Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2020) 28: 591 – 605
© TÜBİTAK
doi:10.3906/elk-1810-46

Adaptive blind equalization for a MIMO chaotic communication system

Gökçen ÇETİNEL[1,*], **Cabir VURAL**[2]
[1]Department of Electrical and Electronics Engineering, Faculty of Engineering, Sakarya University, Sakarya, Turkey
[2]Department of Electrical and Electronics Engineering, Faculty of Engineering,
Marmara University, İstanbul, Turkey

Received: 09.10.2019 • **Accepted/Published Online:** 07.08.2019 • **Final Version:** 28.03.2020

Abstract: There exist few blind solutions for chaotic MIMO channel equalization. In this work, a chaotic MIMO channel equalization framework is proposed. The objective function to be minimized in the proposed solution is obtained by adopting the objective function developed for chaotic SISO channel equalization. Furthermore, an optimum filter that minimizes the proposed cost function is designed to recover chaotic input signals assuming that the channel is known. The stationary point of the adaptive solution is equal to the optimal filter if the adaptive filter coefficients change sufficiently slowly. The adaptive solution is contrasted with the optimum filter in terms of mean-square error and bit error rate performances. In addition, the proposed solution reconstructs chaotic input signals at the same time. Consequently, it can be applied to multiple signal separation problems as well.

Key words: Adaptive equalizers, blind equalizers, chaotic communication, MIMO equalization

1. Introduction

Chaos has attracted attention from researchers including physicists and engineers over the last two decades. Signal processing and communications are two of the most popular areas for researchers interested in chaos. The earliest techniques for chaos-based communication systems are chaotic modulation and chaotic masking [1, 2].

Irregularity, aperiodicty, and difficult accurate prediction over long periods are the main characteristics of chaotic signals. In the continuous case, differential equations can be used to produce chaotic signals, while iterative maps can be used to generate them in the discrete case. Additional prominent properties of chaotic signals are that their autocorrelation functions look like an impulse function and the cross-correlation function between two different chaotic signals has negligible components [2]. Spread spectrum, multiuser, and secure communications are the three main applications of chaotic signals resulting from their aforementioned properties [3–9].

In a chaos-based communication application, an information-carrying message is sent over a channel after being applied to the chaotic modulator. If the propagation channel is not ideal, the transmitted signal is distorted and must be corrected before the chaotic demodulation process. Distortion correction is known as channel equalization. Channel equalization in which the parameters of the channel are unknown is called blind channel equalization. Such methods utilize channel capacity more efficiently compared to the methods using training signals to estimate the unknown channel.

*Correspondence: gcetinel@sakarya.edu.tr

Channel equalization algorithms are classified as single-input single-output (SISO) and multiple-input multiple-output (MIMO). There exist numerous nonchaotic (equivalently classical) blind equalization algorithms. The approach based on inverse filtering criteria (IFC), super-exponential algorithm (SEA), and constant modulus algorithm (CMA) are the well-known blind equalization solutions in the case of SISO [10].

Blind equalization of MIMO channels with a finite impulse response (FIR) is encountered frequently. Second and higher order statistics-based blind equalization algorithms developed by utilizing some properties of the source signals exist for MIMO systems [11].

In classical digital communication systems, proposed statistics-based SISO and MIMO channel equalization algorithms may not yield desired results for chaotic systems since a discrete-time chaotic signal is deterministic. Therefore, algorithms must be derived by exploiting inherent properties of chaotic signals to obtain acceptable performance for chaotic communication systems.

Recently, chaos-based blind equalization algorithms have been developed. However, a SISO model is assumed for the channel in most works [12–17]. For the MIMO channel case, there are only a few nonblind algorithms developed by using training sequences [18, 19]. In previous studies [18, 20], a trellis diagram is substituted for a multiuser communication system. The corresponding trellis diagram is obtained by using a special representation for chaos signals called symbolic dynamic representation. Then the maximum likelihood estimates of the transmitted signals are obtained by the Viterbi algorithm. Correlation delay shift keying (CDSK) using the 2×2 MIMO technique to increase the capacity of data is proposed [19]. Two distinct state-space models are combined with a dual unscented Kalman filter for MIMO systems [21]. The filter estimates the channel coefficients undergoing fading.

The chaotic MIMO blind equalization problem was addressed first by Cetinel and Vural [22]. The present study is an extended version of that previous study in several respects such as inclusion of the derivation of the optimum filter design, stationary point analysis, and more simulation studies.

In MIMO communication systems, multiuser interference (MUI) is another interference that degrades the receiver performance together with intersymbol interference (ISI). Hence, channel equalization must be performed by eliminating the effects of both ISI and MUI to achieve reliable and high-speed communication.

In the present study, a chaotic MIMO blind equalization algorithm is proposed. The proposed objective function is composed of two terms. The first one is obtained from nonlinear predictability of chaotic signals and overcomes the effect of ISI. The second one, which minimizes the effect of MUI, is based on the orthogonality property of chaotic signals. Furthermore, an optimum filter that gives the best equalization results is designed to evaluate the performance of the proposed adaptive algorithm. Finally, the conditions under which the stationary points of the proposed algorithm are equal to those of the optimum filter are obtained.

The present manuscript consists of the following sections. Section 2 states the problem to be solved. If the impulse response of the channel is known, an optimum equalization filter, which is discussed in Section 3, can be derived. The proposed chaos-based MIMO adaptive blind equalization algorithm is discussed in Section 4 and its stationary point analysis is performed in Section 5. Section 6 gives the results obtained by computer simulations. Conclusions are drawn in Section 7.

2. MIMO chaotic blind channel equalization problem

Chaos-based communication systems take into account the inherent properties of chaotic signals to achieve optimum transmission accuracy. In these systems chaotic input signals can be generated by differential equations or iterative maps depending on the working principle. Similar to the classical communication systems, chaos-based communication systems consist of three main blocks: modulator, channel, and demodulator. A model of a chaos-based MIMO communication system is given in Figure 1. First, chaotic modulators convert information carrying messages $m_j[n]$ into chaotic signals $s_j[n]$, where j=1, 2, ...p. Then modulated signals are sent over FIR channels with impulse responses $c_{ij}[n]$, where i=1,2,...,M. Zero-mean additive white Gaussian noise (AWGN) $z_i[n]$ corrupts the output of the channels. The tent, saw tooth, logistic, or Chebyshev mappings are the most frequently used maps to realize chaotic modulators. A nonlinear dynamical equation such as $s[n] = f(s[n-1], ..., s[n-d])$ is used to express these maps, where d is the embedding dimension of the system.

