SAKARYA UNIVERSITY
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DESIGN OF DISCRETE TIME CONTROLLERS FOR DC-DC BOOST CONVERTER

M.Sc. THESIS

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Department	**:**	**ELECTRICAL & ELECTRONICS ENGINEERING**
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This thesis has been accepted unanimously by the examination committee on 05.06.2015

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DECLARATION

I declare that all the data in this thesis was obtained by myself in academic rules, all visual and written information and results were presented in accordance with academic and ethical rules, there is no distortion in the presented data, in case of utilizing other people's works they were refereed properly to scientific norms, the data presented in this thesis has not been used in any other thesis in this university or in any other university.

Mohammed ALKRUNZ

05.06.2015

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LIST OF SYMBOLS AND ABBREVIATIONS

v	:	Actuating Error Vector
ARE	:	Algebraic Riccati Equation
AC	:	Alternative Current
T_A	:	Ambient Temperature
ADC	:	Analog to Digital Converter
BJT	:	Bipolar Junction Transistor
C	:	Capacitor
I_C	:	Capacitor Current
r_C	:	Capacitor Equivalent Series Resistance
$P_{C(loss)}$	:	Capacitor Power Loss
V_C	:	Capacitor Voltage
ΔQ	:	Change in Capacitor Charge
ΔP_D	:	Change in Power Dissipation
r	:	Command Input Vector
CMOS	:	Complementary Metal-Oxide Semiconductor
CCM	:	Continuous Conduction Mode
C_M	:	Controllability Matrix
$i(t)$	:	Current
ζ	:	Damping Ratio
$R_{in(DC)}$	:	DC Input Resistance
K_d	:	Derivative Gain
DAC	:	Digital to Analog Converter
D	:	Diode
I_D	:	Diode Current
R_F	:	Diode Forward Resistance

$V_{F(diode)}$	:	Diode Forward Voltage
V_F	:	Diode Forward Voltage
$P_{R_F(loss)}$	:	Diode Power Loss due to R_F
$P_{V_F(loss)}$	:	Diode Power Loss due to V_F
$P_{D(loss)}$	:	Diode Total Power Loss
V_D	:	Diode Voltage
DC	:	Direct Current
DCM	:	Discontinuous Conduction Mode
V_{DO}	:	Drop-Out Voltage
η	:	Efficiency
W	:	Energy Dissipation
ESR	:	Equivalent Series Resistance
e	:	Error Signal
$V_{o(FL)}$	:	Full-load Output Voltage
K_e	:	Gain Matrix
GTO	:	Gate Turn Off Thyristors
L	:	Inductor
I_L	:	Inductor Current
r_L	:	Inductor Equivalent Series Resistance
$P_{L(loss)}$	:	Inductor Power Loss
V_L	:	Inductor Voltage
x_o	:	Initial State
I_{in}	:	Input Current
u	:	Input or Control Vector
P_{in}	:	Input Power
V_{in}	:	Input Voltage
V_g	:	Input Voltage
$W_C(t)$	:	Instantaneous Energy Stored in a capacitor
$W_L(t)$	:	Instantaneous Energy Stored in an inductor
$p(t)$	:	Instantaneous Power
IGBT	:	Insulated-Gate Bipolar Transistors
K_i	:	Integral Gain

LHP	:	Left Half Plane
LQR	:	Linear Quadratic Optimal Regulator
LTI	:	Linear Time Invariant System
I_R	:	Load Current
R_L	:	Load Resistor
R	:	Load Resistor
LDO	:	Low Drop-Out Voltage Regulator
I_{Lmax}	:	Maximum Inductor Current
I_{omax}	:	Maximum Output Current
MOSFET	:	Metal Oxide Semiconductor Field Effect Transistor
ms	:	millisecond
C_{min}	:	Minimum Capacitor Value
I_{Lmin}	:	Minimum Inductor Current
$V_{in(\mathrm{min})}$	:	Minimum Input Voltage
I_{omin}	:	Minimum Load Current
MCT	:	MOS-Controlled Thyristors
$P_{DS(loss)}$	:	MOSFET Conduction Loss
r_{DS}	:	MOSFET On-Resistance
C_o	:	MOSFET Output Capacitance
$P_{s(loss)}$	:	MOSFET Total Power Loss
MIMO	:	Multiple Input-Multiple Output Systems
ω_n	:	Natural Frequency
$V_{o(NL)}$	:	No-load Output Voltage
V_{onom}	:	Nominal Output Voltage
n	:	Number of State Variables
O_M	:	Observability Matrix
t_{off}	:	Off-Time Interval
t_{on}	:	On-Time Interval
I_o	:	Output Current
P_o	:	Output Power
V_r	:	Output Ripple Voltage
y	:	Output Vector

