes, calling rate is used aserlang' for each subscriber to show the traffic.

The holding time for timed trunk calls usally equals or exceeds three minutes, and average holding time for an exchange will therefore vary with the proportion of timed trunk traffic.

Consider an exchange of 2000 subscribers with an average calling rate of 0,8 in a particular hour, and an average holding time of 3 minutes. The total traffic originated by the subscribers is given by

$$\frac{2000 \times 0{,}8 \times 3}{60} = 80$$

erlangs in the hour considered.

A telephone administration must distribute the possibilities in a correct place, in a correct time and in correct amounts to offer a service of high quality to all subscribers.

Usage of equipment is independent not only from calling rate, but at the same time From the average holding time for each call. Average

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1) Bedri Karafakioğlu, Komütasyon Alet Sayısı Hesabı Notları, İstanbul, 1971, p.p. 2-3

2) T. J. Morgan, Telecommunication Economics, London, 1958, p. 126.

holding time varies from hour to hour, because the callee or the network is busy. In consequence of this kind of situation, causes the network is used less, respectively.

PTT accepted the average holding time as 60 - 200 seconds. Naturally, this is a very symbolic period. In practice, the register records the call only when the call just made. After that, there is no addition charge in continuing local calls, and the subscriber charged only for one call. In this condition, equipment is unnecessarily occupied and the exchange remained continuously loaded. Morover, the income of PTT decrease constantly because of this situation (3). This situation is shown below, figure 1.1.