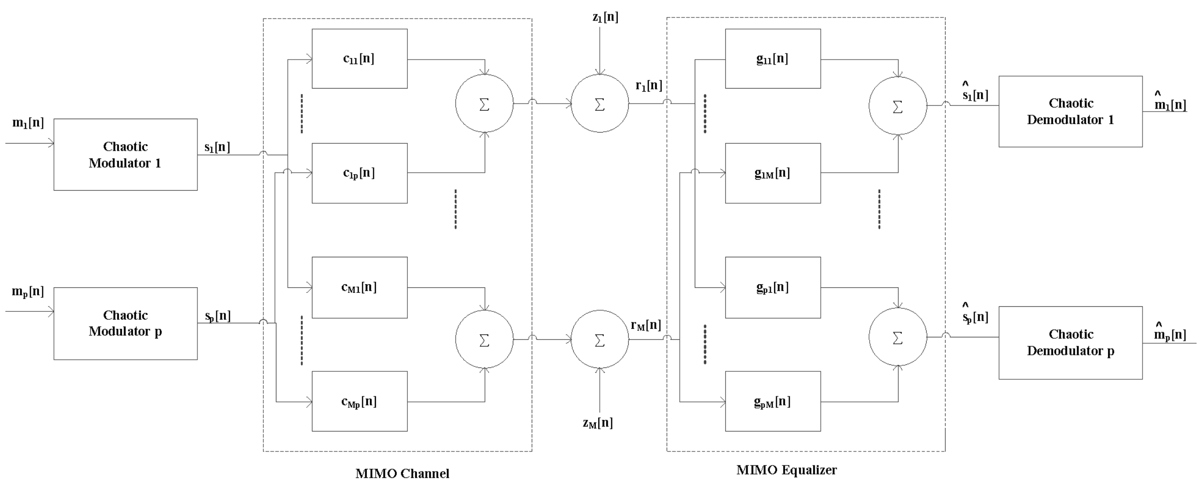


Figure 1. MIMO chaotic communication system model.

At time n, let $s_j[n]$ and $c_{ij}[n]$, where i=1,2,...M and j=1, 2, ...p, denote the j-th input vector and the channel response coefficient vector, respectively. They are given by

$$\boldsymbol{s}_j[n] := [s_j[n]\ s_j[n-1]\ ...\ s_j[n-L_1+1]]^T \tag{1}$$

$$\boldsymbol{c}_{ij} := [c_{ij}[0]\ c_{ij}[1]\ ...\ c_{ij}[L_1-1]]^T \tag{2}$$

In (1) and (2), L_1 is the length of the channel. With the definitions in (1) and (2), the i-th received signal can be expressed as

$$r_i[n] = \sum_{j=1}^{p} \boldsymbol{c}_{ij}^T\, \boldsymbol{s}_j[n] + z_i[n] \tag{3}$$

(3) can be reorganized in the following form:

$$r_i[n] = \boldsymbol{c}_{i1}^T\, \boldsymbol{s}_1[n] + \sum_{j=2}^{p} \boldsymbol{c}_{ij}^T\, \boldsymbol{s}_j[n] + z_i[n] \tag{4}$$

Substituting (1) and (2) in (4) gives

$$r_i[n] = c_{i1}[0]\, s_1[n] + \sum_{k=1}^{L_1-1} c_{i1}[k]\, s_1[n-k] + \sum_{\substack{j=1\\ j\neq 1}}^{p} \boldsymbol{c}_{ij}^T\, \boldsymbol{s}_j[n] \; + z_i[n], \tag{5}$$

where the first term on the right-hand side of the equality is the desired term for the corresponding transmitted signal. However, the second and third terms are undesired terms resulting from a nonideal transmission channel and other users. They are called ISI and MUI for the corresponding signal, respectively. ISI and MUI can be defined for other transmitted signals similarly. Undesired terms must be eliminated to reconstruct the transmitted messages reliably.

As illustrated in Figure 1, a MIMO equalizer composed of FIR filters with length L_2 is realized to obtain reliable signals for chaotic demodulators. In the rest of this section, relationships among the signals in Figure 1 will be written in vector-matrix form by defining appropriate vectors and matrices. The vector-matrix representation will be useful when deriving the optimum and the proposed adaptive filters. Let the received signal vector at the input of the MIMO equalizer be given as

$$\boldsymbol{r}[n] = [r_1[n] ... r_M[n] ... r_1[n-L_2+1] ... r_M[n-L_2+1]] \tag{6}$$

It is possible to write the received signal vector with respect to the channel coefficients and the chaotic input signals by using (3). At time n let $\mathbf{C}$, $\boldsymbol{s}[n]$, and $\boldsymbol{z}[n]$ denote the channel coefficient matrix, input signal vector, and noise vector, respectively. They are defined as

$$\mathbf{C} := \begin{bmatrix} \mathbf{C}[0] & \ldots & \mathbf{C}[L_1-1] & & \ldots & & 0 \\ \vdots & & \mathbf{C}[0] & \ldots & \mathbf{C}[L_1-1] & & 0 \\ 0 & & \ldots & & \boldsymbol{C}[0] & \ldots & \mathbf{C}[L_1-1] \end{bmatrix}^T, \mathbf{C}[k] := \begin{bmatrix} c_{11}[k] & \cdots & c_{1p}[k] \\ \vdots & & \vdots \\ c_{M1}[k] & \cdots & c_{Mp}[k] \end{bmatrix} \tag{7}$$

$$\boldsymbol{s}\,[\text{n}]: = [s_1[n]\ \ldots\ s_p[n], \ldots, s_1[n-L_2-L_1+2]\ \ldots s_p[n-L_2-L_1+2]] \tag{8}$$

$$\boldsymbol{z}\,[n] := [z_1[n]\ \cdots z_M[n] \cdots z_1[n-L_2+1] \cdots z_M[n-L_2+1]] \tag{9}$$

Then the vector corresponding to the received signal can be expressed as

$$\boldsymbol{r}[n] = \boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}\,[n] \tag{10}$$

It is also possible to write the connection between the estimated chaotic input signals and the received signals in vector-matrix notation. In Figure 1, M corrupted signals are applied to the MIMO equalizer block to recover chaotic input signals. Let the MIMO equalizer coefficient vector $\boldsymbol{g}_{ji}$ and i-th received signal vector $\boldsymbol{s}_i[n]$ be defined as

$$\boldsymbol{g}_{ji} := [g_{ji}[0]\ g_{ji}[1]\ ...\ g_{ji}[L_2-1]]^T,\ j = 1, 2, ...p. \tag{11}$$

$$\boldsymbol{r}_i[n] = [r_i[n] \cdots r_i[n-L_2+1]]^T,\ i = 1, 2, ...M. \tag{12}$$

Using these definitions leads to the following expressions for output of the *j*-th equalizer:

$$\hat{s}_j[n] = \sum_{i=1}^{M} \boldsymbol{g}_{ji}^T \, \boldsymbol{r}_i[n], \;\; j = 1, 2, ...p. \tag{13}$$

The estimated chaotic input signals vector at the output of the equalizer is given below

$$\hat{\boldsymbol{s}}[n] := [\hat{\boldsymbol{s}}_1[n] \, \ldots \, \hat{s}_p[n]] \tag{14}$$

By defining equalizer coefficient matrix **G** as

$$\mathbf{G}\boldsymbol{:} = [\mathbf{G}\,[0] \;\, \mathbf{G}\,[1] \; ...\mathbf{G}\,[L_2 - 1]^T, \quad \mathbf{G}\,[k] := \begin{bmatrix} g_{11}[k] & \cdots & g_{1M}[k] \\ \vdots & & \vdots \\ g_{p1}[k] & \cdots & g_{pM}[k] \end{bmatrix} \tag{15}$$

the equalizer output vector is expressed as

$$\hat{\boldsymbol{s}}[n] = \boldsymbol{r}[n] \, \mathbf{G} \tag{16}$$

Finally, substituting (10) in (16) yields

$$\hat{\boldsymbol{s}}[n] = \boldsymbol{s}[n] \, \mathbf{C} \, \mathbf{G} + \boldsymbol{z}[n] \, \mathbf{G} \tag{17}$$

In the remainder of the paper, (17) will be the basis to describe the optimum filter in the case of a known channel and to derive the adaptive filter in the case of an unknown channel.

