Sabit kesitli bir boruda türbülanslı difüzyon

Turbulent diffusion in a pipe of constant cross section

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Bu makalede, sabit dik kesitli bir boı unun dik kesitindeki akım ile eksenel akımın boyuna dispersiyona etkileri incelenmektedir. Genel bir tetkik, boyuna difüzyon ile boyuna konveksiyonun enine difüzyonla kombine etkisinin toplanabilir olduğunu vermektedir. Difüzyon katsayıları ve hızın bileşenleri, sadece, dik kesit değişkenlerine bağlı ise, asimptotik halde, maddenin kütle merkezinin akımın ortalama hızıyla hareket ettiği ve akımın ortalama hızıyla hareket eden bir noktaya göre tarif edilmiş variansın zamana göre lineer olarak değiştiği gösterilmiştir.

In this paper the effects of the cross sectional flow and the axial flow in a pipe of constant cross section on the longitudinal dispersion are considered. A general analysis gives that the longitudinal diffusion, and the combined effect of the longitudinal convection and the lateral diffusion are superposable. The asymptotic forms of the centre of mass and the variance of the soluble matter show that, when the diffusivities lepend only on the cross sectional variables and the components of the velocity do not change along the pipe of constant cross section, the centre of mass ultimately moves at the mean speed of the flow and the variance of the cloud about the point moving at the mean speed of the flow ultimately varies as linear in time.

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1. Introduction

When a solute is released in a solvent which is in a steady laminar flow through a circular pipe it spreads out longitudinally under the combined effect of lateral molecular diffusion and longitudinal convection, and longitudinal molecular diffusion. Experimentally and theoretically it has been shown |1| that the combined effect disperses the matter longitudinally about a plane which moves at the mean speed of the flow and the concentration averaged over the cross section of the pipe satisfies a diffusion equation asymptotically with a certain effective longitudinal diffusion coefficient. The most important feature of the theory of dispersion introduced by Taylor |1| is that it enables one to describe the average concentration in a three - dimensional system by the solution of the one - dimensional diffusion equation. This fact has been confirmed by many authors both experimentally and theoretically (see references given in |2|).

The analysis used for laminar flow has been extended to the cases of turbulent flow in a circular pipe |3| and turbulent flow in a wide channel with free surface |4|. A conclusion follows the fact that the combined action of turbulent lateral diffusion and convection by the mean flow, and longitudinal turbulent diffusion are ultimalety to make the matter spread out symmetrically about a point moving with the discharge velocity. A virtual diffusion coefficient may be defined if the statistical properties of the flow do not change within a cylindrical boundary [5].

The present paper describes the application of the analysis used for turbulent flow in a circular pipe and an open channel to the case of turbulent flow in a pipe of constant but arbitrary cross section. It is found that (i) the combined effect of the lateral turbulent diffusion and longitudinal convection, and the longitudinal turbulent diffusion are superposable; (ii) the centre of mass ultimately moves with the discharge velocity of the flow; (iii) the variance of the cloud changes asymptotically linear in time. Although the results are given in the case of turbulent flow, they can be readily applied to the case of laminar flow.

2. Concentration equation and boundary conditions

A mixture with two components is considered. The composition of the mixture is described by the concentration c, defined as the total

density of the fluid times the ratio of the mass of one component to the total mass of the fluid in a given volume element. c is measured as gr/ml and it is usually small.

A control surface enclosed a volume in the fluid is considered. The increase per unit time in the mass of fluid in this volume is balanced by the flux $c\mathbf{u}$ of the component as it moves with the fluid and the diffusion flux vector $\mathbf{j}$ which exists even when the fluid as a whole is at rest. This balance can be written, in the integral form, as

$$\frac{d}{dt}\int_V c\,dV = -\int_S c\,\mathbf{u}\cdot\mathbf{n}\,dS - \int_S \mathbf{j}\cdot\mathbf{n}\,dS \qquad (2.1)$$

where $\mathbf{n}$ unit outward normal to control surface, V is the control volume and S is the control surface. Using divergence theorem (2.1) can be written in differential form as

$$\frac{\partial c}{\partial t} + \nabla\cdot(c\,\mathbf{u}) = -\nabla\cdot\mathbf{j},$$

or, for incompressible fluid,

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = -\nabla\cdot\mathbf{j}. \qquad (2.2)$$

The left - hand side of (2.2) is the material derivative of c. If there is no diffusion, then the material derivative of c is zero. This mans that the composition of any given fluid element would remain unchanged as it moves about.

