The General Outlook Of The Mixing Ratios Of Transitions In Nuclei At The Onset Of The Deformed Region ($150 \subseteq A \subseteq 190$)

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ABSTRACT

The multipole mixing ratios (S) of transitions in ^{152}Gd, ^{152}Sm, ^{154}Gd and ^{156}Gd were measured by $\gamma - \gamma(\theta)$, $e_K - \gamma(\theta)$ directional correlation and nuclear orientation experiments to see the effect of deformation the sign and the magnitude of S at the onset of the deformed region. The magnitudes of S imply that the transition connecting the levels of same parity are mainly E2 ; and the sign of S is negative for the $2_\beta - 2_{gr}$ in almost spherical ^{152}Gd but positive in the rest of the 4 nuclei. The opposite happens for the transitions deexciting the γ - band. The sign of S is negative for $3_\gamma - 2_{gr}$, $3_\gamma - 4_{gr}$ in ^{152}Sm, ^{154}Gd and ^{156}Gd; it is also negative for $4_\gamma - 4_{gr}$ in ^{152}Sm, ^{156}Gd. The transitions connecting the levels of opposite parity are mainly E1, and it has been observed that the mixing ratios are identical for all nuclei in each cascade.

ÖZET

$\gamma - \gamma(\theta)$, $e_K - \gamma(\theta)$ *yöne bağlı ilişkiler ve çekirdek orientasyonu deneyleri ile ^{152}Gd, ^{152}Sm, ^{154}Gd ve ^{156}Gd'da multipol karışımları (S) ölçülerek deformasyonun, S'nın işaret ve normu üzerindeki etkisi incelendi. S'nın normaları gösterir ki aynı pariteyi bağlayan geçişler E2 dirler; ve S'nın işareti hemen hemen küresel ^{152}Gd'un $2_\gamma - 2_{gr}$ geçişi için negatif, diğer çekirdeklerin aynı geçişleri için pozitiftir. γ bandını ilk hale dönüştüren geçişler için ise bunun aksi olmaktadır. S'nın işareti ^{152}Sm, ^{154}Gd ve ^{156}Gd'da negatif olup; ^{152}Sm ve ^{156}Gd'da $4_\gamma - 4_{gr}$ için yine negatiftir.*

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Aksi pariteleri birleştiren geçişler daha ziyade E1'dirler, ve karışım oranlarını işaretleri, her öçülen geçişte bütün izotoplar için aynıdır.

The shell model is particularly successful in the case of a closed shell nucleus where the shape is spherical. As valence particles are added to a closed shell nucleus, the residual interactions between particles tend to stabilise the spherical shape (Bohr and Mottelson, 1953) [1]. However the spherical symmetry breaks down when more particles are added and the nucleus acquires a permanent deformation. Such a nucleus may be considered in terms of the collective model. This model describes the collective motion of nucleons in terms of vibrations about the equilibrium shape and the rotation of the nuclear orientation which maintains the deformed shape. The significance of the shape was first pointed out by Rainwater (1950) [2] and the mathematical development was carried out by Bohr and Mottelson (1953) [1].

Kumar and Barranger (1968) [4] illustrated this very clearly as shown in Fig. 1. where the potential energy V is drawn as a function of deformation. In this case the closed shell of interest corresponds to the magic number $N = 82$. Near the closed shell ($N = 84$), the pairing forces favour the grouping of the nucleons to give spherical equilibrium shape. As the number of neutrons increase to 88 the long range forces overcome the pairing and the nucleus begins to deform. To study the energy levels of the deformed nuclei one considers the even - even nuclei and the odd nuclei separately. For the even - even nuclei the intrinsic angular momentum is zero, and the Hamiltonean is written to include three basic terms :

$$H = H_{\gamma} + H_{\beta} + H_r$$

The first two terms correspond to vibrations and the last term to rotations. Thus the solution for the Schrödinger equation has two parts: The vibrational and the rotational energy. The rotational energy is given by (Preston, 1962).

