## The Hindrance Factors For Some Transitions In "'Er

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${ }^{1 s}$ Er'un 20 geçişi için multipol karşımları bulunmuştu. Bu izotobun bazı state'leri için Nilsson dalga fonksiyonları da bilinmektedir. Böylece Hindrans faktörleri hesaplantp aynı multipol için daha önce diḡer izotopların geçişlerine bulunan değerlerle karşılaştırılabilir. Aynı multipollar için bu şekilde bulunan değerlerin çok iyi uyuştukları tesbit edilmiştir.

The multipolc mixing ratios for 20 transitions weve determined for ${ }^{165} E r$. The Nilsson wave functions for some of these states are also known. Thus the hindrance factors may be determined and a comparision of the results with the previous values for the same multipolarities in other nuclei can bc plotted. It is found that there is a good agrement omongst the hindrance factors for the same multipolarities.

## INTRODUCTION :

The ratio of the theoretical and the experimental transition probahilities are known as hindrance factors $F$. The theoretical transition propabilities may be given by the Weisskopf estimate, and the hindrance factors ( $F_{H}$ ) with respect to this are:

$$
\left.F_{W}=T_{\left(\frac{1}{2}\right)}(\pi L)_{\text {oxperiment }} / T_{\left(\frac{1}{2}\right)}\right) \gamma(\pi L)_{\text {Woinokopl }}
$$

The partial gamma-ray halflives $T_{\left(\frac{1}{2}\right)}(\pi L)$ with respect to the Weisskopf estimate arc listed in table 1 (Löbner, 1974), and the experimental values may be calculated from

$$
T_{\left(\frac{1}{2}\right) \text { <er }}(\pi L)=T_{\left(\frac{1}{2}\right)}(\text { level })\left[\sum_{d} N_{d} / N_{\mathrm{B}}(\pi L)\right]
$$

[^0]where $N_{\gamma}(\pi L)$ is the intensity of the gamma-ray with multipolarity $L$ and $\Sigma_{d} N_{d}$ is the sum of the intensities of all transitions depopulating the level of interest.

Table 1. Partial gamma-ray half-lives according to the Welsnkopf estimate for slifferent multipole transitions. ( $A=$ mass number, $E=$ transition energy in MeV).

| $T_{\left(\frac{1}{2}\right) \gamma}(E 1)=6.76 A^{-3 / 3}$ | $E_{\bar{Y}}^{-3} \times 10^{-5} \mathrm{sec}$ |
| :--- | :--- |
| $T_{\left(\frac{1}{2}\right) \gamma}(E 2)=9.52 A^{-1 / 3}$ | $E_{\bar{\gamma}}^{-5} \times 10^{-9} \mathrm{sec}$ |
| $T_{\left(\frac{1}{3}\right) \gamma}(M 1)=2.20$ | $E_{\bar{\gamma}}^{-3} \times 10^{-14} \mathrm{sec}$ |
| $T_{\left(\frac{1}{2}\right) \gamma}(M 2)=3.10 A^{-2 / 3}$ | $E_{\bar{\gamma}} \times 10^{-8} \mathrm{sec}$ |

The calculated hindrance factors $F_{W}$ for same transitions in ${ }^{15} \mathrm{Er}$ are shown in table2.

The reduced transition probabilities calculated by Nathan and Nelsson (1965) imply that, if the multipolarity of a gamma-ray between different intrinsic states is $L<\left|K_{\mathrm{t}}-K_{f}\right|$, then such a transition is strictly forbidden within the framework of the Nilsson model, and their transition probabilities are determined by the presence of $K$-admixtures in the wave functions. On the other hand, for $\left|K_{t}-K_{f}\right| \leq L \leq K_{i}+K_{f}$ the ratio of the reduced transition probabilities for gamma-rays for same multipolarity from an arbitrary state to any two members of a final state $K$, is given by

$$
A(\pi L)=\frac{B\left(\pi L, J_{i} K_{i} J_{f} K_{f}\right)}{B\left(\pi L, J_{i} K_{i} J_{f} K_{f}\right)}=\frac{\left\langle J_{i} L K_{i}\left(K_{f}-K_{i}\right) \mid J_{i} K_{f}\right\rangle^{2}}{\left\langle J_{i} L K_{i}\left(K_{f}-K_{i}\right) \mid J_{f} K_{f}\right\rangle^{2}}
$$