Experiment shows that the flux vector $\mathbf{j}$ depends on local proper-ties of c and $|\nabla c|$. The flux vector is known to vanish with $|\nabla c|$ and for sufficiently small values of the magnitude $|\nabla c|$ the flux vector may be written as

$$-j_i = k_{ij}\frac{\partial c}{\partial x_j},$$

where k_{ij} is a second - order tensor and it depends on the local properties of the medium and c, but not on the gradient of concentration. The minus sign for the flux vector is used for a later convenience. Many

materials show isotropy, namely there is no directional distribution. Therefore in an isotropic medium **j** must be parallel to ∇c. In an isotropic medium the diffusivity tensor is written only as $k_{ij} = k \, \delta_{ij}$, where k is the diffusion coefficient. The expression for the flux vector becomes

$$- \mathbf{j} = k \, \nabla c \, . \tag{2.3}$$

The minus sign shows that the direction of the flux vector is in the direction of the decrease of concentration.

Substituting equation (2.3) into equation (2.2) the concentration equation is found in the following form

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (k \nabla c)$$

or, in the case of constant k,

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = k \, \nabla^2 c. \tag{2.4}$$

The boundary condition depends on the conditions at the wall. If the wall is insoluble in the fluid then, since there is no flux across the wall, the normal component of diffusion flux to the wall must vanish; thus the boundary condition is

$$k \, \frac{\partial c}{\partial n} = 0. \tag{2.5}$$

The other boundary conditions are $c = c_0$ and $c = 0$ at wall. In the former, c_0 is the saturation concentration, the wall dissolves in the fluid and equilibrium is rapidly established near its surface. In the latter, the wall absorbs the diffusing substance incedent on it. The boundary condition (2.5) is used throughout this paper.

3. Equation of turbulent diffusion

For an incompressible turbulent flow, far from the laminar region, the concentration is given by the equation

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = 0 \, , \tag{3.1}$$

c being instantaneous value of concentration. Substituting for c and $\mathbf{u}$, $c = C + c'$ and $\mathbf{u} = \mathbf{v} + \mathbf{v}'$ in equation (3.1), and then taking the average of it we obtain

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = - \nabla \cdot \overline{\mathbf{v}' c'} , \tag{3.2}$$

where C and $\mathbf{v}$ are the mean values of the concentration and the velocity, and prime denotes the fluctuating quantities. The term on the right-hand side of equation (3.2) shows the turbulent diffusion flux and it is unknown. A general argument which is similar to momentum transfer gives a relation

$$- \overline{v_i' c'} = \varepsilon_{ij} \frac{\partial C}{\partial x_j} , \tag{3.3}$$

where ε_{ij} is a second-order tensor. For a special case in which the diagonal elements of ε_{ij} are ε_1, ε_2 and ε_3 respectively, equation (3.3) becomes

$$- \overline{u' c'} = \varepsilon_1 \frac{\partial C}{\partial x} , \qquad - \overline{v' c'} = \varepsilon_2 \frac{\partial C}{\partial y} , \qquad - \overline{w' c'} = \varepsilon_3 \frac{\partial C}{\partial z} .$$

