Kuru Sürtünme İhtiva Eden Hidrolik Pozisyonlama Sistemlerinin Stabilitesi Üzerine Teorik Bir İnceleme

A Theoretical Analysis On The Stability Of Hydraulic Positioning Systems Comprising Dry Friction

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Açık merkezli valfle çalışan stabil olmayan lineer bir hidrolik pozisyonlama sisteminin, sükûnet halinde ve sonlu hareket miktarlarındaki limit osilasyonlarını ihata eden, dinamik davranışı üzerine, lineer olmayan, kuru (Coulomb), kızak yolu sürtünmesi tesirinin tayini.

The determination of the influence of the non-linear dry (Coulomb) guideway friction, on the dynamic behaviour (involving limit cycling about stand-still and finite movement rates) of a hydraulic instable linear positioning hydraulic servo-system (with open-centre valve) by describing function analysis.

The present paper deals with the study of integral positional hydraulic servo systems, that is, in practice, the study of hydrocopying devices.

The stability and positioning accuracy of a hydraulic positioning servo system are governed both by the linear dynamic coupling between the various elements and the non-linearities present therein.

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Only the elasticity of the oil in the cylinder (hydraulic motor) will be allowed for, considering the active or passive (resisting) structures as infinitely rigid; the more so, because any structural elasticity can be directly added to that of the oil.

Guideway dry friction is one of the most important non-linearities of such a system. The friction may be considered as a pure Coulomb friction.

The theoretical study, consisting in obtaining the relations between the Coulomb friction of the guideways for different input conditions, and the linear hydraulic and mechanical parameters, when the system is at the stability border, and also the calculation of the steady-state oscillations of the system.

The analysis of the non-linear hydraulic-servo-system has been carried out using sinusoidal input describing function and dual input describing function methods applied to the non-linear element to replace the system non-linearity by a linear gain.

The servo-system, which is here examined, may be represented by the functional block diagram of Fig. 1.

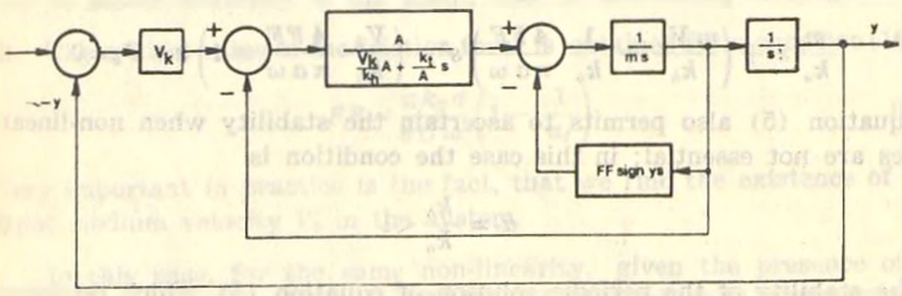


Fig. 1

The equations for the different links of the system are as follows.

The flow toward the hydraulic motor is a function of the valve displacement and the load differential pressure :

$$\frac{Q}{A} = V_k \frac{1}{s} (xs - ys) - \frac{V_k}{\kappa_h} A p_L \tag{1}$$

The output position of the hydraulic motor can be expressed in terms of the flow toward the hydraulic motor and the load differential pressure:

$$y = \frac{Q}{As} - \frac{k_t}{A} p_L \quad (2)$$

Allowing for the dry friction of the guideways and the hydraulic motor being loaded only by inertial mass, and given the large inertia of the load during self-oscillations, the output link will not stop when $y=0$. The equation of the forces acting on the piston of the hydraulic motor is:

$$Ap_l = m\, y\, s^2 + FF \text{ sign } ys \quad (3)$$

Applying now DF analysis to the Coulomb (dry) friction, which has the same characteristics of an ideal relay, this static non-linearity is memory-less and possesses odd symmetry; and one obtains the *DF* expressed as:

$$N(a, \omega) = \frac{4FF}{\pi a \omega} \quad (4)$$

The characteristic linearized equation of the loaded hydraulic drive formed according to the family of equations (1) — (4) is:

$$\frac{m}{k_o} s^3 + \left(\frac{m V_k}{k_h} + \frac{1}{k_o} \frac{4FF}{\pi a \omega} \right) s^2 + \left(\frac{V_k}{k_h} \frac{4FF}{\pi a \omega} + 1 \right) s + V_k = 0 \quad (5)$$

