Açısal Korrelasyonlarda Partikül Parametreleri

The Importance Of Particle Parameters In Angular Correlations

İhsan ULUER **Abdulkadir AKSOY**

Sakarya D.M.M. Akademisi

Partikül parametreleri açısal korrelasyonlarda yapılan hesaplamalarda önemli bir rol oynamaktadır. Son zamanlarda bu parametrelerin hesabında yeni metotlar geliştirilmiş olup yapılan ölçümlerin daha sıhhatli olması sağlanmıştır. Bu yayında partikül parametrelerin özellikleri ve çıkarılışında kullanılan yeni metotlar belirtilmektedir.

The particle parameters play an important role in the calculations of angular correlations. New methods have been designed recently to calculate the particle parameters so that the measurements will be more aligible. In the present paper the particle parameters and the new methods to calculate them have been shown.

INTRODUCTION

The Radiation Parameters are defined by the statistical tensors which depend on eigen - functions of the total angular momentum of the radiation and Z component with eigen - values K and ν,

$$C_{k\nu}(LL') = \sum_{\sigma\sigma'} \sum_{\mu\mu'} (-1)^{L-\mu} (2k+1)^{1/2} <0\sigma \mid L\mu\pi> <0\sigma' \mid L'\mu'\pi'>^{*}$$

$$\times <\sigma' \mid \in \mid \sigma> \begin{pmatrix} L & L' & K \\ \mu & -\eta' & \nu \end{pmatrix}$$

If the detectors are insensitive to polarization then :

$$<\sigma' \mid \in \mid \sigma> \approx \delta\sigma\sigma' \text{ and}$$

$$C_{k\nu}(LL') = \sum_{\sigma} \sum_{\mu\mu'} (-1)^{L-\mu} (2k+1)^{1/2} <0\sigma \mid L\mu\pi> <0\sigma \mid L'\mu'\pi'>^* \begin{pmatrix} L & L' & k \\ \eta & -\mu' & \nu \end{pmatrix}$$

For a given σ, μ can have only one of the values of either -1 or $+1$ corresponding to the left or right circular polarization i.e. $\mu=\mu'$ which also requires that $\nu=0$ because otherwise the $3\,j$ symbol will vanish. Thus :

$$C_{k0}(LL') = \sum_{\sigma} \sum_{\mu} (-1)^{L-\mu} (2k+1)^{1/2} <0\sigma \mid L\mu\pi> <0\sigma \mid L'\mu\pi'>^* \begin{pmatrix} L & L' & k \\ \mu & -\mu & 0 \end{pmatrix} \qquad \ldots\ldots\ldots\ldots (1)$$

In the semi - classical theory of gamma radiation the vector potential A, is treated as a wave function for particles of spin $s=1$ which represents the photons. The components of the vector potential related to the projections of s on the Z axis, $\sigma=1, 0, -1$. For a given L the orbital quantum number may be $l=L\mp1$ for electric multipoles and $l=L$ for magnetic multipoles. The amplitudes $<0\sigma \mid L\mu\pi>$ are zero for $\sigma=0$ because of the transverse characteristics of the electromagnetic waves; therefore for the electric multipoles the amplitudes for $l=L\mp1$ are combined to give zero for $\sigma=0$ component. Keeping this in wiev, Devons and Goldfaby give the amplitudes for a plane wave along Z - axis :

$$<0\sigma \mid L\mu\pi>_{\text{magnetic}} = \sigma\delta_{\sigma\mu} (2L+1)^{1/2}/\sqrt{8\pi}$$

$$<0\sigma \mid L\eta\pi>_{\text{electric}} = \delta_{\sigma\mu} (2L+1)^{1/2}/\sqrt{8\pi}$$

The parities for the electric and magnetic radiations are:

$$\pi \text{ (magnetic)} = (-1)^{L+1}$$

$$\pi \text{ (electric)} = (-1)^{L}$$

Let p be zero for electric and one for magnetic radiation. Then for both types of radiation :

$$<0\sigma \mid L\mu\pi> = (\sigma)^{p} \delta_{\sigma\mu} (2L+1)^{1/2}/\sqrt{8\pi}$$

Substituting this in equation (1) :

$$C_{k0}=\sum_{\sigma}\sum_{\mu}(-1)^{L-\mu}(2k+1)^{1/2}(\sigma)^{P}\delta_{\sigma\mu}\left[(2L+1)^{1/2}/\sqrt{8\pi}\right](\sigma)^{P'}\delta_{\sigma\mu}\ldots$$
$$\times\left[(2L'+1)^{1/2}/\sqrt{8\pi}\right]\begin{pmatrix}L & L' & K\\ \mu & \mu' & 0\end{pmatrix}$$