$$E_{rot} = \hbar^2 \frac{(J(+1) - K^2)}{2 I_1} + \frac{\hbar^2 K^2}{2 I_3}$$

$$J = K + 1 \ , \ K + 2 \cdots$$

If $K = 0$ then only even J are possible and

$$E_{rot} = \frac{\hbar^2}{2 I} (J + 1) J$$

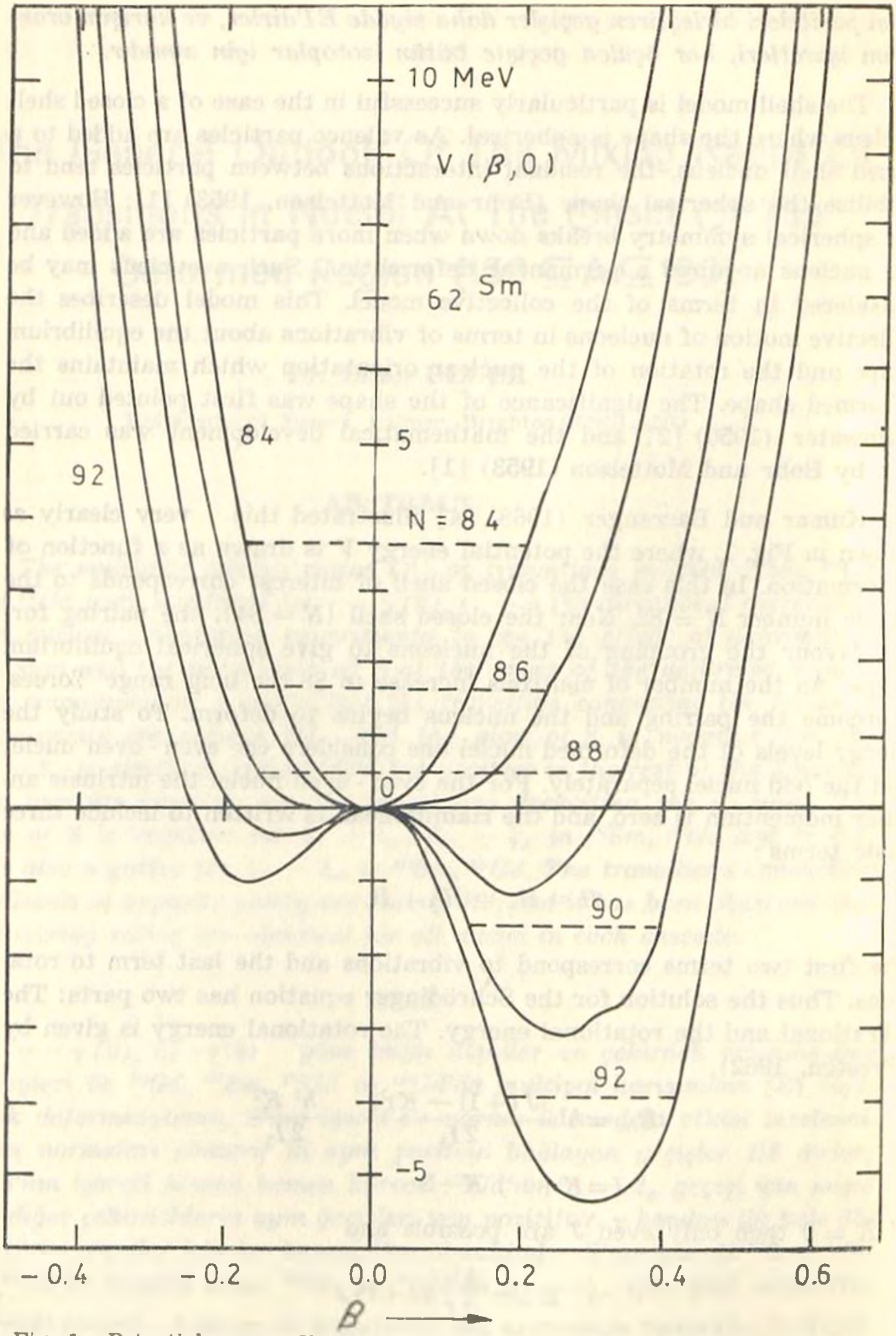

Fig - 1. Potential energy V versus deformation parameter β for asymmetry $\gamma = 0$. The approximate location of the ground state is shown by dashed lines. [Taken from KUMAR and BARANGER (1968)]

The collective motion requires that this energy should be small which means that the moment of inertia I should be large. Thus one expects to detect rotational states in highly destorted even - even nuclei. It is interesting to note that $E_{4+}/E_{2+} = 10/3$, $E_{6+}/E_{2+} = 7$ and these fit well in the deformed region $150 \leqslant A \leqslant 190$. As one comes nearer to the closed shells the above formula has to be corrected.

As far as the vibrational energies are concerned :