Equation (5) also permits to ascertain the stability when non-linearities are not essential; in this case the condition is

$$a_l = \frac{k_h}{k_o} < 1 \quad (6)$$

The stability of the periodic solution of equation (5), which represents the system, would be determined by the criterion

$$\left(\frac{\partial X}{\partial a} \right) \left(\frac{\partial Y}{\partial \omega} \right) - \left(\frac{\partial X}{\partial \omega} \right) \left(\frac{\partial Y}{\partial a} \right) > 0 \quad (7)$$

where

$$X = V_k - \left(\frac{m V_k}{k_h} + \frac{1}{k_o} \frac{4FF}{\pi a \omega} \right) \omega^2 \qquad Y = -\frac{m}{k_o} \omega^3 + \omega \quad (8)$$

It can be seen that the criterion (7) is not statisfied and the periodic solution merely corresponds to the limit of stability in small amplitude.

The frequency of the periodic solution is determined from the second element of equation (8) as follows :

$$\omega=\sqrt{\frac{k_o}{m}} \tag{9}$$

Substituting now this value in the first element of equation (8), we obtain the amplitude of the periodic solution (limit cycling) :

$$a=\frac{FF\,e\,C\,\omega}{\pi k_2}\left(\frac{\alpha_l}{\alpha_l-1}\right) \tag{10}$$

and it can be seen that the periodic solution is possible if

$$\alpha_l>1 \quad \text{or} \quad k_h>k_0 \tag{11}$$

This shows that the stability criterion α_l separates the region in which a periodic solution is possible from that in which it is impossible.

The system has an asympotic stability.

The amplitude of the periodic solution is a diverging limit cycling amplitude, so that accidental amplitudes larger than (10) will be unstable, i.e., increasing to infinity; accidental amplitudes smaller than (10) will be stable, stability in the small, that is, decreasing to zero.

The critical value of the friction force is obtained from equation (10) :

$$FF=\frac{\pi k_2 a}{e\,C\,\omega}\left(1-\frac{1}{\alpha_l}\right) \tag{12}$$

Very important in practice is the fact, that we find the existence of an input medium velocity V_0 in the system.

In this case, for the same non-linearity, given the presence of a positive bias component V_0, the DIDF has been determined by the following expression :

$$N(a,\omega,V_o)=\frac{4\,FF}{\pi a\omega}\sqrt{1-\left(\frac{V_o}{a\,\omega}\right)^2} \tag{13}$$

The limit cycling DIDF is indeed non-phase-shifting, as one expects in the case of this memory-less non-linearity.

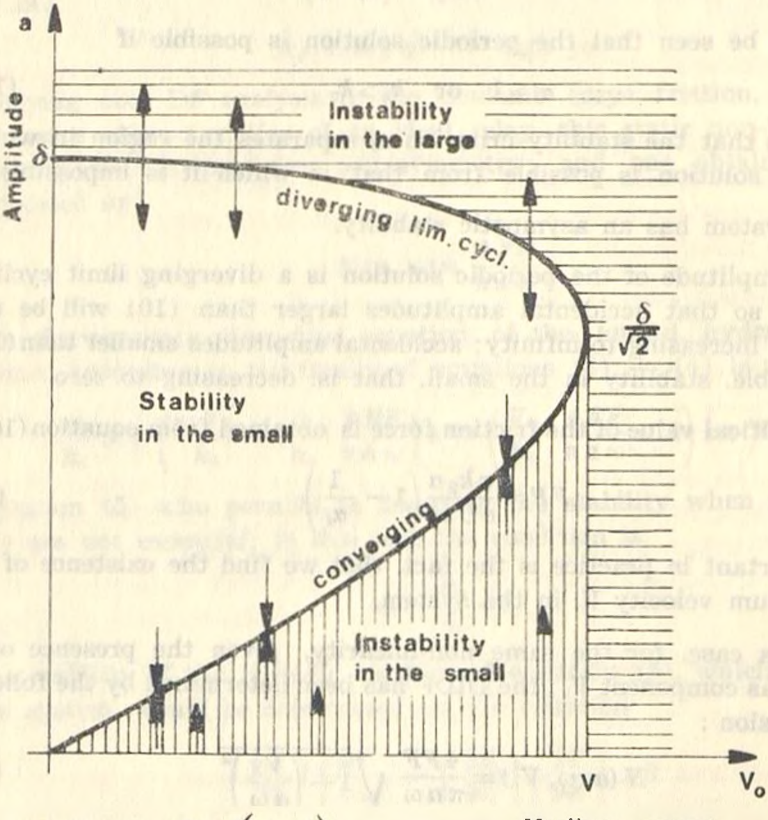
Hence, the limit cycling amplitude is obtained as :