Thus if there is no polarization or parity mixture

$$C_{k0}=\sum(-1)^{L-\mu}(2k+1)^{1/2}(\mu)^{P}(\eta)^{P'}(2L+1)^{1/2}(2L'+1)^{1/2}\frac{1}{8\pi}\begin{pmatrix}L & L' & K\\ \mu & -\mu & 0\end{pmatrix}$$

Clearly These Parameters are characteristic of the radiation emitted and are independent of the properties of the nuclear states involved in the transition.

$$C_{k0}=\frac{1}{8\pi}(-1)^{L-1}(1k+1)^{1/2}(2L+1)^{1/2}(2L'+1)^{1/2}\begin{pmatrix}L & L & K\\ 1 & -1 & 0\end{pmatrix}$$

APPLICATION TO THE ANGULAR CORRELATION FUNCTION

In making a transition from one excited state to another state, the nucleus can transfer, energy, angular momentum and parity to one of the shell electrons in the bound state. This internal conversion process occurs through the interaction of the nuclear currents and charges with the electron via the electromagnetic field. Thus if instead of a γ-ray of multipolarity L, an electron is converted. From an itinial state of total and orbital angular momenta $\zeta_i L_i$ into a state of $\zeta_f L_f$, then

$\zeta_i \zeta_f L$ form a triangle
$L_i L_f L$ form a triangle

Since ζ_f contributes in the description of the final state of the electron, it influences the particle parameters. And the value of ζ_f is limited by these selection rules. On the other hand the parity of the electron wave function depends on the total angular momentum, and when the parity of the multipole is assigned, its parity is also set.

The parity selection rules are :

L_i+L_f+L even integer, electric radiation.

L_i+L_f+L odd integer, magnetic radiation.

Thus if the result of the experiment proves that the radiation (internal conversion) is electric it implies no parity change, and vice versa for a magnetic one. The γ - γ direction correlation experiments do not give such relative parity information because, the electric or magnetic character of a γ - ray can be changed by transforming the electric and magnetic field, but this does not effect the direction of the γ - ray.

For the γ - γ directional Correlations the correlation function is

$$W(\theta) = 1 + A_2 P_2 (\cos\theta) + A_4 P_4 (\cos\theta)$$

And for the electron - γ correlations :

$$W(\theta) = 1 + b_2 A_2 P_2 (\cos\theta) + b_4 A_4 P_4 (\cos\theta)$$

Where b_ν $(LL', X$ any particle$) = C_{\nu r}(LL', X)$ $C_{\nu r}(LL', \gamma)$.

The difference between the two are the particle parameters. There fore in order to get information from e - γ correlation experiments accurate calculation of the particle parameters is necessary. However a rigorous treatment of the electromagnetic interaction between a nucleus and its surrounding electrons requires the formalism of quantum electrodynamics. This has been dore by many people. A very recent one is by R.M. Steffen (1969).

For the K shell Biedenharn and Rose calculated that

$$b\nu = 1 + \frac{\nu(\nu+1)}{2L(L+1) - \nu(\nu+1)} \, \frac{L}{2L+1} \, \frac{|L + | + T_e|^2}{L(L+1) + 1 T_e|^2} \text{ electric.}$$

$$b\nu = 1 + \frac{\nu(\nu+1)}{2L(L+1) - \nu(\nu+1)} \, \frac{L(L+1)}{2L+1} \, \frac{|1 - T_m|^2}{L + 1 + L|T_m|^2} \text{ magnetic.}$$

$$\text{where } T_m = \frac{e^{i\delta_1} R_2(m)}{e^{i\delta-1} R_{-1}(m)} \text{ and } T_e = 2 \frac{e^{i\delta_2}}{e^{i\delta-3}} \frac{R_2(e)}{R_{-3}(e)}$$

in which δ_k are appropriate Coulomb Phases and R_k (m/e) the electron radial matrix elements.

Since they are dominant $M1$ and $M2$ and $E1$ transitions are of interest :

$$b_2\,(M) = 1-2\,\frac{|1-T_m|^2}{6+|T_m|^2}$$

$$b_2\,(E2)\,1 + \frac{2}{5}\,\frac{|3+T_e|^2}{6+|T_e|^2} \quad ; \quad b_2\,(\mathrm{E1}) = 1 - \frac{|2+T|_e}{2+|T_e|^2}$$

These Particle Parameters depend on the ratio of the radial matrix elements i.e. on their relative amplitudes. Hence the directional correlation is more sentitive to minor variations in the radial matrix elements. Such effects may be hidden in experimental errors.