This is called the Alaga branching rule (Alaga et al., 1955).
THE HINDRANCE FACTORS FYR SOME TRANSITIONS IN ${ }^{165} \mathbf{E r}$ :
The hindrance factors $F_{w}$ with respect to the Weisskopf estimate evaluated for some transitions in ${ }^{155} \mathrm{Er}$ are listed in table 2. It can be seen from fig. 2. that these values fall in the range of hindrance factors $\left(F_{w}\right)$ evaluated for $(\Delta K=1, E 1)(\Delta K=1, E 2),(\Delta K=1, M 1)$, ( $\Delta K=1, M 2$ ) and ( $\Delta K=0, E 1$ ) (Löbner, 1974). These values are also consistent with the selection rules for the Nilsson states. Thus in the case of $55 \mathrm{keV}, 60 \mathrm{keV}, 114 \mathrm{keV}$ and 218 keV transitions, the $M 1 \mathrm{multi}$ -


Fig. 1. The decay scheme of ${ }^{10} \mathrm{Er}$ following the EC decay of ${ }^{16 s} \mathrm{Tm}$ ( 29.6 h ) based on the work of Marquer and Chery (1971).
polarity is hindered by $F_{W} \sim 10^{2}-F_{W} \sim 3,5 \times 10^{2}$ whereas the $E 2$ multipolarity is hindered by $F_{i f} \sim 2 \times 10^{-1}-F_{W} \sim 2.7 \times 10^{-2}$. In the case of $249 \mathrm{keV}, 265 \mathrm{keV}$ and 389 keV the $E 1$ multipolarity is hindered by $F_{W} \sim 2.7 \times 10^{4}-F_{W} \sim 9.3 \times 10^{4}$ and the $M 2$ multipolarity is hindered by $F_{\mathrm{KI}} \sim 1.9 \times 10^{-1}-8.9 \times 10^{-3}$. From Löbner's (1974) results it can be seen that the smallest value for $(\Delta K=4, M 2)$ is of the order of $2 \times 10^{3}$, for $(\Delta K=3, M 2) \quad F_{u} \sim 8 \times 10^{2}$, for $(\Delta K=2, M 2) F_{w} \sim 10$, for $(\Delta K=1, M 2)$


Fig. 2. Range of hindrance factors relative to the Weiskopp estimate $F_{\mathrm{w}}$. The dashed lines show the dependence of $F$. on $|\Delta K|$ according to the emprical rule $\log F_{w}=2(|\Delta K|-I)$. The circles show present values. [Taken from Lobbner (1974)]

Table 2. The calculated hindrance factors for some transitions in ${ }^{16} \mathbf{E r}$.

| Y-ray | Initial and Einal | Multi | $T(\overline{. L})^{\circ}$ | $T(x, L)$ | $F_{W}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Enery (keV) | Nilsson States $K\left(N_{n_{3}} \wedge\right)$ | $\begin{aligned} & \text { polarity } \\ & (\pi L) \end{aligned}$ | $\left(\frac{1}{2}\right)$ Y exp | $\left(\frac{1}{2}\right) \mathrm{YWe}$ |  |
| 55 | $\frac{1-}{2}(521), \frac{3-}{2}(521)$ | MI | $4.1 \times 10^{-y}$ | $1.3 \times 10^{-11}$ | $3.15 \times 10^{2}$ |
| 55 |  | $E 2$ | $1.6 \times 10^{-7}$ | $2.4 \times 10^{-5}$ | $6.63 \times 10^{-2}$ |
| 60 | $\frac{1-}{2}(521), \frac{3-}{2}(521)$ | M 1 | $1.1 \times 10^{-9}$ | $1.0 \times 10^{-31}$ | $1.10 \times 10^{2}$ |
| 60 |  | $E 2$ | $3.5 \times 10^{-8}$ | $1.3 \times 10^{-6}$ | $2.69 \times 10^{-2}$ |
| 114 | $\frac{1-}{2}(521), \frac{3-}{2}(521)$ | M1 | $1.0 \times 10^{-9}$ | $0.5 \times 10^{-11}$ | $2.00 \times 10^{2}$ |
| 114 |  | $E 2$ | $1.6 \times 10^{-1}$ | $5.7 \times 10^{-7}$ | $7.80 \times 10^{-2}$ |
| 218 | $\frac{3-}{2}(521), \frac{5-}{2}(523)$ | M1 | $0.1 \times 10^{-9}$ | $0.2 \times 10^{-11}$ | $350 \times 10^{2}$ |
| 218 |  | $E 2$ | $1.4 \times 10^{-1}$ | $0.6 \times 10^{-7}$ | $2.30 \times 10^{-1}$ |
| 249 | $\frac{3-}{2}(521), \frac{5+}{2}(642)$ | $E 1$ | $4.1 \times 10^{-9}$ | $1.5 \times 10^{-13}$ | $2.7 \times 10^{4}$ |
| 249 |  | M2 | $9.8 \times 10^{-9}$ | $1.1 \times 10^{-6}$ | $8.91 \times 10{ }^{3}$ |
| 264 | $\frac{1+}{2}(500), \frac{3-}{2}(521)$ | E1 | $11.3 \times 10^{-8}$ | $1.2 \times 10^{-13}$ | $9.26 \times 10^{4}$ |
| 264 |  | M2 | $24.2 \times 10^{-9}$ | $8.0 \times 10^{-7}$ | $3.03 \times 10-{ }^{2}$ |
| 389 | $\frac{1+}{2}(660), \frac{1-}{2}(521)$ | $E 1$ | $2.6 \times 10^{-9}$ | $3.8 \times 10^{-14}$ | $6.80 \times 10^{4}$ |
| 389 |  | M2 | $2.1 \times 10^{-8}$ | $1.1 \times 20^{-7}$ | $1.91 \times 10^{-}$ |