In this case, the turbulent diffusion flux can be written as

$$- \nabla \cdot (\overline{\mathbf{v}' c'}) = \frac{\partial}{\partial x} \left(\varepsilon_1 \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(\varepsilon_2 \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(\varepsilon_3 \frac{\partial C}{\partial z} \right) ,$$

and the use of it is restricted due to some conditions which are not given here because of the purpose in the present paper. In the case of the diffusion in a pipe of constant cross section the turbulent diffusion flux can be written as

$$- \nabla \cdot (\overline{\mathbf{v}' c'}) = \nabla_s \cdot (\varepsilon \nabla_s C) + \frac{\partial}{\partial x} \left(\varepsilon^* \frac{\partial C}{\partial x} \right) , \tag{3.4}$$

where ε and ε^*, which depend on the cross sectional variables, are diffusivities; s denotes the cross sectional derivatives and x is the coordinate which is taken along the pipe. Inserting equation (3.4) in equation (3.2) we have

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \nabla_s \cdot (\varepsilon \nabla_s C) + \frac{\partial}{\partial x} \left(\varepsilon^* \frac{\partial C}{\partial x} \right) ,$$

or,

$$\frac{\partial C}{\partial t} + \mathbf{v}_s \cdot \nabla_s C + u^* \frac{\partial C}{\partial x} = \nabla_s \cdot (\varepsilon \nabla_s C) + \frac{\partial}{\partial x} \left(\varepsilon^* \frac{\partial C}{\partial x} \right), \tag{3.5}$$

where, $\mathbf{v}_s$ is the cross sectional velocity and u^* is the axial velocity which depend only on the cross sectional variables. In general, the dependence of the components of the velocity on the cross sectional variables alone may not be true. However, this general case will not be considered in the present paper.

4. Superposability of the combined effect and the longitudinal diffusion

For the purpose in this paper we assume that the velocity components and the diffusivities depend only on the cross sectional variables. Using this property and the continuity equation for velocity field, equation (3.5) can be written in the following form

$$\frac{\partial C}{\partial t} + \nabla_s \cdot (C \mathbf{v}_s) + \frac{\partial}{\partial x} (C u^*) = \nabla_s \cdot (\varepsilon \nabla_s C) + \frac{\partial}{\partial x} \left(\varepsilon^* \frac{\partial C}{\partial x} \right). \tag{4.1}$$

We write equation (4.1) in a frame which moves at the mean speed of the flow. For this we put

$$X = x - U_m t, \qquad \tau = t,$$

where $X = X(x, t)$, $\tau = \tau(x, t)$ and U_m is the mean velocity. Using the properties of partial derivatives we have

$$\frac{\partial C}{\partial x} = \frac{\partial C}{\partial X} \frac{\partial X}{\partial x} + \frac{\partial C}{\partial \tau} \frac{\partial \tau}{\partial x} = \frac{\partial C}{\partial X},$$

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial X} \frac{\partial X}{\partial t} + \frac{\partial C}{\partial \tau} \frac{\partial \tau}{\partial t} = -U_m \frac{\partial C}{\partial X} + \frac{\partial C}{\partial \tau};$$

and inserting into equation (4.1) we find

$$\frac{\partial C}{\partial t} + \nabla_s \cdot (C \mathbf{v}_s) + \frac{\partial}{\partial X} (u C) = \nabla_s \cdot (\varepsilon \nabla_s C) + \frac{\partial^2}{\partial X^2} (\varepsilon^* C), \tag{4.2}$$

where $u = u^* - U_m$ is the velocity with respect to the moving frame and it has zero mean, and τ is replaced by t.

Taking the average of equation (4.2) over a cross section and using the identities

$$\int_A \nabla_s \cdot (C\, \mathbf{v}_s)\, dA = \int_\Gamma (\mathbf{n} \cdot \mathbf{v}_s)\, C\, dl = 0, \tag{4.3}$$

since $\mathbf{n} \cdot \mathbf{v}_s = 0$ on Γ and

$$\int_A \nabla_s \cdot (\varepsilon\, \nabla_s C)\, dA = \int_\Gamma \varepsilon \frac{\partial C}{\partial n}\, dl = 0 \tag{4.4}$$

since $\varepsilon \dfrac{\partial C}{\partial n} = 0$ on Γ, one finds

$$\frac{\partial C_m}{\partial t} = - \frac{\partial}{\partial X} \{u\, C\}_m + \frac{\partial^2}{\partial X^2} \{\varepsilon^* C\}_m . \tag{4.5}$$

where Γ denotes the boundary, $\mathbf{n}$ is the unit normal vector of Γ and $\partial/\partial n$ is the normal derivative to wall; C_m is the mean concentration over a cross section and $\{\ \}_m$ shows the mean of any quantity.

