İstif Edilmiş Malzeme Yığınları ve Yatakları Üzerindeki Akıma Tesir Eden Temel Faktörler

Fundamental Factors Gowerning the Flow over Packings and Beds

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Damlatmalı filtrelerdeki gibi bir taneli ortamda tanelerin üzerinde meydana gelen akım, uzun zamandan beri inceleme konusu olmuştur. Ancak, bir hayli zaman ve emek sarfedilmiş olmasına rağmen, genel olarak kabul edilmiş bir teori henüz mevcut değildir. Bu yazıda akıma tesir eden faktörler gözden geçirilecek, ve malzemenin yüzey alanının ve taneler arasındaki temas noktalarında kapiler kuvvetler etkisiyle tutulan sıvı hacminin akış zamanına etkisi etraflı olarak incelenecektir.

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Flow over granular media as in a trickling filter has long been the subject of discussion. However, although considerable time and effort has been expended, there is no commonly accepted theory. In this paper the factors gowerning the flow will be reviewed and the influence of media surface area and the liquid held at the points of contacts between the grains of fill materials by capillary forces upon the travel time will be investigated in detail.

Review of Present Knowledge on Time of Flow over Packings and Beds

Time of Flow over a Single Sphere. Assuming that all the liquid comes in at the very top and leaves at the very bottom of the sphere and that the effect of accelaration and certain effects of curvature are negligible, W. E. Howland derived the following equation for the total time of travel from the top to the bottom of a single sphere with a radious *a* as shown in Figure 1. (1)

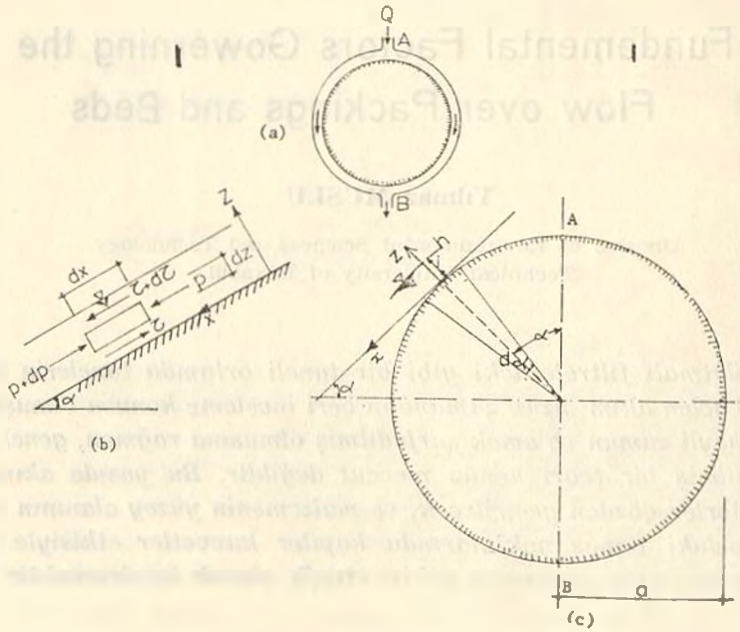

Figure 1. Flow over a single sphere

$$t = 2.6 \left(\frac{3\nu}{g}\right)^{1/3} \frac{(2\pi)^{2/3} a^{5/3}}{Q^{2/3}} \qquad (1)$$

Where

Q = Rate of flow appliéd to sphere (vol. per unit time)

ν = Kinematic viscosity of liquid = $\frac{\mu}{\rho}$

g = Acceleration of gravity

Kinematic viscosity of liquid is a function of temperature. Sewage temperature varies between 12 and 15°C and its kinematic viscosity changes accordingly (2). According to the recommendations of *A T V*, German Association of Sewage Purification, a value of $\nu = 1.31 \times 10^{-6}$ m^2/sec. can be accepted for the kinematic viscosity of sewage. (2).

If all quantities are expressed in metric units and $g = 9.81$ m/sec^2 is substituted into Eq. 1, the following equation is obtained :

$$t = 2.6 \left(\frac{3 \times 1.31 \times 10^{-6}}{9.81} \right)^{1/3} \frac{(2\pi)^{2/3} a^{5/3}}{Q^{2/3}} = 6.5263 \times 10^{-2} \frac{a^{5/3}}{Q^{2/3}} \qquad (2)$$

Where

t = Time of travel in sec

a = Radious of sphere in m

Q = Rate of flow in m^3/sec.

Time of Flow over a Vertical Column of Spheres. If a vertical column of *n* spheres shown in *Figure 2* is considered, time of flow then becomes

$$t = 6.5263 \times 10^{-2}\, n \frac{a^{5/3}}{Q^{2/3}} \qquad (3)$$

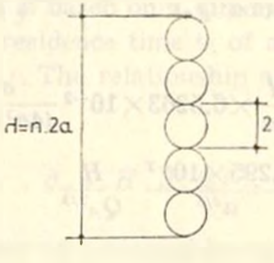

Figure 2. Flow over a vertical column of spheres

where $n = H/2a$ and H denotes the length of the column. This theoretical flow time was also confirmed experimentally (3).

Time of Flow over a Packing of Spheres of Rectangular Arrangement. The porosity of this arrangement is 0.4764. The rest, $(1 - 0,4764) = 0.5236 = \frac{\pi}{6}$ represents the ratio of solid volume to the total volume.