$F_{W} \sim 10^{-2}$. However, there is no experimental value available for ( $\Delta K=0$, M2). In the present case this is found to be $F_{w} \sim 2 \times 10^{-1}$ which may fit rather well amongst the previous values.

The 249 keV occurs in between $\frac{3-}{2}$ (521) and $\frac{5+}{2}$ (642) Nilsson states which are defined as $K\left(N n_{3} \Lambda\right)$ thus for this transition $\Delta K=1$,

* The hall lives of the states are taken from Andrejtacheff et al (1974) and they are listed in table 4., The mixing ratlos are listed in table 9 reference 1.
$\Delta N=-1, \Delta n_{3}=-2$ and $\Delta \Lambda=-1$. The 264 keV transition is in between $\frac{1+}{2}(400)$ and $\frac{3-}{\frac{2}{2}}$ (521) states and has the same values for $\Delta K, \Delta N$, and $\Delta \Lambda$. The selection rules for these agree with the large hindrance factors for $E 1$ multipolarity. These hindrance factors are

$$
F_{\mathrm{W}}(249 \mathrm{keV} ; E 1)=2.77 \times 10^{4}
$$

and

$$
F_{\mathrm{W}}(249 \mathrm{keV}: \mathrm{E} 1)=9.26 \times 10^{4}
$$

Anderstscheff et al (1972) evaluated the $F_{W^{W}}$ for the $E 1$ transitions between the $\frac{5+}{2}(512)-\frac{7-}{2}(633)$ states in a number of nuclei, and they found that

$$
\begin{array}{ll} 
& F_{w} \sim 5 \times 10^{4} \text { in }{ }^{16} \cdot \mathrm{Dy} \\
\text { and } & F_{W} \sim 2 \times 10^{5} \text { in }{ }^{167} \mathrm{Er}
\end{array}
$$

Although these do not correspond to the same states in ${ }^{165} \mathrm{Er}$ they have the same selection rules of $\Delta K=1, \Delta N=-1, \Delta n_{3}=-2$ and $\Delta \Lambda=-1$ as the 249 keV and 264 keV and from this point may be compared and increed they are consistent with the values of $F_{w}$ for ${ }^{165} \mathrm{Dy}$ and ${ }^{167} \mathrm{Er}$.

## REFERENCES

1) Uluer I, 1976, Sakarya DMMA Dergisl, mma-1, 41-57.
2) Marquer G. and Chery R, 1972, Le Journal de physique, No 4, 301-314.
3) Löbner KEG, 1974, in «The Electromagnetic Interaction in Nuclear Physics» ed. W D Hamilton - North Holland, Amsterdam, pp. 140-170.
4) Alaga G. Alder K, Bohr A, and Mottelson B R, 1955, Mat. Fys. Medd. Da. Vid. Selk., 29, No 3.
5) Andrejtsceff W, Manfrass P, Parade H, Schilling K D, Winter D, Fula H, Ionmihal R, Khalikulov A B, Morozov V A, Marupov N Z, A and Mumin ov T M, 1974, Nuclear Physics.
6) Nathan $O$ and Nilsson $S G, 1965$, in \&Alpha-. Beta-, and Gamma-Ray Spectroscopyy. ed. K Slegbahn. North Holland, Amsterdam.

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