$$a=\frac{FF\,eC\,\omega}{\sqrt{2}\,\pi k_2}\left(\frac{\alpha_l}{\alpha_l-1}\right)\sqrt{1\mp\sqrt{1-\left(\frac{2\pi k_2 V_o}{FF\,e\,C\,\omega^2}\right)^2\left(1-\frac{1}{\alpha_l}\right)^2}} \tag{14}$$

It is seen from equation (14), that there results a higher diverging limit cycling curve and a lower converging limit cycling curve, both ending at limit velocity : Fig. 2

$$V = \frac{FF\,e\,C\,\omega^2}{2\pi k_2}\left(\frac{\alpha_l}{\alpha_l - 1}\right) \tag{15}$$

$$a = \frac{FF\,e\,C\omega}{\sqrt{2}\,\pi k_2}\left(\frac{\alpha_L}{\alpha_l - 1}\right)\sqrt{1 \pm \sqrt{1 - \left(\frac{2\pi k_2 V_0}{FF\,e\,C\omega^2}\right)^2\left(1 - \frac{1}{\alpha_l}\right)^2}}$$



$$\delta = \frac{4FF}{\pi\sqrt{m}V_k}\,\frac{1}{\sqrt{k_0}}\left(\frac{\alpha_l}{\alpha_l - 1}\right)$$

$$V = \frac{2FF}{\pi\, m V_k}\left(\frac{\alpha_l}{\alpha_l - 1}\right)$$

Fig. 2

CONCLUDING REMARKS

It may be concluded that Coulomb friction generates stability at zero amplitudes (stability in the small) at rest, and at small amplitudes in the lower speed range (steady-state limit cycling as described by the lower converging curve) also for accidental amplitudes smaller than the higher diverging curve.

In practice, it is always possible to find a minimum damping of other origin which, even if the real servo-system is insufficiently damped, will position the converging limit cycling curve along the zero amplitudes, thus resulting in a practically stable servo-system at low speeds.

The system, because of asymptotic stability, has only a conditional stability depending on the critical friction, limit velocity and the system parameters.

The stabilizing effect of the friction on the system is approaching zero, and becoming weaker, while the medium speed V_0 is increasing to limit velocity V.

In The region $V_0 \geq V$ the system stability is no longer governed by the friction, but by the stability criterion α_f.

On the whole, it may be stated that Coulomb friction has a stabilizing effect upon unstable servo-systems at low input speeds.

NOTATIONS

Q (m³ sec⁻¹) = Flow

A (m²) = Actuating cylinder section

p_L (kg m⁻²) = Load differential pressure

V_K (sec⁻¹) = Speed gain

x (m) = Spool displacement

y (m) = Slide displacement

k_h (kg m⁻¹) = Force gain

k_t (m^5 kg^{-1}) = Hydraulic compressibility number

s (d/dt) = Shorthand notations

m (kgm^{-1} sec^2) = Mass of slide

FF (kg) = Coulomb friction force

a (m) = Limit cycling amplitude

ω (sec^{-1}) = Natural frequency

k_0 (kg m^{-1}) = Stiffness of oil columns

α_l = Stability criterion

k_2 (m^2 sec^{-1}) = Flow gain

e (kg^{-1} m^2) = Oil specific compressibility

C (m) = Effective stroke of actuating cylinder

V (m sec^{-1}) = Limit velocity of the slide

V_0 (m sec^{-1}) = Medium speed

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