Consider a cylindirical bed of rectangular arrangement of depth H and diameter D. The number of spheres in the bed becomes:

$$n = \frac{(\pi D^2/4) \cdot H \cdot \pi/6}{(4/3)\pi a^3} \tag{4}$$

Since the number of the layers is $H/2a$, the number of spheres in a layer can be written as

$$\frac{(\pi D^2/4) H \pi/6}{(4/3)\pi a^3 H/2}$$

If Q_A is the hydraulic loading rate, the total rate of flow over the bed is therefore $Q_A \pi D^2/4$. Then the rate of flow Q applied to a chain of spheres as shown in Figure 2 becomes :

$$Q = \frac{\pi D^2}{4} Q_A \frac{(4/3)\pi a^3 \cdot (H/2a)}{(\pi D^2/4) H \pi/6} = 4a^2 Q_A \tag{5}$$

Since such a chain of spheres has $n = H/2a$ spheres, the time of flow over the entire bed can be written by substituting Eq. 5 into Eq. 1 or Eq. 2:

$$t = \frac{H}{2a} \cdot 2.6 \left(\frac{3\nu}{g}\right)^{1/3} \frac{(2\pi)^{2/3} a^{5/3}}{(4a^2 Q_A)^{2/3}} = 1.3 \cdot H \left(\frac{3\nu}{g}\right)^{1/3} \frac{(2\pi)^{2/3}}{4^{2/3} a^{2/3} Q_A^{2/3}} \tag{6}$$

or

$$t = \frac{H}{2a} \times 6.5263 \times 10^{-2} \frac{a^{5/3}}{(4a^2 Q_A)^{2/3}}$$

$$= \frac{1.295 \times 10^{-2}}{a^{2/3}} \frac{H}{Q_A^{2/3}} \tag{7}$$

Where

t = time of flow in sec

H = Depth of bed in m

Q_A = Hydraulic loading rate in $m^3/sec/m^2$

a = Radius of sphere in m

As hydraulic loading rate Q_A is more generally expressed in $m^3/d/m^2$, one writes Eq. 6 in the following form :

$$t = \frac{1.295 \times 10^{-2}}{a^{2/3}} \frac{H}{\left(\frac{Q_A}{86400}\right)^{2/3}}$$

$$= \frac{25.308}{a^{2/3}} \frac{H}{Q_A^{2/3}} \tag{8}$$

Where

$[t] = \text{sec}$

$[H] = \text{m}$

$[Q_A] = \text{m}^3/\text{d}/\text{m}^2$

$[a] = \text{m}$

Time of flow can also be expressed in minutes :

$$t = \frac{25.308}{60} \frac{H}{a^{2/3} Q_A^{2/3}} = \frac{0.4218 \text{ H}}{a^{2/3} Q_A^{2/3}} \tag{8 a}$$

Time of Flow over a Randomly Packed Assemblage of Spheres.

An excellent example of this type work is the experimental study by M. D. Sinkoff, et al (6). It is based on a dimensional analysis of variables that effect the mean residence time t_G of a fluid in a packed bed of clean media (see Figure 6.). The relationship among the variables is indicated by

$$f(Q_A \ , g , \nu , H , S , t_G) = 0$$

where S is the media surface area divided by volume occupied. It is. called as specific surface area.

In order to evaluate the functional relationship set by dimensional analysis, M. D. Sinkoff, et al, performed a set of experiments on a column of glass and porcelain spheres while varying the hydraulic loading rate and measuring t_G. The result of this analysis has been given with the following equation :

$$t_G = k \frac{H}{g^{1/3} \nu^{1/3}} \cdot \left(\frac{S}{Q_A}\right)^n \cdot \nu^n = CH \left(\frac{S}{Q_A}\right)^n \tag{9}$$

in which k and n are constants. The analysis of the measured values showed that for the glass spheres $k = 3.0$ and $n = 0.83$ and for the porcelain spheres $k = 1.5$ and $n = 0.53$. Thus for glass spheres of diameters of 1/2 - in., 3/4 - in., and 1 - in.

$$t_G = 3.0\, H \frac{(\nu)^{0.50}}{g^{1/3}} \left(\frac{S}{Q_A}\right)^{0.83} \tag{10}$$

or with $\nu = 1.31 \times 10^{-6}$ m²/sec and $g = 9.81$ m/sec²

$$t_G = 1.604 \times 10^{-3} H \left(\frac{S}{Q_A}\right)^{0.83} \tag{11}$$

where $[t_G]$ = sec ; $[S]$ = m²/m³ ; $[Q_A]$ = m³/m²/sec ; $[H]$ = m
or

$$t_G = 20.07\, H \left(\frac{S}{Q_A}\right)^{0.83} \tag{12}$$

where $[t_G]$ = sec ; $[S]$ = m²/m³ ; $[Q_A]$ = m³/m²/d ; $[H]$ = m

or

$$t_G = 0.335\, H \left(\frac{S}{Q_A}\right)^{0.83} \tag{12a}$$

where $[t_G]$ = min ; $[S]$ = m²/m³ ; $[Q_A]$ = m³/m²/d ; $[H]$ = m

For the porcelain spheres of 3 - in diameter

$$t_G = 1.5\, H \frac{(\nu)^{0.70}}{g^{1/3}} \left(\frac{S}{Q_A}\right)^{0.53} \tag{13}$$

or with $\nu = 1.31 \times 10^{-6}$ m²/sec and $g = 9.81$ m/sec²

$$t_G = 46.7 \times 10^{-3} H \left(\frac{S}{Q_A}\right)^{0.53} \tag{14}$$

where $[t_G]$ = sec ; $[S]$ = m²/m³ ; $[Q_A]$ = m³/m²/sec ; $[H]$ = m

or

$$t_G = 19.29\ H\left(\frac{S}{Q_A}\right)^{0.53} \tag{15}$$

where $[t_G]$ = sec ; $[S]$ = m^2/m^3 ; $[Q_A]$ = $m^3/m^2/d$; $[H]$ = m

or

$$t_G = 0.321\ H\left(\frac{S}{Q_A}\right)^{0.53} \tag{15a}$$

where $[t_G]$ = min ; $[S]$ = m^2/m^3 ; $[Q_A]$ = $m^3/m^2/d$; $[H]$ = m

Howland's equation for a bed of spheres of rectangular arrangement can also be written in this form by introducing the concept of specific surface area S. Consider again a cyclindirical bed of depth H and diameter D. The number of spheres in the bed is given by Eq. 4. Percentage of the void volume $=\left(1-\frac{\pi}{6}\right)\times 100$