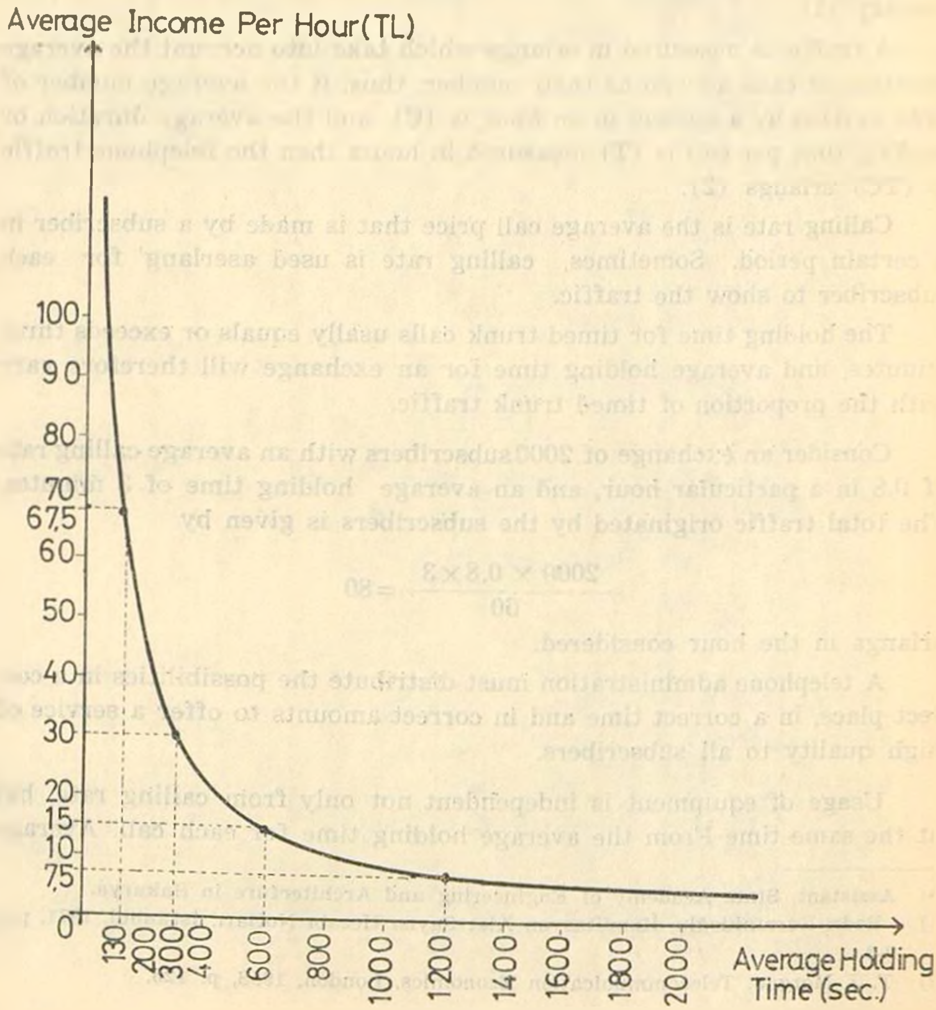

FIGURE 1.1

One of the important property of the telephone traffic is the costant change in its volume. Traffic is the result of the independent calls of subscribers. The traffic varies according to the properties of districts. Also it varies form day to day and time to time. Usually, when the distribution which is made by the central exchange to a business area and a residential area is compared exact differences will be conspicious. This situation is shown in the figure 1.2, below.

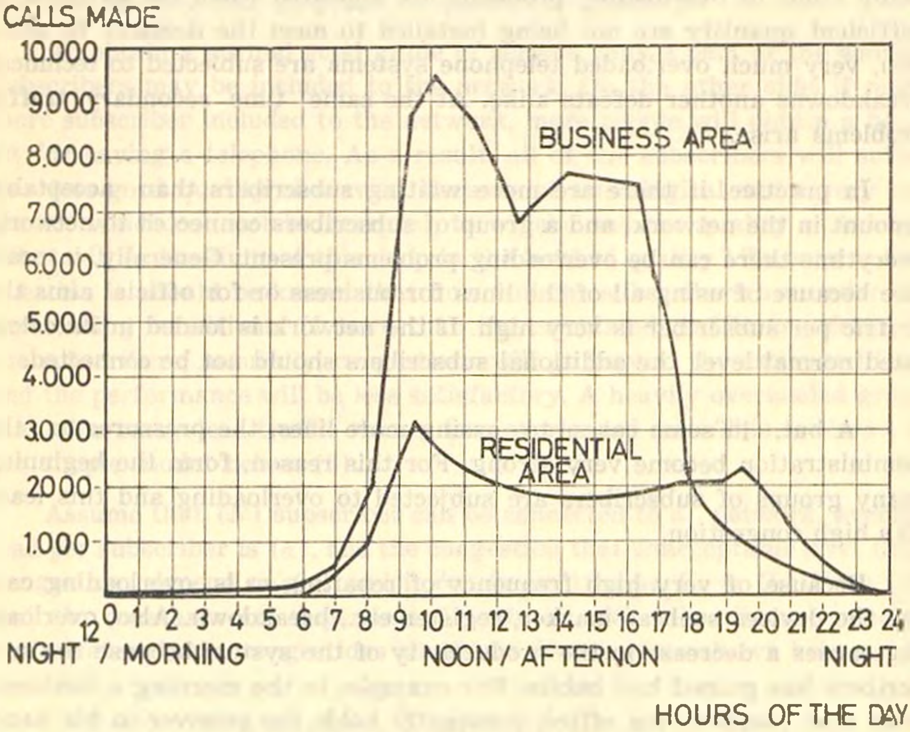

FIGURE 1.2

3) The average holding time accepted by PTT is 60 - 200 sec.

$$m_i = \frac{l_1 + l_2}{2}, \quad m_i = \frac{60 + 200}{2} = 130 \text{ sec.}$$

1. Average H. T. 130 sec.

Average call per hour : $\frac{3600}{130}$ = 27, Average income per hour : 27x 2.5 = 67,5 TL.

2. Average H.T. 300 sec.

Average call per hour : $\frac{3600}{300}$ = 12, Average income per hour : 12 x 2.5 = 30 TL.

3. Average H.T. 600 sec.

Average call per hour : $\frac{3600}{600}$ = 6, Average income per hour : 6 x 2.5 = 15 TL.

4. Average H.T. 1200 sec.

Average call per hour : $\frac{3600}{1200}$ = 3, Average income per hour : 3 x 2.5 = 7.5 TL.

In the dial system, the subscribers tend to carry on their habits that they were used to in manuel system. The results of the researches about the behavior of the subscribers who hear the busy signal show that, the 90 % of subscribers give a call to the same number after awhile (4).

One of the important projects of the telephone traffic is to fix the loading capacity. Therefore, there must be reliable demand estimations. Many kinds of overloading problems are appeared when the switches in sufficient quantity are not being installed to meet the demand. In addition, very much overloaded telephone systems are subjected to technical breakdowns another defeats alike. At the same time secondary traffic problems arise.