3. Optimum filter design for known channels

In this section, an optimum filter will be designed when the characteristics of the channel are known. The first step is to construct an appropriate objective (cost) function. The objective function should be designed such that both ISI and MUI are eliminated when it attains its minimum value. The proposed cost function consists of two terms. One term tries to eliminate ISI while the other term aims to get rid of MUI. In the following discussion, d (embedding dimension) is assumed to be 1. Note that the corresponding chaotic map equation should be approximately satisfied if each equalizer output $\hat{s}_i[n]$, i=1,2,...p is a reliable estimate of the *i*-th transmitted signal. Under this condition, $f(\hat{s}_i[n-1])$ provides an estimate for $\hat{s}_i[n]$. Thus, the difference $\hat{s}_i[n] - f(\hat{s}_i[n-1])$ is a suitable error signal for ISI. The first term of the objective function is the sum of the squared error signals over all chaotic signals and it is called the nonlinear prediction error (NPE). Minimizing the NPE overcomes the effect of ISI as discussed previously [23]. Even if the NPE goes to zero, it is not possible to recover transmitted signals reliably because of MUI resulting from the contribution of other chaotic input signals. Thus, a term called cross-correlation term (CCT) is added to the NPE to eliminate the effect of MUI. The CCT is derived by exploiting the orthogonality of chaotic signals and it is given by

$$\sum_{\substack{i,j=1 \\ i \neq j}}^{p} \sum_{\delta=\delta_1}^{\delta_2} (\hat{s}_i[n] \, \hat{s}_j[n-\delta] \,)^2, \tag{18}$$

where $[\delta_1, \delta_2]$ is the channel delay span interval obtained from achievable delays among all chaotic inputs [24]. It is clear that the CCT is zero if and only if the outputs of the equalizer are orthogonal to each other. Thus, the offered cost function that eliminates ISI and MUI effects simultaneously is

$$J(\mathbf{G}) = \frac{1}{2}\sum_{i=1}^{p} \left(\hat{s}_i[n] - f(\hat{s}_i[n-1])\right)^2 + 2\sum_{\substack{i,j=1\\ i\neq j}}^{p}\sum_{\delta=\delta_1}^{\delta_2} \left(\hat{s}_i[n]\,\hat{s}_j[n-\delta]\right)^2, \tag{19}$$

where $\hat{s}_i[n]$ is the estimate of the i-th chaotic signal and $f(.)$ is a nonlinear chaotic mapping function used to generate chaotic information signals.

The standard process will be performed to design the optimum filter. First, the objective function is going to be reorganized in terms of the equalizer coefficients. For this purpose, (17) obtained in Section 2 can be used. Then the mean of the objective function will be calculated. Finally, stationary points of the objective function mean are the possible solutions. By (17), $\hat{\boldsymbol{s}}_i[n]$ can be reorganized in the following form:

$$\hat{\boldsymbol{s}}_i[n] = \boldsymbol{r}[n]\,\mathbf{G}_{i,n} = \boldsymbol{s}[n]\,\mathbf{C}\,\mathbf{G}_{i,n} + \boldsymbol{z}[n]\,\mathbf{G}_{i,n}, \tag{20}$$

where $\mathbf{G}_{i,n}$ represents the i-th column of $\mathbf{G}$. Substituting (20) in (19) yields

$$\begin{aligned} J(\mathbf{G}) = {} & \frac{1}{2}\sum_{i=1}^{p}\left\{\, (\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}[n])\mathbf{G}_{i,n} - f((\boldsymbol{s}[n-1]\,\mathbf{C} + \boldsymbol{z}[n-1])\mathbf{G}_{i,n-1})\,\right\}^2 \\ & + 2\sum_{\substack{i\neq j\\ j=1}}^{p}\sum_{\delta=\delta_1}^{\delta_2}\left\{\, (\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}[n])\mathbf{G}_{i,n}(\boldsymbol{s}[n-\delta]\,\mathbf{C} + \boldsymbol{z}[n-\delta])\,\mathbf{G}_{j,n-\delta}\right\}^2 \end{aligned} \tag{21}$$

The mean of $J(\boldsymbol{G})$ needs to be calculated. Then stationary points are obtained by equating the derivative of E $[J(\boldsymbol{G})]$ with respect to $\mathbf{G}_{i,n}$ to zero. However, it is possible to change the order of expectation and derivative operators for simplicity as discussed previously [25]. (21) is a function of a matrix. The derivative with respect to a matrix must be defined to proceed. Let $\mathbf{G}$ be a matrix whose column vector representation is $[\boldsymbol{G}_1, \boldsymbol{G}_{2,\dots,}\boldsymbol{G}_p]$, where p is the number of columns, and let $J(\boldsymbol{G})$ be a function of $\boldsymbol{G}$. Then $\mathrm{d}J(\mathbf{G})/\mathrm{d}\mathbf{G}$ is defined as $\frac{dJ(\boldsymbol{G})}{d\boldsymbol{G}} = \left[\frac{dJ(\boldsymbol{G})}{d\boldsymbol{G}_1}\dots\frac{dJ(\boldsymbol{G})}{d\boldsymbol{G}_p}\right]$. In other words, the derivative with respect to a matrix is also a matrix constructed by calculating derivatives of its columns. With this definition, we get

$$\begin{aligned} \frac{d\,\boldsymbol{J}(\mathbf{G})}{d\,\mathbf{G}_{i,n}} = {} & \left\{\, (\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}[n])\mathbf{G}_{i,n} - f((\boldsymbol{s}[n-1]\,\mathbf{C} + \boldsymbol{z}[n-1])\mathbf{G}_{i,n-1})\,\right\} \\ & (\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}[n])^T + 4\sum_{\substack{i\neq j\\ j=1}}^{p}\sum_{\delta=\delta_1}^{\delta_2}\left\{\, (\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}[n])\mathbf{G}_{i,n}(\boldsymbol{s}[n-\delta]\,\mathbf{C} + \boldsymbol{z}[n-\delta])\,\mathbf{G}_{j,n-\delta}\right\}(\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}[n])^T, \end{aligned} \tag{22}$$

where $\mathbf{G}_{i,n-1}$ and $\mathbf{G}_{i,n-\delta}$ are assumed to be independent of $\mathbf{G}_{i,n}$. The exact form of the optimum filter equation for a given chaotic map is obtained by substituting its mapping function in (22). Calculating $E\,[J(\mathbf{G})/d\mathbf{G}_{i,n}]$