Once the particle parameter b_2 is calculated then, for both electric and magnetic radiation:

$$b_\nu = 1 + \frac{\nu\,(\nu+1)\,[L\,(L+1)-3]}{3\,[2L\,(L+1) - \nu\,(\nu+1]} \times [b_2-1]\ ;\ b_0 = 1$$

THE RECENT WORK ON PARTICLE PARAMETERS :

Biedenharn and Rose tabulated the directional particle parameters for the K - shell using the point nucleus approximation (There is a sign error in these tables.) Later Band et al. and Listengarten et al. computed tables of particle parameters for K, L_{I} and L_{II} shell using electron wave functions appropriate for finite nuclei and assuming that the nuclear currents interact with the electrons only at the nuclear surface (Model of Sliv.) Directional particle parameters for the K, L and M shell have been tabulated for $Z=60$ to 96 in steps of $Z=4$ by Pauli, who calculated the radial integrals on the basis of electron wave functions obtained with a Thomas - Fermi - Dirac potential with the inclusion of finite nuclear size effects. The most complete tables of directional particle parameters have been published by Hager and seltzer. These tables give the particle parameters for the K. L. and M shells for every Z from 30 to 103 for four lowest electric and magnetic multipoles. These computations were based on a relativistic self constitent - field (Hartree - Fock - Slater) calculation to obtain the electron wave functions and the Fermi nuclear charge distribution was used to take the finite nuclear size into account. A computer program is written by Pauli to calculate the normalized directional particle parameters with the inclusion of penetration effects. These parameters are given by:

$$\widetilde{b}_{\nu}(EL)=\frac{b_{\nu}(EL)(1+C_1\lambda_1+C_2\lambda_1^2+C_3\lambda_2+C_4\lambda_4^2+C_5\lambda_1\lambda_2)}{1+A_1\lambda_1+A_2\lambda_1^2+A_3\lambda_2+A_4\lambda_2^2+A_5\lambda_1\lambda_2}$$

$$b_{\nu}(ML)=\frac{(1+D_1\lambda+D_2\lambda^2)}{1+B_1\lambda+B_2\lambda^2}$$

$$\widetilde{b}_{\nu}(EL,\ ML+1)=\frac{b\nu(EL,ML+1)(1+E_1\lambda_1+E_2\lambda_2)}{(1+A_1\lambda_1+A_2\lambda_1^2+A_3\lambda_2+A_4\lambda_2^2+A_5\lambda_1\lambda_2)^{1/2}}$$

$$\widetilde{b}_{\nu}(ML,\ EL+1)=\frac{b\nu(ML,\ EL+1)(1+F_1\lambda)}{(1+B_1\lambda+B_2\lambda^2)^{1/2}}$$

where only the effects of the lowest multipole are included in the interference particle parameters. C_i, D_i, E_i, and F_i are the penetration coefficients, which have been computed by Hager and Seltzer. The penetration coefficients A_i and B_i, The nuclear structure parameters λ, λ_1, λ_2 ... are the parameters used in calculating the conversion coefficients assuming there are penetration effects :

i. e. $\alpha_x(ML)=\alpha_x(ML)(1+B_1(Y)\lambda+B_2(x)\lambda^2)$ (magnetic)

and $\alpha_x(EL)=\alpha_x(EL)(1+A_1\lambda_1+A_2\lambda_1^2+A_3\lambda_2+A_4\lambda_2^2+A_5\lambda_1\lambda_2)$ (electric)

(X denotes the shell, K, L_I, ...).

The authors are indepted to Prof. R.M. Steffen for providing his recent work on the subject.

References:

Biedenharn and M.E. Rose, Rev. Mod. Phys 25 (1953) 755.

H.C. Pauli, Helv. Phys. Acta 40, 713 (1967)

Rose, M.E., 1953, Phys Rev. 91, 610.

H.C. Pauli, A computer Program for internal conversion Coefficients and Particle parameters. (1969)

H.C. Pauli, K. Alder, and R.M. Steffen, «The theory of Internal Conversion» in «The Electromagnetic Interaction in Nuclear Spectroscopy» North - Holland, 1975 ed. W.D. Hamilton, 341.

R.M. Steffen and K. Alder, «Angular Distribution and Correlation of Gamma Rays (1) Theoretical Basis», in «The Electromagnetic in Nuclear Spectroscopy» North - Holland, 1975 ed. W. D. Hamilton, 505.

R.M. Steffen and K. Alder, «Extranuclear Perturbations of Angular Distributions and Correlations» in «The Electromagnetic Interaction in Nuclear Spectroscopy» North - Holland, 1975 ed. W.D. Hamilton, 583.