Volume of a sphere $= \frac{4}{3}\pi a^3$

Surface area of a sphere $= 4\pi a^2$

Hence the total surface area of spheres per unit volume of the bed becomes :

$$S = \frac{(\pi D^2/4)\, H\, \pi/6}{(4/3)\,\pi a^3} \cdot \frac{4\pi a^2}{(\pi D^2/4)\, H} = \frac{\pi}{2a} \tag{16}$$

or

$$a = \frac{\pi}{2S} \tag{16a}$$

Substituting Eq. 16 a into Eq. 6 it follows :

$$t = 1.3\, H\left(\frac{3\nu}{g}\right)^{1/3} \frac{(2\pi)^{2/3}}{4^{2/3}\left(\frac{\pi}{2}\right)^{2/3}} \left(\frac{S}{Q_A}\right)^{2/3}$$

$$= 1.3\, H\left(\frac{3\nu}{g}\right)^{1/3}\left(\frac{S}{Q_A}\right)^{2/3} \tag{17}$$

If the numerical values of ν and g are introduced here, one obtains :

$$t = 0.958 \times 10^{-2} H \left(\frac{S}{Q_A} \right)^{2/3} \qquad (18)$$

where

$$[t] = \text{sec} \; ; \; [S] = \text{m}^2/\text{m}^3 \; ; \; [Q_A] = \text{m}^3/\text{m}^2/\text{sec.} \; ; \; [H] = \text{m}$$

or

$$t = 18.729 \, H \left(\frac{S}{Q_A} \right)^{2/3} \qquad (19)$$

where

$$[t] = \text{sec} \; ; \; [S] = \text{m}^2/\text{m}^3 \; ; \; [Q_A] = \text{m}^3/\text{m}^2/\text{d} \; ; \; [H] = \text{m}$$

or

$$t = 0.312 \, H \left(\frac{S}{Q_A} \right)^{2/3}$$

where

$$[t] = \text{min} \; ; \; [S] = \text{m}^2/\text{m}^3 \; ; \; [Q_A] = \text{m}^3/\text{m}^2/\text{d} \; ; \; [H] = \text{m}$$

A Mathematical Model to Study the Effect of the Liquid Held at the Points of Contacts between the Grains of Fill Material by Capillary Forces upon the Flow Time

Any analysis of flow over a bed must be based upon ideal particles, hence the various modes of packing of uniform spheres must be carefully investigated.

From the six modes of packing of spheres studied by Graton and Fraser, the only arrangement that Eq.1 is aplicable to, is a rectangular arrangement. (4), (5). There are those countless variaties of assembly in which no orderly or systematic repetitive arrangement can be discovered. All those terms may be grouped together under the general term of chance packing. However experiments show that in containers of simple shape and tend to hundreds of times the sphere diameter, rhombic colonies of complete stability predominate in the bed. From this point of view only such a model as shown in *Figure 3 a* and *Figure 3 b* can represent the porous media (5). In this modified rhombic arrangement the spacing between the spheres is adjusted to the observed porosity. The flow phenomenon in this packing is extremely complex and quite different from that of rectangular arrangement. Since no theoretical study known to the author exists on this subject, this study has been carried on for this purpose.

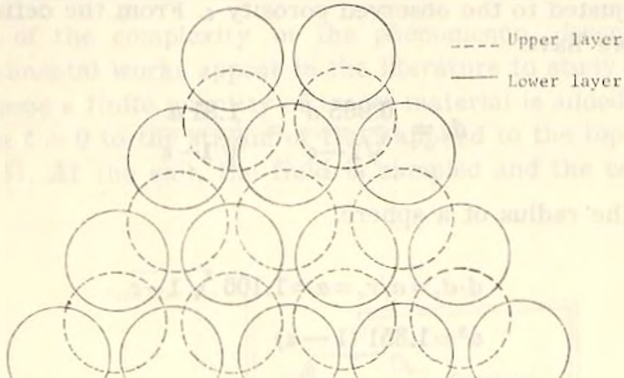

Figure 3 a. The proposed model for flow over filter media

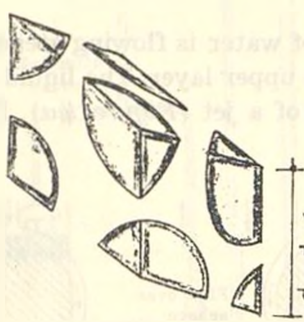

Figure 3. b. The unit cell of the idealized assemblage of spheres studied by the present theory

Let us consider two layers of such an assemblage of spheres lying on a horizontal starting plane (Figure 3 a). Spheres plotted with the heavy lines indicate the first lower layer and those designated with the dotted lines show the second layer which is on the top. The circles in Figure 3 a are the horizontal projections of all layers of the assemblage, because the projections of the other layers coincide with those shown in the figure.

We now consider a porous media represented by spheres placed in hexagonal array according to the rhombic arrangement. The unit cell of this idealized assemblage has been shown in Figure 3 b. The adjacent grain centers are equidistant and at a distance of $d_e = 2\,r_e = d + \delta$, in which d is the sphere diameter and δ is the distance between sphere surfaces, adjusted to the observed porosity ε. From the definition of the porosity ε, we have

$$d_e = \frac{0\,905\,d}{\sqrt[3]{1-\varepsilon}} = \frac{1.81\,a}{\sqrt[3]{1-\varepsilon}} \tag{20}$$

where a is the radius of a sphere.

$$d/d_e = a/r_e = \alpha = 1.106\,\sqrt[3]{1-\varepsilon} \tag{21}$$

$$\alpha^3 = 1.351\,(1-\varepsilon) \tag{22}$$

The distance between the centers of the two layers is

$$h = 2\,r_e\sqrt{\frac{2}{3}} = \frac{1.477\,a}{\sqrt[3]{1-\varepsilon}} \tag{23}$$