The first term on the right-hand side of equation (4.5) denotes the combined effect of lateral turbulent diffusion and longitudinal convection, and the second is the longitudinal turbulent diffusion. Thus we may conclude that both effects are superposable. This property provides the fact that first the combined effect is considered and then the effect of longitudinal diffusion is simply added.

Equation (4.5) is written in a frame which moves at the mean speed of the flow, for all values of time. The term on the left-hand side of (4.5) shows the changes of the mean concentration in time. The fist term on the right-hand side of equation (4.5) denotes the combined effect of lateral diffusion and longitudinal convection. This term vanishes if there is no lateral diffusion, in other words, concentration does not change in the cross section or there is no longitudinal convection, namely the velocity component along the pipe does not change in the cross section. The second term on the right-hand side of equation (4.5) is longitudinal turbulent diffusion which is characterized by the turbulent diffusivity ε^* and it usually is smaller than the combined effect. It is very interesting that there is no direct effect of the diffusivity ε on the equation satisfied by the mean concentration. The effect of the diffusivity ε on the mean concentration comes from the boundary condition which depends on ε.

Experiment shows that the combined effect depends on the local properties of the mean concentration and it goes to zero when the mean concentration gradient vanishes. For small values of the mean concentration gradient a linear relation between the combined effect and the mean concentration gradient can be written as

$$- \{u\,C\}_m = K_V\,\frac{\partial C_m}{\partial X}\;,\tag{4.6}$$

where K_V is a virtual diffusion coefficient. K_V is an effective diffusion coefficient for longitudinal dispersion of solute if the statistical properties of the flow do not change along the pipe.

Since ε^* depends only on the cross sectional variables $\{\varepsilon^* C\}_m$ may be written as

$$\{\varepsilon^*\,C\}_m = K_L\,C_m\;,$$

or, since the cross section of the pipe does not vary along the pipe,

$$\left\{\varepsilon^*\,\frac{\partial C}{\partial X}\right\}_m = K_L\,\frac{\partial C_m}{\partial X}\;,\tag{4.7}$$

where K_L denotes the longitudinal diffusivity although the definition of K_L is not obvious as K_V. Substituting (4.6) and (4.7) into equation (4.5) we have

$$\frac{\partial C_m}{\partial t} = K\,\frac{\partial^2 C_m}{\partial X^2}\;,\tag{4.8}$$

where K is equal to $K_V + K_L$ and is the total effective diffusivity. Equation (4.8) shows clearly that it is possible to describe the average concentration in a three-dimensional diffusion system by the solution of the one-dimensional diffusion equation. This property is true for laminar and turbulent flows.

5. The centre of mass and the variance of a soluble matter

Let us take into account equation (4.1) and multiply both sides by x and integrate in the interval $(-\infty,\infty)$ and over the cross section of the pipe of constant cross section we have

$$\int_A dA \int_{-\infty}^{\infty} x \frac{\partial C}{\partial t} dx + \int_A dA \int_{-\infty}^{\infty} x [\nabla_s \cdot (\mathbf{v}_s C)] dx + \int_A dA \int_{-\infty}^{\infty} u^* x \frac{\partial C}{\partial x} dx$$

$$= \int_A dA \int_{-\infty}^{\infty} x \nabla_s \cdot (\varepsilon \nabla_s C) dx + \int_A dA \int_{-\infty}^{\infty} x \varepsilon^* \frac{\partial^2 C}{\partial x^2} dx .$$