Vibrations about a stable deformation value β are called β - vibrations, and the oscillations in the shape with constant deformation are called γ - vibrations. In other words, for fixed γ the β - vibration describes an axially symmetric vibration in the eccentricity of the elliptic cross - section of the nucleus, and a γ - vibration for fixed β leads to loss of axial symmetry. The quantum numbers of the deformed nucleus are shown in Fig. 2. The lowest order γ - vibration has $K = 2$ and even parity and the lowest order β - vibration in even - even nuclei is expected

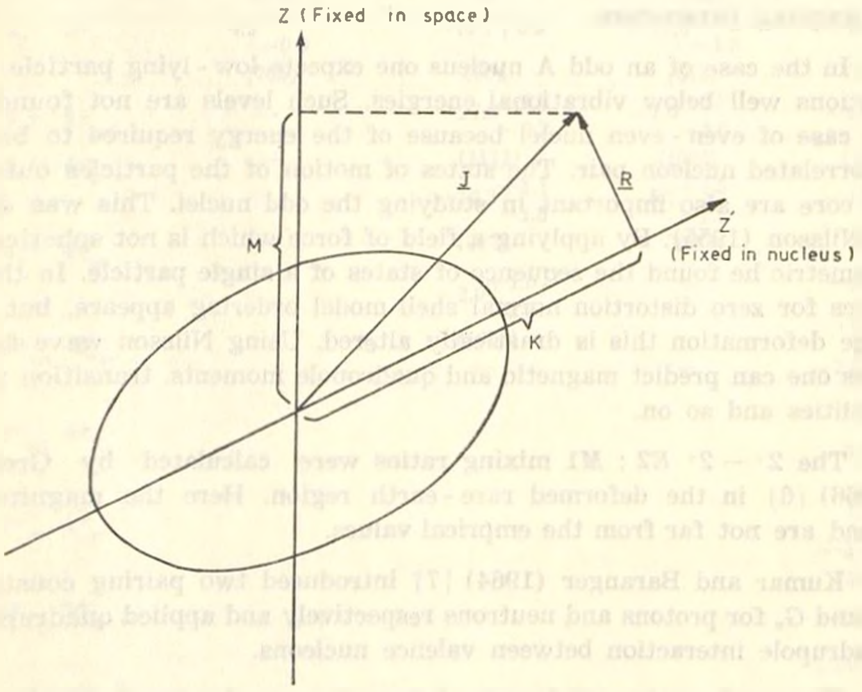

Fig - 2. The Quantum Numbers of a Deformed Nucleus.

to have $K = 0$, even parity. When higher order deformations occur the nucleus is pear shaped rather than an ellipsoid. This band is based on octupole vibrations of the shape. Such vibrations can carry from 0 to 3 units of angular momentum along the Z' axis of the nucleus. Various conditions restrict the spin and parity to odd, negative values.