In practice, if there are more waiting subscribers than acceptable amount in the network, and a group of subscribers connected to network, everytime there can be overloading problems present. Generally, in practice because of using all of the lines for business or for official aims the traffic per subscriber is very high. If the network is loaded in its calculated normal level, the additional subscribers should not be connected.

A but, in some cases to assing more lines, the pressures on the administration become very strong. For this reason, form the beginning many groups of subscribers are subjected to overloading and this leads to a high congestion.

Because of very high frequency of repaiting calls, overloading causes the devices such as marker, register etc., breakdown. Also, overloading causes a decrease in the productivity of the system because the subscribers has gained bad habits. For example, in the morning a businessman that comes to his office, constantly holds the receiver in his hand. Because he knows that he will wait awhile to hear the signal.

Another kind of overloading problem arises when the service is improved suddenly. An unstatisfied accumulated demand is present. When the difficulty is over, the traffic increases in huge amounts. This is a known condition. Long distance and international routes can incease their traffic expressed in erlang, 10 times more when more trunk provided and service is improved. This is called «service improvement jump». It is easy

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4 Theory of Traffic Engineering, Published by Northern Electric Company Limited undated, p. 7.

to guess this situation. An improvement may not remove overloading but lead to a new overloading situation (5).

Difficulty comes from the estimation of traffic demand correctly from the cases of heavy overloading. When the supply does not meet the demand, this question arises : Are the obtained sources used to obtain a normal good grade of service to a few subscriber or to give a less good quality of service to a lot of subscribers? Briefly, the question is, whether the priority will be given to quality or quantity.

To obtain a normal good grade of service, only a part of the waiting subscribers may be included to the network. On the other side· if much more subscriber included to the network, more people will obtain a benefit for having a telephone. As a result, all of the subscribers will suffer to a less good quality of service. Because of a nonlogical and a very bad service which can not be accepted, to which level the quality will drop? What will be the limit of this drop in the level of quality? To onswer this question it should be known that how will the subscribers act under the overloading conditions. If more than the calculated traffic is offered to the subscribers, naturally, congestions will go beyond the estimated level and the performance will be less satisfactory. A heavily overloaded group may also carry less traffic than a less overloaded one. This is shown in the simple model below (6).

Assume that (N) subscriber can be connected to a network, average load per subscriber is (α), and the congestion that unacceptable level (B_0). Congestions depend on (x) and it con be written as $B_0 = C_n{}^k$.Here (C) and (k) constants are dependent. If only $x(<N)$ subscribers connected to the network· the congestions should be decreased as shown below :

$$B_x = C\left(\frac{X}{N}\alpha\right)^k = C\left(\frac{X}{N}\right)^k (\alpha)^k$$

$$B_x = B_0\left(\frac{X}{N}\right)^k \tag{1}$$

Then traffic carried by the network should be

$$A_t = x\alpha\ (1 - B_x) \tag{2}$$

5) A. Elldin , «Traffic Engineering in Developing Countries», Telecommunication Journal, vol. 44, September 1977, pp. 427 - 29.

6) Ibid., p. 430.

Also this can be written as below

$$A_c = \alpha \frac{N}{N} \chi \left[1 - B_o \left(\frac{\chi}{N} \right)^k \right]$$

$$A_c = \alpha N \frac{\chi}{N} \left[1 - B_o \left(\frac{\chi}{N} \right)^k \right] \tag{3}$$

And (A_c) depends on $\left(\frac{x}{N} \right)$

And (A_c) has a maximum value for

$$\left(\frac{\chi}{N} \right)^k = \frac{1}{(k+1)\, B_o} \tag{4}$$

Here :

$$B_x = B_o \left(\frac{\chi}{N} \right)^k, \qquad B_x = B_o \frac{1}{(k+1)\; B_o} = \frac{1}{k+1}$$

$$\Rightarrow (A_c)_{max} = \alpha N \left(\frac{1}{(k+1)\, B_o} \right)^{1/k} \left(1 - \frac{1}{k+1} \right),$$

$$(A_c)_{max} = \alpha N \; \frac{k}{k+1} \left(\frac{1}{(k+1)\, B_o} \right)^{1/k} \tag{5}$$