and setting it to zero gives

$$\boldsymbol{G}^{*}_{i,n} = \boldsymbol{T}[n,n]^{-1}\frac{E(\,f(\boldsymbol{s}[n-1]\mathbf{C}+\boldsymbol{z}[n-1]\mathbf{G}_{i,n-1})(\mathbf{C}^{T}\boldsymbol{s}^{T}[n]+\boldsymbol{z}^{T}[n])}{1+4\sum\limits_{\substack{i,j=1\\ i\neq j}}^{p}\sum\limits_{\delta=\delta_1}^{\delta_2}\left(\boldsymbol{s}[n-\delta]\,\mathbf{C}+\boldsymbol{z}[n-\delta]\,\mathbf{G}_{j,n-\delta}\right)^{2}}, \tag{23}$$

where

$$\boldsymbol{T}[n,n] = \mathbf{C}\,\boldsymbol{s}^{T}[n]\,\boldsymbol{s}[n]\,\mathbf{C} + \boldsymbol{z}^{T}[n]\,\boldsymbol{z}[n] \tag{24}$$

and $\boldsymbol{G}_{i,n-1}$ and $\boldsymbol{G}_{j,n-\delta}$ are the corresponding columns of the equalizer coefficient matrix at time n-1 and n-δ, respectively. Note that when calculating the expectation $\boldsymbol{s}[n]$ and $\boldsymbol{z}[n]$ are assumed to be uncorrelated and $\boldsymbol{z}[n]$ is assumed to be the zero-mean AWGN vector. Having found $\boldsymbol{G}^{*}_{i,n}$ for i=1,2,...,p, the optimum filter coefficient matrix is obtained as $\mathbf{G}^{*} = \left[\mathbf{G}^{*}_{1,n}...\mathbf{G}^{*}_{p,n}\right]$. The optimum filter is not fixed. Hence, it must be calculated at each time, while the Wiener filter is fixed. The reason can be explained as follows: the Wiener filter uses the statistical characteristics of the transmitted signal that are fixed. In contrast, the chaotic equalizer uses nonlinear dynamics of the transmitted signal, which are time varying.

4. The proposed adaptive algorithm

Terms assumed to be known in the optimum filter design are unknown in practice. Therefore, an adaptive equalizer must be built. The cost function given in (19) is the basis to derive the proposed adaptive MIMO blind equalization algorithm. The steepest descent (SD) method will be exploited to update the MIMO equalizer coefficients because of its simplicity [25]. Thus, the form of the SD algorithm for our problem can be given as

$$\mathbf{G}_{n+1} = \mathbf{G}_{n} - \mu[\,\mathbf{\Lambda}_1(n)\,...\,\mathbf{\Lambda}_p(n)] \tag{25}$$

In (25), $\mathbf{\Lambda}_i(n)$ (i=1,2,...,p) is the derivative of the objective function with respect to the i-th column of the equalizer coefficient matrix $\boldsymbol{G}$ and μ is a small constant upper bounded by the maximum eigenvalue of the covariance matrix of the observed data. $\mathbf{\Lambda}_i(n)$ can be calculated by using the chain rule of the derivative given by

$$\mathbf{\Lambda}_i[n] = \frac{\partial J(\mathbf{G}_n)}{\partial \mathbf{G}_{i,n}} = \frac{\partial J(\mathbf{G}_n)}{\partial \hat{s}_i[n]}\frac{\partial \hat{s}_i[n]}{\partial \mathbf{G}_{i,n}},\; i = 1, 2, ..., p. \tag{26}$$

From (19), the first derivative in (26) is equal to

$$\frac{\partial J(\mathbf{G}_n)}{\partial \hat{s}_i[n]} = (\hat{s}_i[n] - f(\hat{s}_i[n-1])) + 4\sum_{\substack{j=1\\ i\neq j}}^{p}\sum_{\delta=\delta_1}^{\delta_2}(\hat{s}_i[n]\,\hat{s}_j[n-\delta]\,)\hat{s}_j[n-\delta]. \tag{27}$$

The second derivative can be computed by using the relation given in (20). It can be shown to be

$$\frac{\partial \hat{s}_i[n]}{\partial \mathbf{G}_{i,n}} = \boldsymbol{r}^{T}[n] \tag{28}$$

Substituting (27) and (28) in (26) yields the desired derivative term in the adaptive algorithm as follows:

$$\boldsymbol{\Lambda}_i(n) = (\hat{s}_i[n] - f(\hat{s}_i[n-1]))\boldsymbol{r}^T[n] + \sum_{\substack{j=1 \\ i\neq j}}^{p} \sum_{\delta_1}^{\delta_2} (\hat{s}_i[n]\,\hat{s}_j[n-\delta])\hat{s}_j[n-\delta]\,\boldsymbol{r}^T[n] \tag{29}$$

A rectangular window is used for calculating the cross-correlation term $\hat{s}_i[n]\,\hat{s}_j[n-\delta]$ in (29). Its value is computed from the estimated signals within the window. The weight of the cross-correlation term in (19) was set to 2, although it can be made variable.

5. Stationary point analysis

In the proposed algorithm, the equalizer parameters at the next iteration are obtained by adding a correction vector in the opposite direction of the objective function gradient. The purpose of this section is to investigate the conditions under which the stationary point of (25) is equal to the optimum filter.

By definition, $\boldsymbol{G}_{n+1}$ should be equal to $\boldsymbol{G}_n$ for a stationary point at convergence. In other words, the term in square brackets in (25) must be zero. Substituting (29) in (25) and then taking the mean values of both sides at a stationary point yield

$$E\left[(\hat{s}_i[n] - f(\hat{s}_i[n-1]))\boldsymbol{r}^T[n] + 4\sum_{\substack{j=1 \\ i\neq j}}^{p} \sum_{\delta_1}^{\delta_2} (\hat{s}_i[n]\,\hat{s}_j[n-\delta])\hat{s}_j[n-\delta]\,\boldsymbol{r}^T[n]\right] = 0 \tag{30}$$

Substituting (20) in (30) gives

$$E\left[(\boldsymbol{r}[n]\mathbf{G}_{i,n} - f(\boldsymbol{r}[n-1]\,\mathbf{G}_{i,n-1}))\boldsymbol{r}^T[n]\right] + 4\sum_{\substack{i,j=1 \\ i\neq j}}^{p} \sum_{\delta=\delta_1}^{\delta_2} E[\boldsymbol{r}[n]\,\mathbf{G}_{i,n}\,(\boldsymbol{r}[n-\delta]\,\mathbf{G}_{j,n-\delta})^2\boldsymbol{r}^T[n]] = 0 \tag{31}$$

When μ is sufficiently small, $\mathbf{G}_{i,n}$ can be assumed to vary considerably slowly relative to $\boldsymbol{r}$[n] at convergence. In other words, $E[\boldsymbol{r}[n]\boldsymbol{G}_{i,n}] \cong \boldsymbol{r}[n]\boldsymbol{G}_{i,n}$.