Assume that a sheet of water is flowing steadily down over the surfaces of the spheres of the upper layer. The liquid leaves at the very bottom of the sphere in form of a jet (*Figure 4 a*). If the rate of flow Q is

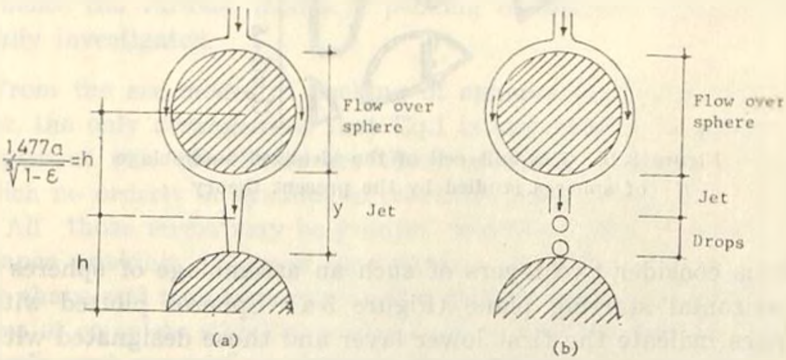

Figure 4. Types of flow over the proposed model of the filter media

sufficiently great, the jet length l will reach the top of sphere of the layer below *(Figure 4 a)*. Otherwise the break - up of the liquid jet will occur and drops will form *(Figure 4 b)*. There is a certain rate of flow above which a jet forms. At low velocities drops will form at the very bottom of the sphere of the upper layer, as soon as the liquid leaves it.

Because of the complexity of the phenomenon described above, mainly experimental works appear in the literature to study the subject. For this purpose a finite quantity of tracer material is added instantaneously at time $t = 0$ to the stream of fluid applied to the top of the bed. (see *Figure 5*). At the exit, the fluid is sampled and the concentration

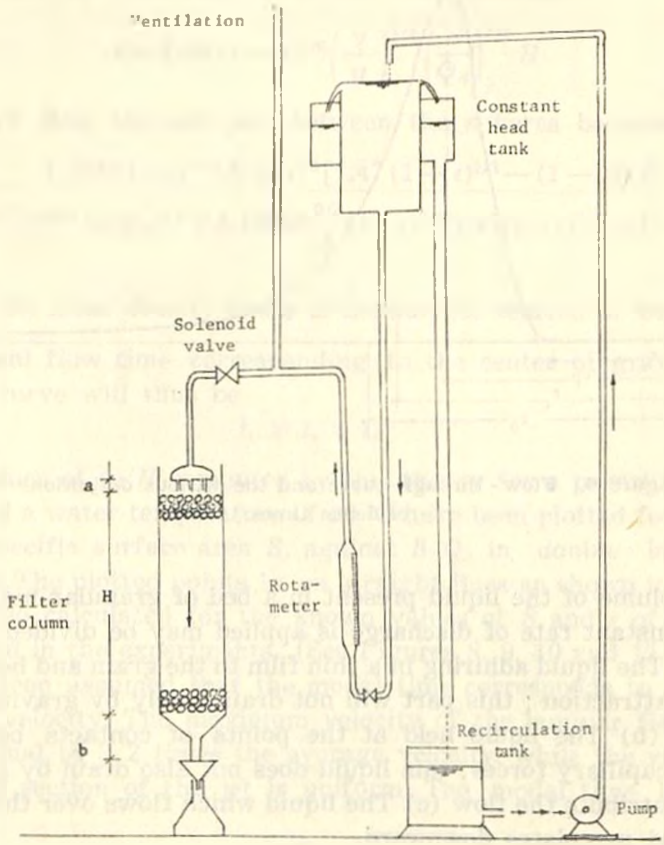

Figure 5. Schematic diagram of flow system

of tracer in each sample is determined. A Curve of tracer concentration c versus the elapsed time t is plotted. (*Figure 6*). Then, the mean residence time is, by definition, the displacement of the centroid of the area under the curve from the c axiss. Of course, other definitions are also possible as shown in (Figure 6). They are the modal time t_p, the median time t_G, and the theoretical detention time t_D which is the volume of liquid over spheres divided by the rate of flow. Relations among these various definitions need to be investigated. This is specially important in order that one can compare the results of a theoretical equation such as Howland's equation with the residence times as defined in (Figure 6).

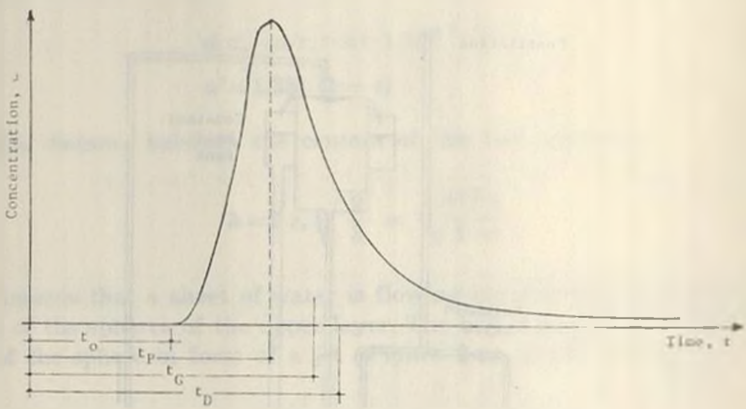

Figure 6. Flow - through curve and the various definitions of residence times

The volume of the liquid present in a bed of granular material over which a constant rate of discharge is applied may be divided into three parts: (a) The liquid adhiring in a thin film to the grain and held there by molecular attraction ; this part will not drain freely by gravity from the material ; (b) The liquid held at the points of contacts between the grains by capillary forces; this liquid does not also drain by gravity and will not contribute the flow (c) The liquid which flows over the grains by gravity and percolates downward.