V_o	:	Output Voltage
$V_{o(minL)}$	:	Output Voltage at the Minimum Load Current
$P_{T(loss)}$	:	Overall Power Loss of the Boost Converter
V_{pk}	:	Peak Value
%OS	:	Percentage Overshoot
J	:	Performance Index or Cost Function
Q, R	:	Positive Definite Symmetric Constant Matrices
pnp	:	Positive-Negative-Positive Transistor
PBJT	:	Power Bipolar Junction Transistor
P_{loss}	:	Power Loss
P_{R_s}	:	Power Loss in the Shunt Resistor
P_{TS}	:	Power Loss in the shunt-transistor
$PSRR$	:	Power Supply Rejection Ratio
K_p	:	Proportional Gain
PID	:	Proportional Integral Derivative
V_{GS}	:	Pule Train Signal
PWM	:	Pulse Width Modulation
V_{oref}	:	Reference Output Voltage
RHP	:	Right Half Plane
rms	:	Root Mean Square
T	:	Sampling Time
s	:	Second
t_s	:	Settling Time
R_s	:	Shunt Resistor
SCR	:	Silicon Controlled Rectifier
^	:	Small Variation
P	:	Solution of Algebraic Riccati Equation
$A_{Off}, B_{Off}, C_{Off}, D_{Off}$	:	State Space Matrices During Off-State Interval
$A_{ON}, B_{ON}, C_{ON}, D_{ON}$	:	State Space Matrices During On-State Interval
A, B, C, D	:	State Space Matrices under Continuous Time Domain

A_d, B_d, C_d, D_d	:	State Space Matrices under Discrete Time Domain
x	:	State Vector
SITH	:	Static Induction Thyristors
SIT	:	Static Induction Thyristors
S	:	Switch
I_s	:	Switch Current
T_s	:	Switch Cycle Period
SMPS	:	Switch Mode Power Supplies
f_s	:	Switching Frequency
$P_{sw(loss)}$	:	Switching Loss
I_{AV}	:	The Average Value of Current
V_{AV}	:	The Average Value of Voltage
G_{IDC}	:	The Current DC Gain
d	:	The Duty Cycle
$i_{C_{max}}$	:	The Root Mean Square value of Capacitor Maximum Current
$i_{C_{min}}$	:	The Root Mean Square value of Capacitor Minimum Current
I_{rms}	:	The Root Mean Square value of Current
$I_{D(rms)}$	:	The Root Mean Square value of diode Current
$I_{s(rms)}$	:	The Root Mean Square value of switch Current
V_{rms}	:	The Root Mean Square value of Voltage
V_s	:	The Switch Drop Voltage
G_{VDC}	:	The Voltage DC Gain
I_C	:	Transistor's Collector Current
V_{CE}	:	Transistor's Collector to Emitter Voltage
TTL	:	Transistor-Transistor Logic
R_T	:	Variable Resistor
$v(t)$	:	Voltage
V_{rc}	:	Voltage across the Capacitor Equivalent Series Resistance
V_{R_s}	:	Voltage across the Shunt Resistor
ZOH	:	Zero Order Hold

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SUMMARY

Keywords: DC-DC Boost Converter, Small Signal Model, Pole Placement, LQR.

DC-DC converters are extensively used in modern power electronic devices due to their high efficiency, high power density, high power levels, low cost, and small size. In general, they can be step-up, step-down or step-up/down converters and can have multiple output voltages. Boost converter, (also known as a step-up converter) is a type of switched-mode dc-dc converter which produces output voltage that is greater than input voltage.