Using the continuity equation and the integral identities (4.3) and (4.4) and assuming that

$$\lim_{x \to \pm\infty} x^n C \to 0 \quad \text{and} \quad \lim_{x \to \pm\infty} x^n \frac{\partial^m C}{\partial x^m} \to 0,$$

and after some algebra (see Appendix) we have

$$\int_A dA \int_{-\infty}^{\infty} x \frac{\partial C}{\partial t} dx = \int_A dA \int_{-\infty}^{\infty} u^* C \, dx. \tag{5.1}$$

The total mass of the soluble matter in the pipe is given by

$$\int_A dA \int_{-\infty}^{\infty} C \, dx = M.$$

For large times we may write

$$\lim_{t \to \infty} \int_{-\infty}^{\infty} C \, dx = \frac{M}{A}, \tag{5.2}$$

since the mass of a cylindrical shell of radius r ultimately becomes independent of the distance from the axis of the pipe and of the azimutal angle. Thus, in the asimptotic case for the time, equation (5.1) can be written as

$$\int_A dA \int_{-\infty}^{\infty} x \frac{\partial C}{\partial t} dx = \left(\int_{-\infty}^{\infty} C \, dx \right) \int_A u^* \, dA$$

$$= \frac{M}{A} \int_A u^* A$$

$$= M U_m$$

The centre of mass is defined by

$$x_0 = \frac{\displaystyle\int_A dA \int_{-\infty}^{\infty} x\,C\,dx}{\displaystyle\int_A dA \int_{-\infty}^{\infty} C\,dx} = \frac{1}{M} \int_A dA \int_{-\infty}^{\infty} x\,C\,dx.$$

Since x and t are idependent variables, then we have

$$\frac{dx_0}{dt} = \frac{1}{M} \int_A dA \int_{-\infty}^{\infty} x\,\frac{\partial C}{\partial t}\,dx.$$

This relation is true for all values of time. For large times it follows that

$$\lim_{t\to\infty} \frac{dx_0}{dt} = U_m. \tag{5.3}$$

Equation (5.3) shows clearly that the centre of mass ultimately moves at the mean speed of the flow, regardless the initial distribution of the concentration.

The variance of the cloud of soluble matter is a parameter which is related to the concentration distribution of soluble matter. For the purpose here the definition of the variance is given about the centre of mass. Thus, the variance is written in the following form

$$\sigma^2 = \frac{1}{M} \int_A dA \int_{-\infty}^{\infty} (x - x_0)^2\,C\,dx. \tag{5.4}$$

Equation (5.4) shows that the variance depends only on t. Taking the derivative of σ^2 with respect to t, since x and t are independent and x_0 depends on t alone, we have

$$\frac{d\sigma^2}{dt} = \frac{1}{M} \int_A dA \left[\int_{-\infty}^{\infty} -2 \frac{dx_0}{dt} (x-x_0) C \, dx + \int_{-\infty}^{\infty} (x-x_0)^2 \frac{\partial C}{\partial t} \, dx \right]$$

$$= -\frac{2}{M} \frac{dx_0}{dt} \left(\int_A dA \int_{-\infty}^{\infty} xC \, dx - x_0 \int_A dA \int_{-\infty}^{\infty} C \, dx \right) + \frac{1}{M} \int_A dA \int_{-\infty}^{\infty} (x-x_0)^2 \frac{\partial C}{\partial t} \, dx$$

$$= -\frac{2}{M} \frac{dx_0}{dt} (M x_0 - x_0 M) + \frac{1}{M} \int_A dA \int_{-\infty}^{\infty} (x-x_0)^2 \frac{\partial C}{\partial t} \, dx$$

$$= \frac{1}{M} \int_A dA \int_{-\infty}^{\infty} (x-x_0)^2 \frac{\partial C}{\partial t} \cdot dx.$$