After a certain time of flow establishment, rate of discharge applied to the bed becomes equal to the outflow rate. (see Figure 5). After this moment, if the influent valve is closed and if the liquid drained freely from the bed is collected, one obtains the volume described above under the item (c). It will be designated V_G. The volume of the liquid which the bed will retain against the pull of gravity is the sum of liquid volumes explained under the items (a) and (b) above. It will be designated V_D. So the total volume of the liquid stored in the bed for a constant rate of flow, becomes

$$V = V_D + V_G \tag{24}$$

In this study it has been assumed that the flow times t_G and t_p defined in Figure 6 can be computed by means of the average and maximum velocity of the laminar flow over spheres and jets (7). Time of flow over the spheres of the model described above has been calculated by Eq. 1 and the following result has been obtained :

$$t_s = 1.65\,(1-\varepsilon)^{1/8}\left(\frac{\nu}{g}\right)^{1/8}\left(\frac{S}{Q_A}\right)^{7/8} H \tag{25}$$

Time of flow through jets between the spheres becomes

$$t_j = \frac{1.3253\,(\nu/g)^{1/4}(S/Q_A)^{7/8}\left[1.47\,(1-\varepsilon)^{2/3}-(1-\varepsilon)\right] H}{(1-\varepsilon)^{2/3}\left\{g^{13/8}(S/Q_A)^{1/8}+8.45952\,\frac{\rho g}{\sigma}(1-\varepsilon)^{2/3}\left[1.47(1-\varepsilon)^{2/3}-(1-\varepsilon)\right](\nu/g)^{1/4}\right\}} \tag{26}$$

where ρ is the mass density and σ is the surface tension of the liquid.

The total flow time corresponding to the center of gravity of the dispersion curve will thus be

$$t_G = t_s + t_j \tag{27}$$

The values of t_G/H computed by this theory for a porosity value of $\varepsilon = 0.40$ and a water temperature of 20 °C have been plotted for different values of specific surface area S, against S/Q_A in double logharitmic coordinates. The plotted points lie on straight lines as shown in Figure 7. They are also calculated for the known values of S and ε of the filter columns used in the experiments. (See Figures 8, 9, 10 and 11).

It has been assumed that the modal time corresponds to the maximum flow velocity. The maximum velocity of the laminar flow over a sphere is equal to 3/2 times the average velocity, while the velocity on a horizontal section of the jet is uniform. The modal time hence becomes

$$t_p = \frac{2}{3}\,t_s + t_j \tag{28}$$

Eq. 26 indicates that t_j/H and hence t_G/H increase with S untill S receives a certain value. When S becomes greater than that, t_j/H and

t_G/H start to decrease. This value can be calculated by differentiating t_i/H with respect to S, and then equalizing it to zero. It is approximately $S \approx 1000$ m²/m³. When $S > 1000$ m²/m³, *i. e.* for fine materials, the first term in the denominator of Eq. 26 predominates and the second term can be neglected. So Eq. 26 can be written as :

$$t_i = \frac{1.3253\left(\frac{\nu}{g}\right)^{1/4}\left[1.47(1-\varepsilon)^{2/3}-(1-\varepsilon)\right]\left(\frac{S}{Q_A}\right)^{3/4} H}{(1-\varepsilon)^{2/3}\, S^{1/4}} \qquad (29)$$

Theoretically it means that time of flow does not depend upon surface tension for large values of S. From Eq. 29 it results that t_i/H and hence t_G/H decrease when S increases. As a limit case $t_i = 0$ and hence $t_G = t_S$ when $S \rightarrow \infty$. However no experimental run has been performed for such large values of S and therefore nothing is known on the validity of this theoretical result.

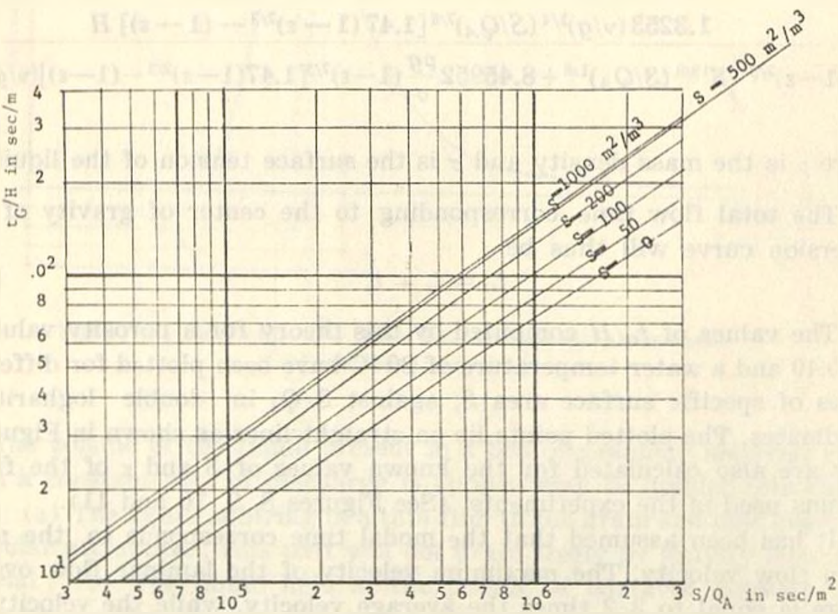

Figure 7. Time of flow t_G/H as a function of S/Q_A for different values of specific surface areas, S. (Porosity = 0.40 ; Water temperature = 20°C)

On the contrary, at low values of S the second term in the denominator of Eq. 26 predominates and the first term can be neglected. That is to say, surface tension plays an important part. In this case Eq. 26 can be expressed as

$$t_l = 0.157 \left(\frac{\nu}{g}\right)^{1/8} \frac{\sigma}{\rho g} \frac{S^{11/8}\left(\frac{S}{Q_A}\right)^{7/8} H}{(1-\varepsilon)^{4/3}} \tag{30}$$

Eq. 30 shows that t_l/H decreases when S becomes smaller. As a limit case $t_l = 0$ and hence $t_G = t_S$ again when $S = 0$ (*See Figure* 7).