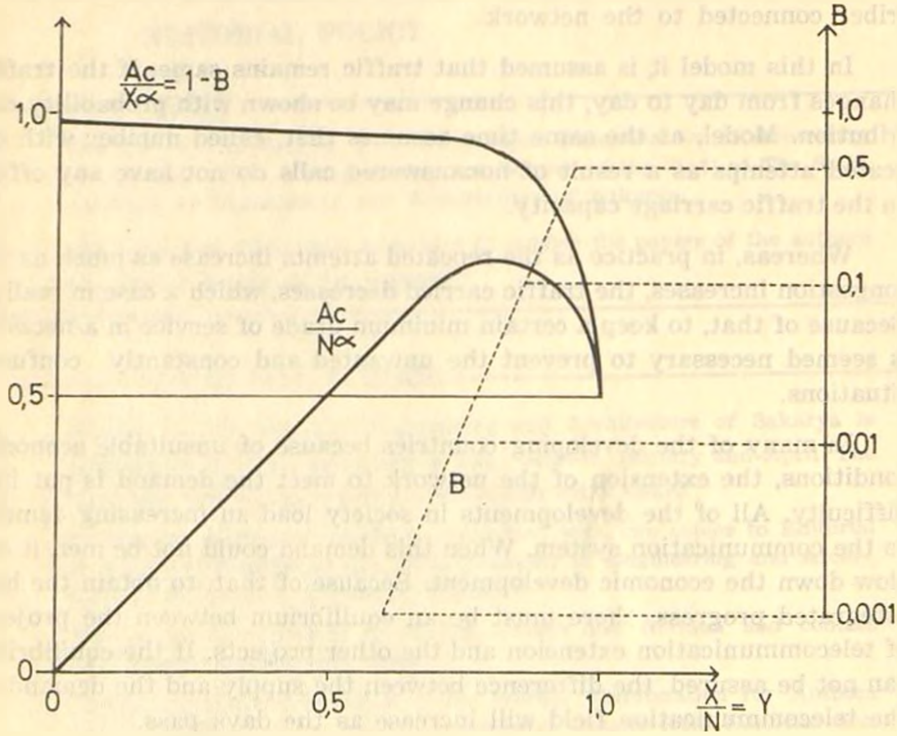
At this maximum value we have

$$B_x = \frac{1}{k+1} \tag{6}$$

As a result ; for $x < N$, if

$$B_o > \frac{1}{k+1} \tag{7}$$

maximum traffic carried obtained.



$(B_0 = 0.5,\ k = 10)$

FIGURE 1.3

Telephone administrations must make a choice between the alternatives below :

1) Connect the subscribers at B_1, which is a really good grade of service. B_1 is of the order 10^{-3} to 10^{-2}.

2) Connect most of the subscribers at B_2, which is a somewhat higher congestion. B_2 may be of the order 10^{-2} to 10^{-1}.

3) As in (4) and (5), connect the subscribers to the network when the traffic carried has reached to its maximum value. The congestions here, which are given by (6), (k) can be accepted if they are large (say, $k > 10$ to 20) as possible.

4) Connection of the maximum number of subscribers ($x = N$) to the network. In this case, the congestion (B_0) will be high, and the traf-

fic carried, and revenue may be less than the congestion when less subscriber connected to the network.

In this model it is assumed that traffic remains same. If the traffic changes from day to day, this change may be shown with probability distribution. Model, at the same time assumes that, called number with repeated attemps as a result of nonanswered calls do not have any effect on the traffic carriage capacity.

Whereas, in practice as the repeated attemts increase as much as the congestion increases, the traffic carried decreases, which a case in reality. Because of that, to keep a certain minimum grade of service in a network is seemed necessary to prevent the unwanted and constantly confused situations.

In many of the developing countries because of unsuitable economic conditions, the extension of the network to meet the demand is put into difficulty. All of the developments in society load an increasing demand to the communication system. When this demand could not be met, it can slow down the economic development. Because of that, to obtain the best integrated progress, here must be an equilibrium between the projects of telecommunication extension and the other projects. If the equilibrium can not be assured, the difference between the supply and the demand on the telecommunication field will increase as the days pass.

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