In the case of an odd nucleus the single particle plays an important role. As can be seen in the following formula (Preston, 1962) [5] a rotational band is built on each particle state E_K :

$$E_{J,K} = E_K + \left(\frac{\hbar^2}{2I}\right)[J(J+1) - 2K^2 + \delta_{K,1/2}\, a\, (-)^{J+1/2}(J+1/2)]$$

(where a is called the decoupling parameter).

That is the energy of a nuclear state for axially symmetric nucleus. When $K = \frac{1}{2}$ the lowest state has $K = J$, and the rest of the states have $J = K + 1$, $K + 2$ ... For $K = 1/2$, J depends on the value of the de coupling parametere.

In the case of an odd A nucleus one expects low - lying particle excitations well below vibrational energies. Such levels are not found in the case of even - even nuclei because of the energy required to break a correlated nucleon pair. The states of motion of the particles outside the core are also important in studying the odd nuclei. This was done by Nilsson (1955). By applying a field of force which is not spherically symmetric he found the sequence of states of a single particle. In these states for zero distortion normal shell model ordering appears, but for large deformation this is drastically altered. Using Nilsson wave functions one can predict magnetic and quadropole moments, transition probabilities and so on.

The $2^+ - 2^-$ $E2 : M1$ mixing ratios were calculated by Greiner (1966) [6] in the deformed rare - earth region. Here the magnitudes found are not far from the emprical values.

Kumar and Baranger (1964) [7] introduced two pairing constants G_p and G_n for protons and neutrons respectively and applied quadrupole - quadrupole interaction between valence nucleons.

The pairing plus quadropole interaction as developed by Kumar (1974) [8, 9] has been particularly successful when applied to nuclei at each end of the deformed region. It is expected that these calculations shall be extended to cover the whole deformed rare earth region.

Experiments have been caried out by many people to measure the multipole mixing ratios of transitions ot the onset of the deformed region (as well as in other places in this region) [10 - 15]. This is done in order to provide experimental data for the confirmation and the varification of the theoretical methods, and the models used. However as discussed above most of the theoretical work have not been done, alt-

Table - 1. E2/M1 mixing ratios of transitions in nuclei near the onset of the deformed region 150 A 190. (The transition energies are given in parantheses, in keV).

J^i	$J^{\pi'}$	^{152}Gd	^{152}Sm	^{154}Gd	^{156}Gd
2_γ^+	2^+_{gr}	3.05 ∓ 0.14 (586)	$8.2^{+8.6}_{-3.0}$ (689)	$7.4^{+10.6}_{-3.0}$ (692)	—
2_γ^+	2^+_{gr}	$4.3^{+0.7}_{-0.6}$ (765)	-10.7 ∓ 0.6 (964)	$-10.0^{+0.7}_{-1.2}$ (873)	$-6.5^{+2.6}_{-7.9}$ (1066)
3_γ^+	2^+_{gr}	—	$-27.8^{+4.2}_{-0.2}$ (1112)	$-7.0^{+2.7}_{-3.0}$ (1005)	$-11.8^{+0.6}_{-0.7}$ (1159)
3_γ^+	4^+_{gr}	—	$-12.3^{+1.1}_{-1.5}$ (867)	$-5.7^{+1.2}_{-1.9}$ (757)	$-11.7^{+2.7}_{-5.3}$ (960)
4_γ^+	4^+_{gr}	—	$-3.0^{+1.0}_{-2.4}$ (1005)	—	$-4.0^{+3.9}_{-1.6}$ (1067)
4^+_{rot}	4^+_{gr}	—	—	—	$-2.07^{+0.13}_{-0.14}$ (1222)
4^+_{rot}	4^+_{γ}				$9.2^{+0.7}_{-0.6}$ (263)
5^+_{rot}	4^+_{gr}				-3.8 ∓ 0.2 (1334)
5^+_{rot}	6^+_{gr}				$-6.7^{+3.0}_{-4.0}$ (1038)
5^+_{rot}	4^+_{rot}				$0.15^{+0.10}_{-0.09}$ (112)

hough there is availible exprimental data. Infact recently Krane (1973) |3| has compiled a very useful set of data including the measured mixing ratios throughout the deformed region.