If the porosity of the natural materials is assumed to be $\varepsilon = 0.40$, Eq. 26 can be written in the following form for a water temperature of 20°C and $\nu = 10^{-6} \times 1.008\ m^2/sec$; $(\nu/g)^{1/4} = 1.7916 \times 10^{-2}$; ; $\sigma/\rho g = 0.074 \times 10^{-4}\ m^2$

$$t_l = \frac{1.4865 \times 10^{-2} S^{11/8} (S/Q_A)^{7/8} H}{S^{13/8} (S/Q_A)^{1/8} + 48345} \tag{31}$$

in which

$$[t_l] = \sec\ ;\ [H] = m\ ;\ [S] = m^2/m^3\ ;\ [S/Q_A] = \sec/m^2$$

With the same temperature ad the data given above, Eq. 25 becomes :

$$t_s = 10^{-3} \times 7.291 (S/Q_A)^{2/3} H \tag{32}$$

where

$$[t_s] = \sec\ ;\ [H] = m\ ;\ [S/Q_A] = \sec/m^2$$

The validity of this model was demonstrated by experiments using NaCl as tracer while draining the liquid by gravity through columns filled with 3.30 mm, 3.56 mm, 9.54 mm, 16.7 mm and 35.0 mm diameter spheres (*See Figure 8, 9, 10 and 11*). In these experiments, several drops of NaCl solution were suddenly added into the inlet of filter, and the electrical conductivity of the effluent was recorded over time. Later a solenoid valve was closed and the drained volume of liquid V_G was measured. The volume of the liquid V_D which the bed will retain against

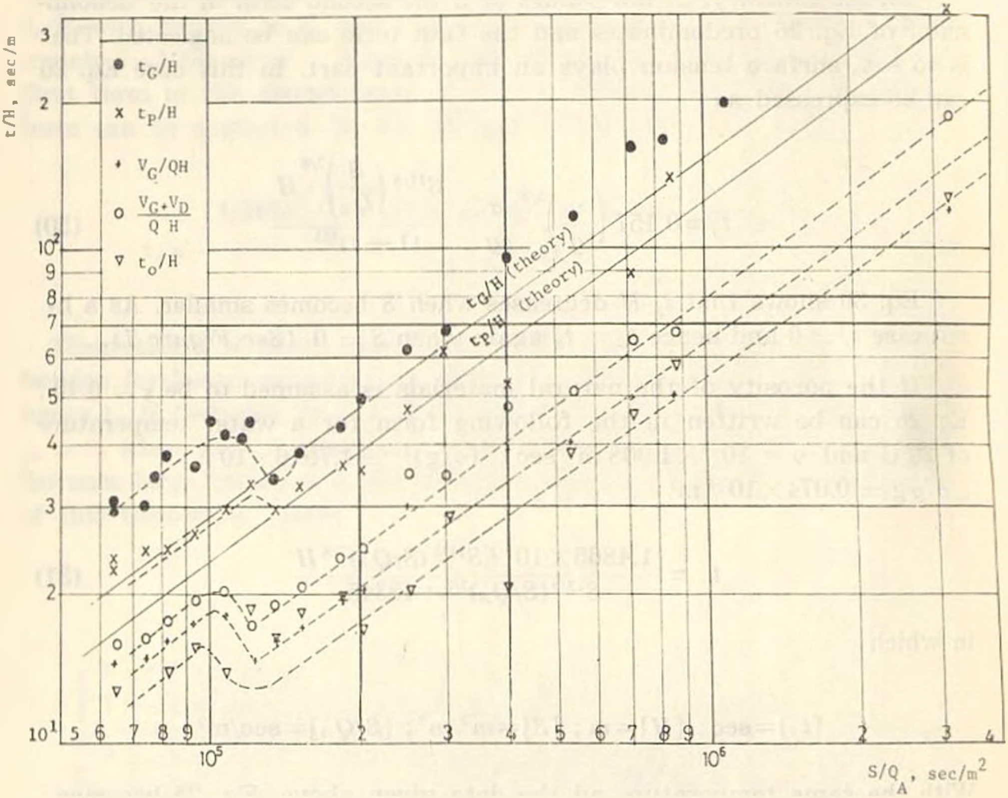

Figure 8. Various flow times measured in a packed column filled with lead shots of 3.3 mm diameter and the results of the present theory developed in this paper. ($\varepsilon = 0.40$; $H = 0.48$ m ; $S = 1090.9$ m²/m³ ; $T = 20$ °C)

the pull of gravity was also measured. The modal time, the time to the center of gravity, the time to the beginning of the dispersion curves and the calculated times of V_G/Q and $(V_G+V_D)/Q$ have been plotted against S/Q_A in double logharitmic coordinates. S is specific surface area, Q is rate of flow, and Q_A is surface load. The plotted points generally lie on straight lines in agreement with the theory developed herein. However they are on a curve concave downward in a certain region corresponding to a transition between the drop formation and jetting, again in confirmation of the theory developed.