This equation gives the variation of the variance with time and it is independent of the concentration equation. In order to relate the variance and the concentration equation it is necessary to put $x-x_0=\xi$ and $u=u^*-U_m$ in equation (4.1) and then we have

$$\frac{\partial C}{\partial t} + \nabla_s \cdot (v_s C) + u \frac{\partial C}{\partial \xi} = \nabla_s \cdot (\varepsilon \nabla_s C) + \varepsilon^* \frac{\partial^2 C}{\partial \xi^2}. \tag{5.5}$$

Multiplying (5.5) by ξ^2 and integrating in the interval $(-\infty, \infty)$ and over the cross section we obtain

$$\int_A dA \int_{-\infty}^{\infty} \xi^2 \frac{\partial C}{\partial t} \, d\xi + \int_A dA \int_{-\infty}^{\infty} \xi^2 \nabla_s \cdot (v_s C) \, d\xi + \int_A dA \int_{-\infty}^{\infty} \xi^2 u \frac{\partial C}{\partial \xi} \, d\xi$$

$$= \int_A dA \int_{-\infty}^{\infty} \xi^2 \nabla_s \cdot (\varepsilon \nabla_s C) \, d\xi + \int_A dA \int_{-\infty}^{\infty} \xi^2 \varepsilon^* \frac{\partial^2 C}{\partial \xi^2} \, d\xi.$$

Using the continuity equation and the integral identities (4.3) and (4.4) and assuming that

$$\lim_{\xi \to \pm\infty} \xi^n C \to 0 \quad \text{and} \quad \lim_{\xi \to \pm\infty} \xi^n \frac{\partial^m C}{\partial \xi^m} \to 0,$$

and after some algebra (see Appendix) we have

$$\int_A dA \int_{-\infty}^{\infty} \xi^2 \frac{\partial C}{\partial t} \, d\xi = 2 \int_A dA \, u \int_{-\infty}^{\infty} \xi C \, d\xi + 2 \int_A dA \, \varepsilon^* \int_{-\infty}^{\infty} C \, d\xi.$$

The left - hand side of this equation is related to the derivative of the variance with respect to t. Thus we find that

$$\frac{d\sigma^2}{dt} = \frac{2}{M} \left[\int_A dA \, u \int_{-\infty}^{\infty} \xi C \, d\xi + \int_A dA \varepsilon^* \int_{-\infty}^{\infty} C \, d\xi \right].$$

The right - hand side depends only on t and in the asymptotic case when t goes to infinity the integrals

$$\int_{-\infty}^{\infty} C \xi \, d\xi \quad \text{and} \quad \int_{-\infty}^{\infty} C \, d\xi$$

are independent of time; thus the right - hand side ultimately becomes constant. Ultimately we may write

$$\varkappa = \frac{1}{M} \left[\int_A u \, dA \int_{-\infty}^{\infty} \xi C \, d\xi + \int_A \varepsilon^* \, dA \int_{-\infty}^{\infty} C \, d\xi \right], \tag{5.6}$$

and the variance ultimately takes the form

$$\sigma^2 = 2 \varkappa t. \tag{5.7}$$

Equation (5,7) shows that the variance varies as linear in time. There-fore we may expect that the mean concentration is ultimately distributed, about a point which moves at the mean speed of the flow, according to the normal law of error, when the statistical properties of the flow do not change along the pipe of constant cross section.

APPENDIX

The centre of mass :

$$\int_A dA \int_{-\infty}^{\infty} x \, \nabla_s \cdot (\mathbf{v}_s C) \, dx = \int_A dA \int_{-\infty}^{\infty} \nabla_s \cdot (\mathbf{v}_s \, x \, C) \, dx$$

$$= \int_A dA \left[\nabla_s \cdot \mathbf{v}_s \left(\int_{-\infty}^{\infty} x \, C \, dx \right) \right]$$

$$= \int_\Gamma (\mathbf{n} \cdot \mathbf{v}_s) \left(\int_{-\infty}^{\infty} x \, C \, dx \right) dl = 0 \, ;$$

$$\int_A dA \int_{-\infty}^{\infty} x \, u^* \frac{\partial C}{\partial x} \, dx = \int_A dA \, u^* \int_{-\infty}^{\infty} x \frac{\partial C}{\partial x} \, dx$$

