Nükleer Deformasyonda Eşleşme Kuvvetinin Rolü

The Pairing Force In Nuclear Deformation

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İkileşme kuvveti küresel simetriyi korumaya çalışır, fakat valans nucleonlar ilave oldukça, çekirdek deforme olmaya başlar ve kollektif görünümlü rotasyonel spektraya götürülen quadropol kuvveti etki eder. Dolayısı ile küresel çekirdeklerde quadropol kuvveti ve deforme çekirdeklerde ikileşme kuvveti perturbasyon olarak kabul edilir.

The pairing force tries to hold the spherical symmetry in a nucleus, but as valence nucleouns are added the nucleus begins to deform and the quadropole forces act, leading to rotational spectra in collective features. Therefore when spherical nuclei are considered the quadropole force is the perturbation and when the deformed nuclei are considered the pairing force is assumed to be the perturbation.

The short range force between two nucleons in the same energy state, effecting primaryly the particles in unfilled shells in nuclei, is named as pairing force.

In a $(j^2)_J$ configuration the attraction is less for high J values, and they are depressed to have zero energy for all $J \neq 0$. This suggests the pairing force of the form :

$V = -\frac{1}{4} A^+ A$, where A^+ creates and A destroys a pair of particles in $J = 0$ state, and G is the strength of the pairing force.

If $|0>$ represents the closed shell then $(A^+)^{N/2}|0>$ is an N particle state. Infact this is the eigenfunction of V.

$$i.e.\ V(A^+)^{N/2}|o> = \left\{\frac{1}{4} GN^2 - \frac{1}{2}\left(j + \frac{3}{2}\right) GN \right\} (A^+)^{N/2}|o> ;$$

in terms of seniority (the number of unpaired particles, v) this becomes:

$$V \mid NvJ> = -\frac{1}{4} G (N-v)(2j+3-N-v) \mid NvJ>$$

which shows that the energy is independent of J. The same result can be obtained by using Quasi - spin description using the analogy of the quasi - spin operator to that of the angular momentum operator. The states v can be excited to higher with $\Delta v \leqslant 2$.

If more than one level has to be filled $(j_1, j_2 \ldots)^N$ the Hamiltonean becomes :

$$H = \frac{1}{4} G A^+ A + \Sigma \in_\nu a^+ a_\nu$$

where a^+_ν ereates and a_ν destroys a particle in state ν and each level has energy $\in_\nu$. There is an approximate solution to this by using quasi - particles. The method involves writing H interms of quasi - particle operators :

$$\beta_\nu = U_\nu a_\nu + P_\nu v_\nu a^+_{-\nu} \text{ and } \beta_\nu^+,$$

where P has the form $(-)^{j-m}$. and $u_\nu^2 + v_\nu^2 = 1$ Choosing u/v and neglecting higher order terms in β_ν and β_ν^+.

$$H = H_{00} + {}_{11} \text{ where}$$

$$H_{00} = \sum \in_\nu v_\nu - \Delta^2/G - \frac{1}{2} G \sum v_\nu^4$$

$$H_{11} = \sum \{ (\in_\nu - G v_\nu^2)(u_\nu^2 - v_\nu^2) + 2\Delta u_\nu v_\nu \} \beta^+ \beta_\nu$$

$$\Delta = \frac{G}{2} \sum u_\nu v_\nu$$

And the total energy of the system :

$$E = H_{00} + \Sigma E_\nu \qquad \text{(sum over all occupied orbits)}$$

The lowest energy corresponding to the quassi - particle vacuum

$$|0> \text{ is } E = H_{00}$$

By neglecting the higher order terms in the Hamiltonean, the effect of some particles have been lost. To compansate for this, add a

$-\lambda N$ to the Hamiltonean. The condition for the second order terms $H_{20}+H_{02}=0$ gives λ and Δ interms of the strenght of the pairing force, number of particles to be fed in and the level energies. However if G is small there is no solutionto λ and Δ.

For even even nucleus the quasiparticle energies are given by :

$$E_{\nu}=\sqrt{\Delta^2+\in(j-\lambda)^2}\,,\quad \Delta \text{ large}$$

The gap between the 0 quasi - particle level $|\widetilde{0}>$ and 2 quasi particle level $\beta_{\nu}^{+}\beta_{\nu}^{+}|\widetilde{0}>$ is of the order of 2Δ and it is named as pairing gap.

Incase of an odd nucleus :

$$E_{\nu}'-E_{\nu}\cong(\in_{\nu}-\in_{\nu'})\left(\frac{\in_{\nu}+\in_{\nu'}}{2}-\lambda\right)$$

where E_{ν}' and E_{ν} are the highest and the lowest energies in each quasi particle level. The pairing gap is about 2Δ between 1 quasi - particle and 3 quasi - particle levels.

In even even nuclei experimentally it is found that the 2^{+} level is pushed down. This is due to the splitting of the 2 quasiparticle level. But even if H_{22} (one of the second order terms in the Hamiltonean) is applied this effect can not be eliminated. This implies that there is an effect of a long range Quadropole force leading to rotational spectra in collective features. Quadropole force is assumed as perturbation when spherical nuclei is corsidered, whereus pairing force is the perturbation when deformed nuclei is considered.

When valence particles are added deformations would set, if there were not the pairing effect, but pairing holds the spherical symmetry as long as possible. However as the number of valance particles increase the nucleus tends to deform. Even after the deformation the pairing will introduce configuration mixing, in which pairs of particles are scattered among the last filled levels. The pairing tries to hang on to any symmetry possible, and even if spherical symmetry must be given up it seems to be able to keep up axial symmetry in the deformed system.

Pairing is the main factor near closed shells. In all regions the ground states of even nuclei are 0^{+}. In the region of pairing an even number of nucleons generally pair off to angular momentum 0 in an odd nucleus, leaving the net spin determined by the odd particle; although there are exceptions where three or more nucleons couple together in a less trivial way to form the ground state spin.

«It is not clear, however, that one can neglect interactions such as those of the pairing type between neutrons and protons in partially filled shells, nor that effects from four body type interactions don't built up.»

The effect of pairing on the moment of inertia can be calculated and it is shown that it dcreases the moment of inertia considerably.

References:

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