The multipole mixing ratios of transitions in ^{152}Sm and ^{152}Gd measured by Kalfas and Hamilton (1973) |10 - 11| and Doubt and Hamilton (1972) |12| are quoted in Table - 1. and Table - 2. together with the results of the present autor (1975) |13 - 14| on ^{154}Gd to study the effect of deformation on the sign and the magnitude of the mixing ratio. The magnitudes of the mixing ratios imply that the transitions connecting the levels of same parity are mainly $E2$ and the transitions between the levels of opposite parity are mainly $E1$.

Table - 1. includes the $E2/M1$ mixing ratios of transitions in ^{152}Gd, ^{152}Sm, ^{154}Gd and ^{156}Gd. As it can be seen from this table : The sign of S

Table - 2. M2/E1 mixing ratios of transitions in nuclei near onset of the deformed region 150 A 190. (The transition energies are given in parantheses, in keV).

J^{π}	$J^{\pi\prime}$	152_{Gd}	152_{Sm}	154_{Gd}	156_{Gd}
2^-_{oct}	2^+_{g}	—	—	0.17 ∓ 0.15 (904)	—
2^-_{oct}	2^+_{gr}	—	$0{\cdot}057 \mp 0.010$ (1408)	0.012 ∓ 0.011 (1274)	—
2^-_{oct}	3^+_{γ}	—	—	0.00 ∓ 0.07 (592)	—
3^-_{oct}	2^+_{gr}	$-0{,}037^{+0.016}_{-0.023}$ (779)	—	—	-0.08 ∓ 0.3 (1187)
4^-_{oct}	4^+_{gr}	—	—	$0.20^{+0.22}_{-0.19}$ (1189)	—
3^-_{oct}	2^+_{gr}	—	—	—	−0.030 (5) (1845)
3^-_{rot}	4^+_{gr}				0.012 (4) (1646)
3^-_{rot}	2^+_{γ}				−0.024 (8) (780)
4^-_{rot}	4^+_{gr}				0.06 (2) (534)
4^-_{rot}	5^+_{rot}				−0.009 (4) (422)

is negative for the $2_{\gamma} - 2_{gr}$ transitions in almost spherical ^{152}Gd, and it becomes positivein the more deformed ^{152}Sm, ^{154}Gd and ^{156}Gd nuclei. The opposite happens for the transitions deexciting the γ band. The mixing ratio found for the $2\gamma - 2_{gr}$ transition in ^{152}Gd is positive whereas it is negative for the rest of the three deformed nuclei. The sign of S is negative for $3\gamma - 2_{gr}$, $3_{\gamma} - 4_{gr}$ in ^{152}Sm, ^{154}Gd and ^{156}Gd, and it is also negative for $4_{\gamma} - 4_{gr}$ in ^{152}Sm and ^{156}Gd.

Table - 2. includes the $E1$ $M2$ mixing ratios of transitions in ^{152}Gd, ^{152}Sm, ^{154}Gd, and ^{156}Gd. It can be observed that the signs of the mixing ratios tabulated are identical for all nuclei in each cascade. Thus it may be concluded that the $E2/M1$ mixing ratios change sign as one goes from spherical to deformed nuclei but $E1$ $M2$ mixing ratios do not. *(The signs of the mixing ratios are consistent with the convention of Krane and Steffen (1970)* [15]).

The rotational band is well populated in ^{156}Gd but in the rest of the nuclei they are not measurably apparent. On the other hand the β band is less populated in the more deformed ^{156}Gd whereas it is strongly populated in more spherical ^{152}Gd.

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