$$= \int_A dA \, u^* \left(\left[x \, C \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} C \, dx \right)$$

$$= - \int_A dA \, u^* \int_{-\infty}^{\infty} C \, dx \, ;$$

$$\int_A dA \int_{-\infty}^{\infty} x \, \nabla_s \cdot (\varepsilon \nabla_s C) \, dx = \int_A dA \int_{-\infty}^{\infty} \nabla_s \cdot (\varepsilon \nabla_s x C) \, dx$$

$$= \int_A dA \left[\nabla_s \cdot \left(\varepsilon \, \nabla_s \int_{-\infty}^{\infty} x \, C \, dx \right) \right]$$

$$= \int_\Gamma \varepsilon \frac{\partial}{\partial n} \left(\int_{-\infty}^{\infty} x \, C \, dx \right) dl$$

$$= \int_\Gamma dl \left[\int_{-\infty}^{\infty} x \left(\varepsilon \frac{\partial C}{\partial n} \right) dx \right] = 0 \, ;$$

$$\int_A dA \int_{-\infty}^{\infty} x\, \varepsilon^* \frac{\partial^2 C}{\partial x^2}\, dx = \int_A dA\, \varepsilon^* \left(\left[x \frac{\partial C}{\partial x} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{\partial C}{\partial x}\, dx \right)$$

$$= - \int_A dA\, \varepsilon^* \left[C \right]_{-\infty}^{\infty} = 0.$$

The variance :

$$\int_A dA \int_{-\infty}^{\infty} \xi^2\, \nabla_s \cdot (\mathbf{v}_s C)\, d\xi = \int_A dA \left[\nabla_s \cdot \left(\mathbf{v}_s \int_{-\infty}^{\infty} \xi^2 C\, d\xi \right) \right]$$

$$= \int_\Gamma (\mathbf{n} \cdot \mathbf{v}_s) \left(\int_{-\infty}^{\infty} \xi^2 C\, d\xi \right) dl = 0 \; ;$$

$$\int_A dA \int_{-\infty}^{\infty} u\, \xi^2 \frac{\partial C}{\partial \xi} = \int_A dA\, u \left(\left[\xi^2 C \right]_{-\infty}^{\infty} - 2 \int_{-\infty}^{\infty} \xi C\, d\xi \right)$$

$$= -2 \int_A dA\, u \int_{-\infty}^{\infty} \xi C\, d\xi \; ;$$

$$\int_A dA \int_{-\infty}^{\infty} \xi^2\, \nabla_s \cdot (\varepsilon \nabla_s C) d\xi = \int_A dA \left[\nabla_s \cdot \left(\varepsilon \nabla_s \int_{-\infty}^{\infty} \xi^2 C\, d\xi \right) \right]$$

$$= \int_\Gamma dl \left[\varepsilon \frac{\partial}{\partial n} \int_{-\infty}^{\infty} \xi^2 C\, d\xi \right]$$

$$= \int_\Gamma dl \left(\int_{-\infty}^{\infty} \xi^2 \varepsilon \frac{\partial C}{\partial n}\, d\xi \right) = 0 \; ;$$

$$\int_A dA \int_{-\infty}^{\infty} \xi^2\, \varepsilon^* \frac{\partial^2 C}{\partial \xi^2}\, d\xi = \int_A dA\, \varepsilon^* \left(\left[\xi^2 \frac{\partial C}{\partial \xi} \right]_{-\infty}^{\infty} - 2 \int_{-\infty}^{\infty} \xi \frac{\partial C}{\partial \xi}\, d\xi \right)$$

$$= \int_A dA \; \varepsilon^* \left(-2 \left[\xi C \right]_{-\infty}^{\infty} + 2 \int_{-\infty}^{\infty} C \, d\xi \right)$$

$$= 2 \int_A dA \; \varepsilon^* \int_{-\infty}^{\infty} C \, d\xi.$$

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