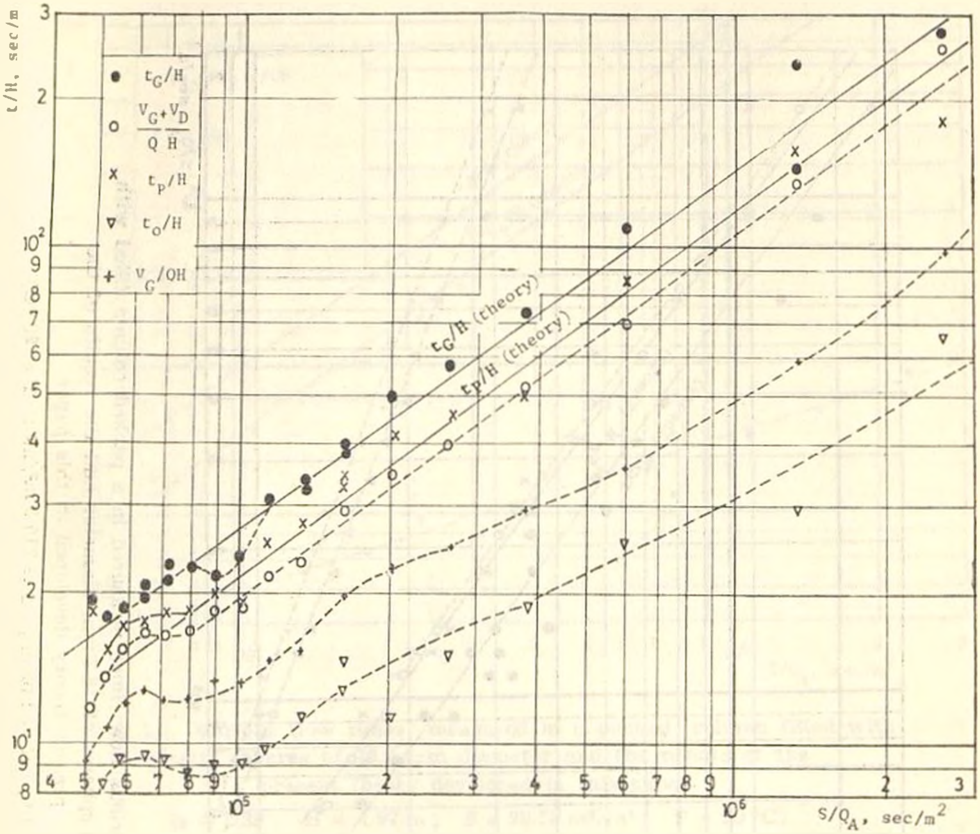

Figure 9. Various flow times measured in a packed column filled with steel spheres of 9.54 mm diameter and the results of the peresent theory developed in this paper. ($\varepsilon = 0.423$; $H = 0.80$ m ; $S = 362.9$ m^2/m^3 ; $T = 20$ °C)

Experiments show that the times to the beginning of the dispersion curve are related to the volume of liquid freely drained from the bed. ($t_0 \cong V_G/Q$). On contrast, the modal time corresponds to the total volume of the liquid contained inside the filter divided by the rate of flow [$t_p \cong (V_G+V_D)/Q$].

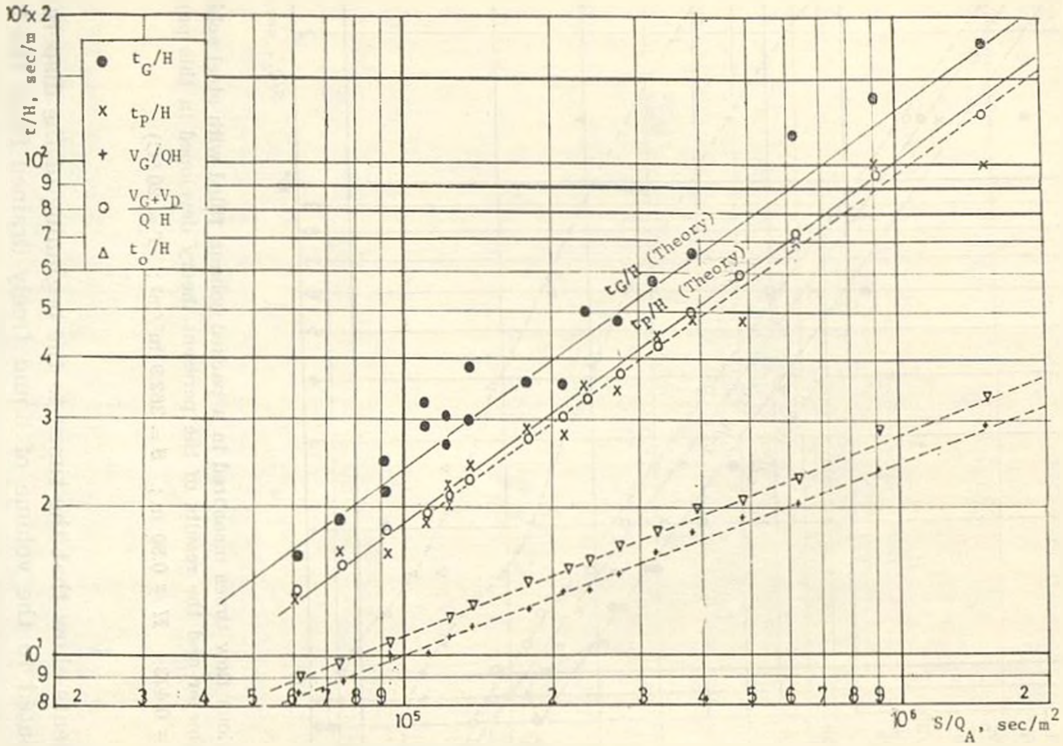

Figure 10. Various flow times measured in a packed column filled with porcelain spheres of 16.7 mm diameter and the results of the present theory developed in this paper.
($\varepsilon = 0.40$; $H = 1.02$ m ; $S = 215.3$ m^2/m^3 ; $T = 20$ °C)