Consider the following facts:

i. $\boldsymbol{s}$[n] and $\boldsymbol{z}$[n] are uncorrelated and so E[$\boldsymbol{s}$[n]$\boldsymbol{z}$[n]]=$\boldsymbol{s}$[n]E[$\boldsymbol{z}$[n]]=0.

ii. $\boldsymbol{s}$[n] is a deterministic signal, i.e. E[$\boldsymbol{s}$[n]$\boldsymbol{s}^T$[n]]=$\boldsymbol{s}$[n] $\boldsymbol{s}^T$[n]. Using these facts in (31) and going through intermediate steps give

$$\mathbf{T}[n,n]\,\mathbf{G}_{i,n} - E\left[f(\boldsymbol{r}[n-1]\,\mathbf{G}_{i,n-1})\boldsymbol{r}^T[n]\right] + 4\,\mathbf{T}[n,n]\,\mathbf{G}_{i,n}\left(\sum_{\delta=\delta_1}^{\delta_2}(\boldsymbol{r}[n-\delta]\,\mathbf{G}_{j,n-\delta})^2\right) = 0, \tag{32}$$

where $\boldsymbol{T}$[n,n] is defined in (24). Now $\mathbf{G}_{i,n}$ can be calculated from (33), easily leading to

$$\mathbf{G}_{i,n} = \boldsymbol{T}[n,n]^{-1}\frac{E\left[f(\boldsymbol{s}[n-1]\mathbf{G} + n[n-1])\right]\mathbf{G}_{i,n-1}(\mathbf{C}^T\boldsymbol{s}^T[n] + \boldsymbol{z}^T[n])]}{1 + 4\sum_{\delta=\delta_1}^{\delta_2}(\boldsymbol{s}[n-\delta]\,\mathbf{C} + \boldsymbol{z}[n-\delta]\,\mathbf{G}_{j,n-\delta})^2} \tag{33}$$

It is recognized that (33) is equal to the optimum filter derived in Section 3. Note that slow variation in $\mathbf{G}_{i,n}$ relative to $\boldsymbol{r}$[n] is the only assumption for this equivalence to hold.

6. Results

Six experiments were conducted to assess the performance of the proposed solution. We used MIMO FIR channels having different lengths during the simulations. For each simulation, 1000 trials were performed and average results are illustrated in the performance figures. Transmitted chaotic signals were generated by the most commonly used chaotic maps, namely the logistic map, Chebyshev map, tent map, and Henon map. Nonlinear mapping functions of the logistic, Chebyshev, tent, and Henon maps are given in (34)–(37), respectively:

$$s[n] = \lambda s[n-1](1-s[n-1]),\ \lambda \in (3,4) \tag{34}$$

$$s[n] = \cos(4\arccos(s[n-1]) \tag{35}$$

$$s[n] = b-1-bs[n-1],\ b \in [1.3,2] \tag{36}$$

$$s[n] = 1-1.4(s[n-1])^2+0.3s[n-2] \tag{37}$$

As can be seen from the above equations, the embedding dimension of the logistic, Chebyshev and tent maps is one and the embedding dimension of the Henon map is two. In the first and second experiments, convergence behavior of the proposed framework is analyzed and it is contrasted with the classical blind MIMO equalization algorithm given previously [24]. The impulse response of the overall system for a particular pair of input and output signals defined in (38) is used as a performance measure.

$$h_{ij}[n] := \sum_{m=1}^{M} g_{jm}[n] * c_{mi}[n],\ i,j = 1,2,...p. \tag{38}$$

By using the overall impulse responses, output signals can be expressed as

$$\hat{s}_j[n] = \sum_{i=1}^{p} \sum_{l=0}^{L_2+L_1-1} s_i[l]\, h_{ij}[n-l]. \tag{39}$$

From (39), the j-th equalizer output depends only on the j-th chaotic input signal if $h_{ij}[n]=0$ for $i \neq j$ [26]. The proposed blind adaptive filter can recover the input signals provided that this condition is met.

In the first experiment, transmitted chaotic signals are produced with the logistic map by using several initial conditions and a 2-input 3-output FIR model is used for the channel. Length of the FIR equalizer filters was chosen as $L_2 = 20$. As mentioned in Section 4, μ in the SD algorithm has an upper bound determined by the maximum eigenvalue of the covariance matrix of the observed data. In simulations $\mu = 10^{-5}$ was found to satisfy the convergence condition for all observed chaotic signals. Initial values of the equalizer coefficients were $g_{11} = g_{22} = \delta[n-10]$. A 50-point rectangular window was used to calculate the cross-correlation in (18) for $\delta_1 = -24, \delta_2 = 25$. The impulse responses $h_{ij}[n]$, $j = 1,2.$ at convergence are illustrated in Figure 2, from which it can be seen that the condition $h_{12}[n] = h_{21}[n] = 0$ is almost satisfied. In other words, the first equalizer block recovers the first input signal while the second equalizer block recovers the second input.

In the second experiment, the adaptive solution is contrasted with the classical blind MIMO equalization method discussed previously [24]. In this experiment, a 2-input 4-output FIR model is used for the channel and the length of the equalizer is $L_2 = 20$. Overall impulse responses of the chaotic and conventional MIMO communication systems are illustrated in Figures 3a and 3b. Figure 3 shows that signal recovery conditions are not satisfied if conventional MIMO blind equalization is used. The underlying reason for this behavior can be explained as follows. Note that chaotic signals take values in the interval [0, 1]. Hence, $E[s_i^2]$ cannot be zero unless the signal is zero. One of the assumptions made about the transmitted signal previously [24] is violated. Hence, conventional equalization methods are not expected to give satisfactory performance for chaotic signals.

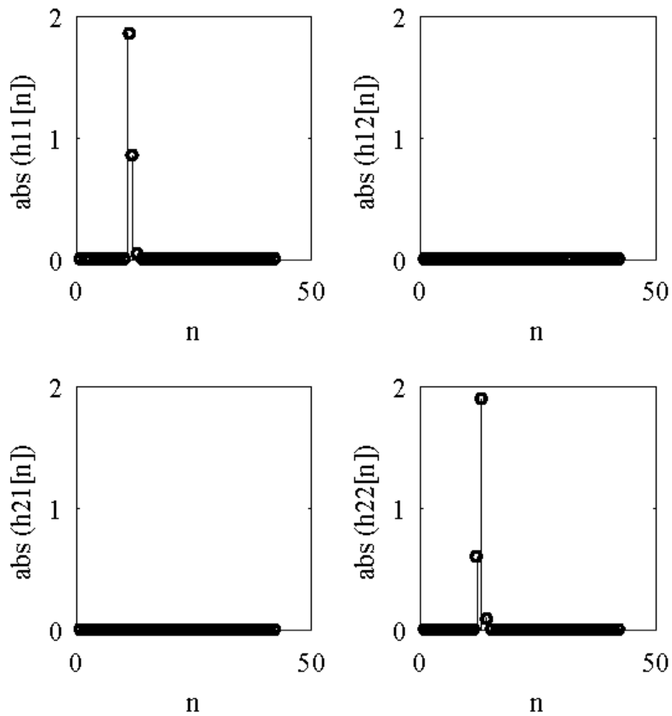


Figure 2. The impulse responses of the overall system at convergence for the first experiment.