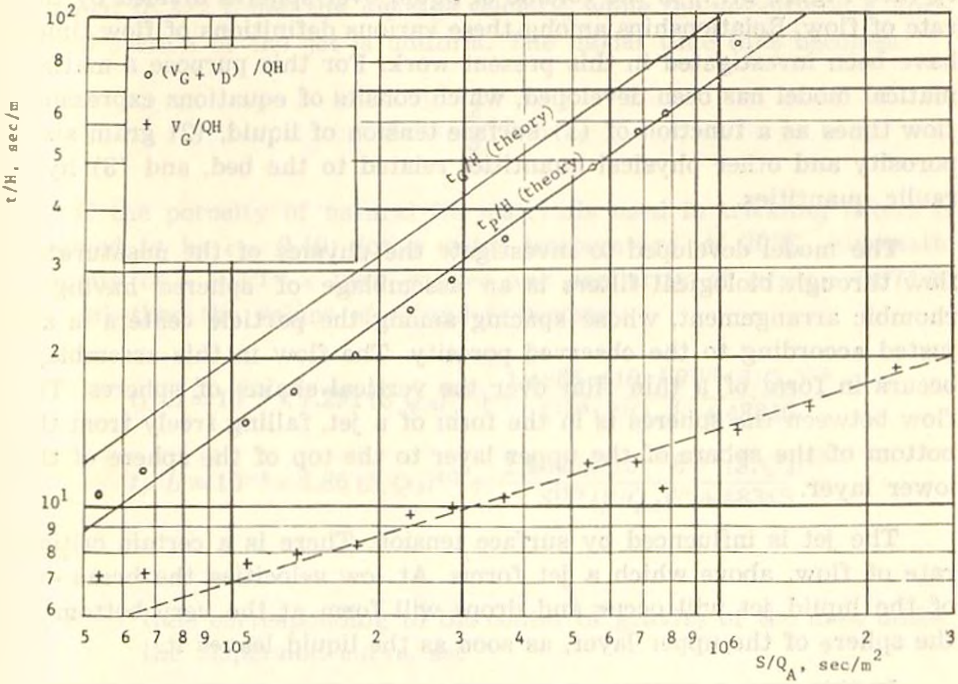

Figure 11. Various flow times measured in a packed column filled with plastic spheres of 36.7 mm diameter and the results of the present theory developed in this paper.
($\varepsilon = 0.39$; $H = 1.97$ m ; $S = 99.73$ m²/m³ ; $T = 20$ °C)

Summary and conclusion

Although a number of methods have been proposed for the design of trickling filters, there remains a need to study the effects of different factors upon the biological efficiency of filters.

In this paper, the parameters characterizing the dispersion curves have been studied because the dispersion of waste matter in the liquid flowing through biological filters has a definite influence on the biological reactions.

These parameters correspond to the characteristic points of a curve of tracer concentration versus the elapsed time. They include the mean residence time t_G, the modal time t_P, and the theoretical detention time

t_D (See Figure 6). t_D is the volume of liquid over spheres divided by the rate of flow. Relationships among these various definitions of flow times have been investigated in this present work. For this purpose a mathematical model has been developed, which consits of equations expressing flow times as a function of (1) surface tension of liquid, (2) grain size, porosity and other physical quantities related to the bed, and (3) hydraulic quantities.

The model developed to investigate the physics of the unsaturated flow through biological filters is an assemblage of spheres having a rhombic arrangement, whose spacing among the particle centers is adjusted according to the observed porosity. The flow in this assemblage occurs in form of a thin film over the vertical chains of spheres. The flow between the spheres is in the form of a jet, falling freely from the bottom of the sphere of the upper layer to the top of the sphere of the lower layer.

The jet is influenced by surface tension. There is a certain critical rate of flow, above which a jet forms. At low velocities the break - up of the liquid jet will occur and drops will form at the very bottom of the sphere of the upper layer, as soon as the liquid leaves it.

In this way, a mutual correspondance has been established between the flow over a single sphere - forming jet under the influence of gravity and the extremely complex flow through filters. With the aid of this model, it has been possible to translate to the unsaturated flow through beds, all results related to a single sphere and a jet falling freely under the influence of gravity. The velocity profile in flow over a sphere is a parabola, while the velocity in a horizontal section of the jet is uniform. So it has been possible to clarify the mechanism for forming the flow, and to express the various parameters characterizing the dispersion of soluble matter through beds in terms of the properties of the media and the liquid.

The time t_G corresponding to the center of gravity of the area under the dispersion curve has been calculated by means of the average velocity of the flow. t_G is the summation of flow time t_S over spheres and flow time t_j through jets:

$$t_G = t_S + t_j$$

It has been assumed that the modal time corresponds to the maximum flow velocity. The maximum velocity of the laminar flow over a sphere is equal to 3/2 times the average velocity, while the velocity on a horizontal section of the jet is uniform. The modal time thus becomes:

$$t_P = \frac{2}{3} t_S + t_i$$

If the porosity of natural fill materials used in trickling filters is assumed to be $\varepsilon = 0.40$; for a water temperature of 20°C, kinematic viscosity of $\nu = 10^{-6} \times 1.008$ m²/sec and capilarity of $\sigma/\rho g = 0.074 \times 10^{-4}$ m², then the values of t_G and t_P become :

$$t_G/H = 10^{-3} \times 7.291\,(S/Q_A)^{2/3} + \frac{1.4865 \times 10^{-2}\, S^{11/8}\,(S/Q_A)^{7/8}}{S^{13/8}\,(S/Q_A)^{1/8} + 48345}$$

$$t_P/H = 10^{-3} \times 4.86\,(S/Q_A)^{2/3} + \frac{1.4865 \times 10^{-2}\, S^{11/8}\,(S/Q_A)^{7/8}}{S^{13/8}\,(S/Q_A)^{1/8} + 48345}$$

where

t_G = time corresponding to the center of gravity of the area under the dispersion curve, sec

t_P = Modal time, sec

S = Specific surface area of fill materials, m²/m³

H = Depth of filter, m

Q_A = Hydraulic loading rate, m³/sec/m²

When $S \geqq 1000$ m²/m³, i.e., for fine materials, the first term in the denominater of the above expression predominates. Therefore when $S \rightarrow \infty$ $t_G = t_S$. Conversely, at low values of S, the second term in the denominator predominates and the surface tension has a significant effect. As a limit case again $t_G = t_S$ when $S \rightarrow 0$

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