The computational complexity of the proposed algorithm for a 2-input 2-output FIR channel is about $14L_2 + 2L_1$ multiplication and $4L_1 + 2L_2 + 50$ summation for each iteration (L_1 and L_2 are the lengths of channel and equalizer, respectively). According to this result we can clearly say that the computational complexity is close to that of conventional equalization algorithms [24].

To the best of our knowledge, there is no study that evaluates the performance of its chaos-based MIMO blind channel equalization method by using metrics similar to those in our paper. Thus, in the third experiment, the performance of the proposed study is compared to that of the optimum filter, which is expected to give the best performance since it has information about the characteristics of the channel. In the comparisons, the mean square error (MSE) is used as the performance measure. At the end of the k-th iteration, let $\hat{\mathbf{s}}_{j,k}$ denote the estimated signal vector for the j-th input signal. Then the MSE between the j-th input and output vectors is defined as

$$MSE_{j,k} = \frac{1}{L_1}\|\mathbf{s}_j - \hat{\mathbf{s}}_{j,k}\|, \tag{40}$$

where L_1 is the length of the vectors and $\|.\|$ denotes the norm operator. Figures 4a and 4b illustrate MSE variations as a function of iteration number for adaptive equalizer outputs obtained from the proposed adaptive

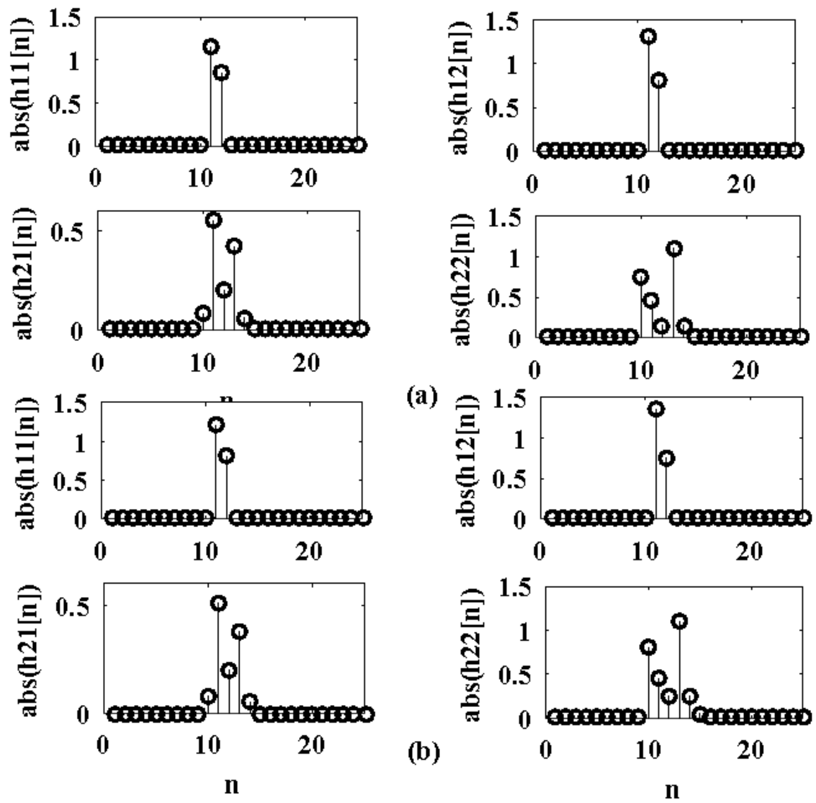


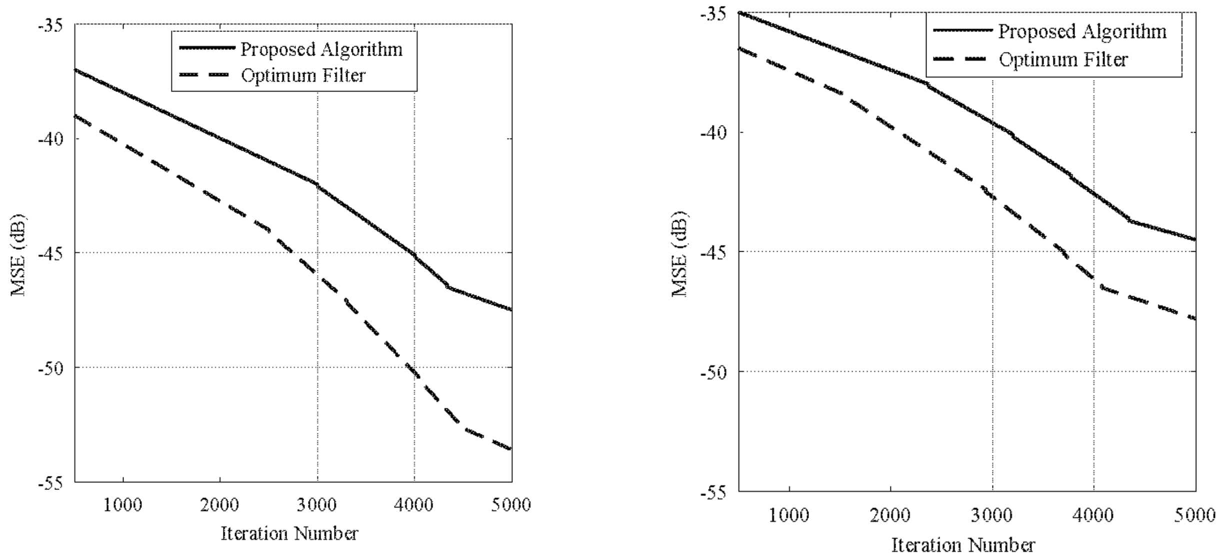
Figure 3. The overall impulse responses for the second experiment (a) chaotic, (b) conventional MIMO communication systems.

and optimum filters, respectively. In this experiment, a 2-input 3-output FIR model with $L_2 = 20$ is used for the channel. Results of the proposed adaptive filter are in harmony with those of the optimum filter.

The proposed adaptive filter is applied to several chaotic communication systems in Experiment 4. Chaotic input signals are generated by logistic, Chebyshev, and tent maps in the simulations. For this and the following experiment, a 2-input, 4-output FIR model is used for the channel. L_2 and μ are chosen as 20 and 10^{-5}, respectively. Figures 5a and 5b show the MSE variations for the corresponding equalizer outputs. As the figures show, the tent map gives better results than the others but, in general, similar behaviors are obtained for different chaotic maps since the proposed algorithm exploits the properties of chaotic maps.

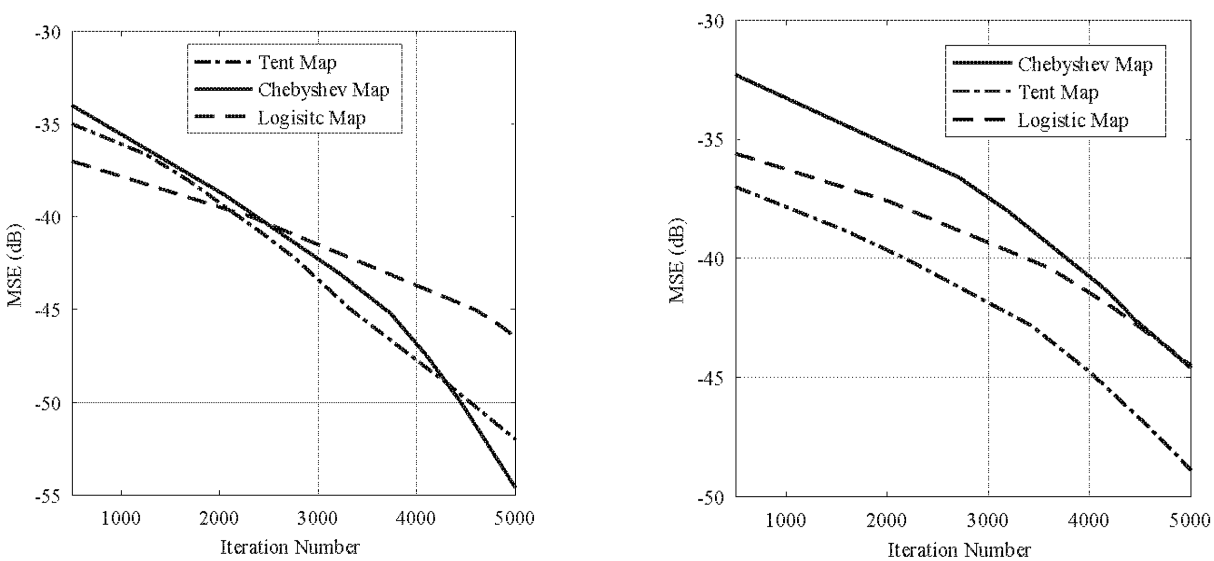
In the fifth experiment, the effect of the embedding dimension of chaotic maps is investigated. For this purpose, two different chaotic communication systems with different embedding dimensions are considered. Transmitted chaotic signals are generated by the Henon map with d=2 in the first system and the logistic maps with d=1 in the second system. Figures 6a and 6b illustrate the MSE variations for the equalizer outputs. Again, behaviors are similar regardless of the chaotic maps and the embedding dimensions since adaptive equalizers are built upon information provided by the chaotic maps used at the transmitter.

In addition to the above experiments, the performance of the proposed algorithm is evaluated by calculating the bit error rate (BER), which can be expressed as the ratio of the number of errors to the total number of bits sent. For this purpose, a chaos shift keying (CSK) modulation–demodulation process is applied as shown at the input and output of Figure 1. The operating principle and block diagram of the CSK system



(a) (b)

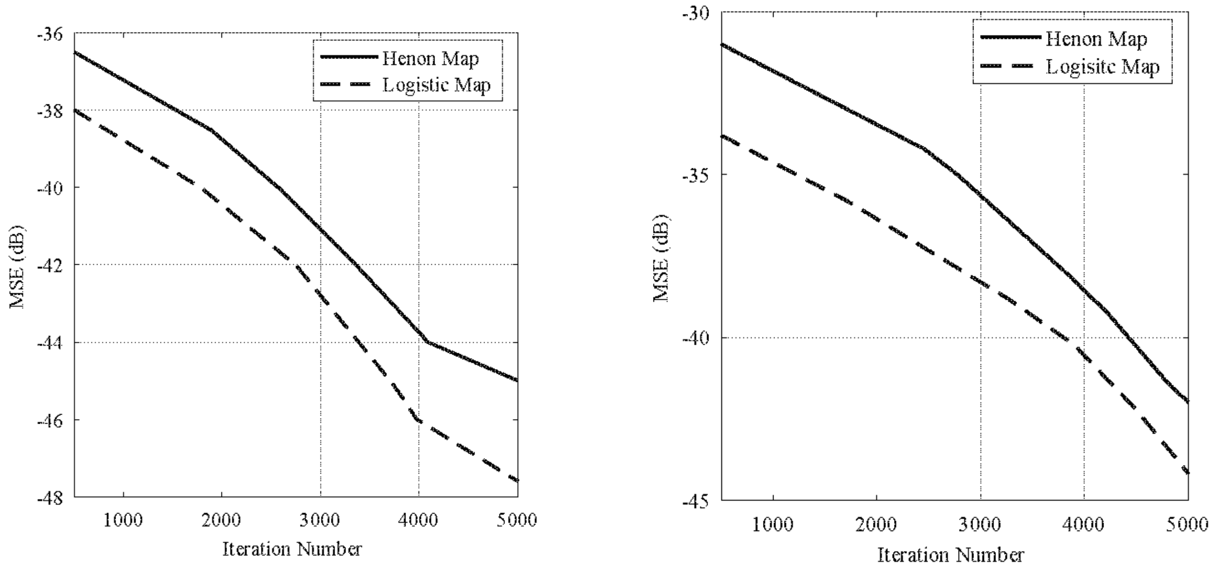
Figure 4. The overall impulse responses for the second experiment (a) chaotic, (b) conventional MIMO communication systems.



(a) (b)

Figure 5. MSE values for different chaotic mapping functions in the fourth experiment: a) between the first information signal and the first equalizer output and b between the second information signal and the second equalizer output.

can be found elsewhere [1]. BER performance curves of the proposed scheme are illustrated in Figures 7 and 8. In Figure 7, BER versus different signal to noise ratio (SNR) values for both the proposed algorithm and the optimum filter is given. BER values are calculated by averaging the results obtained for the first and second outputs. Similarly, in Figure 8, BER values are shown for different chaotic maps. As can be seen from the figures, BERs are at an acceptable level for reliable communication. Note that the BER performance of the proposed algorithm can be improved by applying different modulation–demodulation scenarios. However, this is outside the presented paper's scope.



(a) (b)

Figure 6. MSE values for the Henon map with d=2 and the logistic map with d=1: a) between the first information signal and the first equalizer output and b) between the second information signal and the second equalizer output.

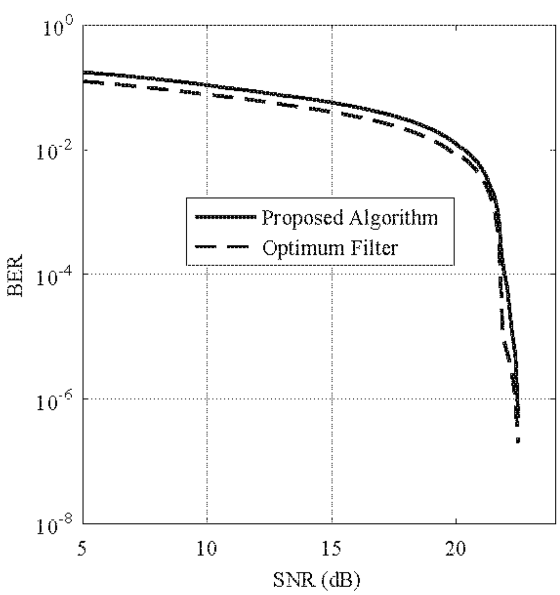


Figure 7. BER versus SNR for the proposed algorithm and optimum filter.

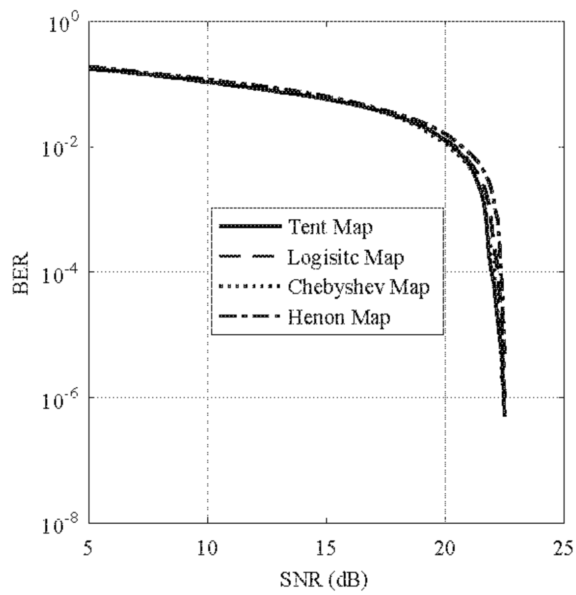


Figure 8. BER versus SNR for different chaotic maps.

7. Discussion

An adaptive chaotic blind equalization approach to combat ISI and MUI for MIMO systems was proposed in this work. In addition, given that the impulse response of the channel is known, an optimum filter was derived. The proposed approach was contrasted with the optimum filter in terms of MSE performance. The stationary point of the adaptive algorithm was shown to be equal to that of the optimum filter provided that the adaptive filter coefficients change sufficiently slowly by means of stationary point analysis. The proposed algorithm gives results close to those obtained by the optimum filter. It can also reconstruct all input signals at the same time.

References

[1] Stavroulakis P. Chaos Applications in Telecommunications. New York, NY, USA: Taylor & Francis Group, 2006.

[2] Abel A, Schwarz W. Chaos communications - principles, schemes and systems analysis. Proceedings of IEEE 2002; 90 (5): 691-710. doi:10.1109/JPROC.2002.1015002

[3] Kennedy MP, Kolumban G, Jako Z. Chaotic modulation schemes. In: Kennedy MP, Rovatti R, Setti G (editors). Chaotic Electronics in Telecommunications. Boca Baton, FL, USA: CRC Press, 2000, pp. 151-180.

[4] Mazzini G, Rovatti R, Setti G. Interference minimization by auto-correlation shaping in asynchronous DS-CDMA systems: chaos-based spreading is nearly optimal. Electronic Letters 1999; 35 (13):1054-1055. doi: 10.1049/el:19990754

[5] Cong L, Shaoqian L. Chaotic spreading sequences with multiple access performance better than random sequences. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 2000; 47 (3): 394-397. doi: 10.1109/81.841922

[6] Chen CC, Yao K. Design of spread spectrum sequences using chaotic dynamical systems and ergodic theory. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 2001; 48 (9): 1110-1114. doi: 10.1109/81.948438

[7] Gotz M, Kelber K, Schwarz W. Discrete-time chaotic encryption systems. Part I: Statistical design approach. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 1997; 44 (10): 963-970. doi: 10.1109/81.633885

[8] Kelber K, Falk T, Schwarz W, Killias T. Discrete-time chaotic encryption systems. Part II: Continuous- and discrete-value realization. In: Proceedings of International Workshop on Nonlinear Dynamics of Electronic Systems; Seville, Spain; 1996. pp. 27-32.

[9] Dachselt F, Kelber K, Schwarz W. Discrete-time chaotic encryption systems. Part III: Crypto graphical analysis. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 1998; 45 (9): 883-888. doi: 10.1109/81.721265

[10] Chen CY, Chi CY, Chen CH, Feng CC. Batch processing algorithms for blind equalization using higher-order statistics. IEEE Signal Processing Magazine 2003; 20 (1): 25-49. doi: 10.1109/MSP.2003.1166627

[11] Via J, Santamaria I, Perez J. Deterministic CCA-based algorithms for blind equalization of FIR-MIMO channels. IEEE Transactions on Signal Processing 2007; 55 (7): 3867-3878. doi: 10.1109/TSP.2007.894273

[12] Zhu Z, Leung H. Adaptive blind equalization for chaotic communications systems using extended Kalman filter. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 2001; 48 (8): 979-989. doi: 10.1109/81.940188

[13] Leung H, Xu X, Guo J. Blind equalization for power-line communications using chaos. IEEE Transactions on Power Delivery 2014; 29 (3): 1103-1110. doi: 10.1109/TPWRD.2013.2296834

[14] Xie N, Leung H. Blind equalization using a predictive radial basis function neural network. IEEE Transactions on Neural Networks 2005; 16 (3): 709-720. doi: 10.1109/TNN.2005.845145

[15] Tsai JS, Lu FC. A new approach for adaptive blind equalization of chaotic communication: the optimal linearization technique. Computers and Mathematics with Applications 2009; 58 (9): 1687-1698. doi: 10.1016/j.camwa.2009.06.054

[16] Maoge X, Yaolinag S, Liwei L. Adaptive blind equalization for chaotic communication systems using particle filtering. In: 8th International Conference on Signal Processing; Beijing, China; 2006. pp. 16-20.

[17] Xie N, Leung H. Blind identification of autoregressive systems using chaos. IEEE Transactions on Circuits and Systems I: Regular Papers 2005; 52 (9): 1953-1964. doi: 10.1109/TCSI.2005.852488

[18] Ciftci M, Williams DB. Channel equalization for multiuser chaotic communications systems. In: IEEE International Conference on Acoustics, Speech, and Signal Processing; Orlando, FL, USA; 2001. pp. 1113-1116.

[19] Yoon S, Lee JH, Ryu HG. Chaos communication system using MIMO technique. In: 16th Conference on Advanced Communication Technology (ICACT); Pyeongchang, South Korea; 2014. pp. 579-583.

[20] Ciftci M, Williams DB. An optimal estimation algorithm for multiuser chaotic communications systems. In: IEEE International Symposium on Circuits and Systems; Phoenix-Scottsdale, AZ, USA; 2002. pp. 397-400.

[21] Hu Z, Feng J. Blind channel equalization algorithm based on dual unscented Kalman filter for chaotic multi-input multi-output communication systems. Transactions of Tianjin University 2012; 18 (1): 33-37. doi: 10.1007/s12209-012-1621-0

[22] Cetinel G, Vural C. Blind source separation and equalization of multiple- input multiple-output FIR channels for chaotic communication systems. In: European Signal Processing Conference; Aalborg, Denmark; 2010. pp. 1135-1139.

[23] Vural C, Cetinel G. Blind equalization of single-input single-output fir channels for chaotic communication systems. Digital Signal Processing; 20 (1): 201-211. doi: 10.1016/j.dsp.2009.06.001

[24] Papadias CB, Paulraj A. A constant modulus algorithm for multiuser signal separation in presence of delay spread using antenna arrays. IEEE Signal Processing Letters 1997; 4 (6): 178-181. doi: 10.1109/97.586042

[25] Johnson JCR, Sethares W A. Telecommunication Breakdown. Upper Saddle River, NJ, USA: Prentice Hall, 2003.

[26] Lu Y, Liu KJR. Adaptive source separation and equalization for multiple-input/multiple-output systems. IEEE Transactions on Information Theory 1998; 44 (7): 2864-2876. doi: 10.1109/18.737518