A small signal modeling based on state space averaging technique for DC-DC Boost converter is carried out. Discrete time controller is designed using two design techniques; frequency domain and state space methods. Root locus technique is used to design an integral controller. A state feedback gain matrix is designed by pole placement technique and Linear Quadratic Optimal Regulator (LQR) methods. The performance of the controlled boost converter are investigated and verified through MATLAB/SIMULINK simulation.

Comparison between the designed controllers related to the design methodology, implementation issues and performance is carried out. It is seen that the designed controllers yielded comparable performances.

In this study, it is aimed to design a controller for DC-DC boost converter to provide satisfactory performance in term of static, dynamic and steady-state characteristics.

YÜKSELTİCİ TİP DC-DC DÖNÜŞTÜCÜLER İÇİN AYRIK-ZAMAN KONTROLÖR TASARIMI

ÖZET

Anahtar kelimeler: Yükseltici tip DC-DC dönüştürücüler, Küçük-İşaret Modeli, Kutup Yerleştirme, LQR.

Güç elektroniği, süreç kontrolü ve elektrik enerjisinin elektronik ekipmanlar, makineler ve diğer cihazlar tarafından kullanılmak için uygun bir forma dönüştürülmesi için yarı iletken cihazları içeren yöntemler kullanan teknoloji olarak tanımlanabilir.

Güç elektroniği devrelerinde, diyot gibi bazı elementler herhangi bir kontrol sinyali olmadan güç devresiyle kontrol edilebilir ve SCR (silikon kontrollü doğrultucu) gibi bazı elementlerin AÇIK konuma getirilmesi için bir kontrol sinyaline ihtiyaç duyulurken, bunlar güç devresi ile KAPALI konuma getirilebilir. Aynı zamanda MOSFET gibi bazı elementler hem AÇIK konuma hem de KAPALI konuma getirilmek için bir kontrol sinyaline ihtiyaç duyar.

Bilimsel üretimdeki hızlanmayla ilişkili olarak, kontrol sinyali ve güç cihazları enerji işleme devrelerinin tasarımı için aynı yarı iletken içinde bulunabilir. Böylelikle güç elektroniği mühendisleri ile dijital olarak entegre devre tasarımcıları arasında tasarım metodolojisinde büyük farklılıklar söz konusu olmaz. Bu yüzden güç elektroniği devrelerinin kontrol sorunu için geniş dijital çözümlerin önümüzdeki yıllarda bulunması beklenmektedir.

DC-DC dönüştürücüler güç elektroniği alanı için çok önemli olan DC gerilim kaynağının farklı gerilim seviyelerine dönüştürülmesi açısından endüstriyel uygulamalarda yaygın bir şekilde kullanılır. DC-DC dönüştürücüler yüksek verimlilikleri, yüksek güç yoğunlukları, yüksek güç seviyeleri, düşük maliyetleri ve küçük boyutlarından dolayı dağıtılmış güç tedarik sistemlerinde ve modern güç elektroniği cihazlarında kapsamlı bir şekilde kullanılır. Aynı zamanda Kesintisiz güç kaynaklarında, güç katsayısını geliştirilmesinde, harmonik eliminasyon, yakıt hücresi uygulamalarında ve fotovoltaik dizilerde kapsamlı olarak kullanılır. Bunların aynı zamanda pille çalışan araçlarda, tramvaylarda, cer motoru kontrollerinde ve DC motorları kontrollerinde kullanımlarına da rastlamak mümkündür.

DC-DC Dönüştürücüler yükseltici ve alçaltıcı dönüştürücüleri olarak kullanılabilir ve çoklu çıkış gerilimine sahip olabilir. Alçaltıcı dönüştürücülerinde, çıkış gerilimi giriş geriliminden daha düşükken, yükseltici dönüştürücülerinde çıkış gerilimi giriş geriliminden daha yüksektir. Bazı dönüştürücüler hem yükseltici hem alçaltıcı

dönüştürücüleri olarak ayarlanabilir. Ayrıca DC-DC dönüştürücüleri tek bir çıkışa sahip olabileceği gibi, çoklu çıkışlara da sahip olabilir. Aynı zamanda çıkış gerilimi sabit veya ayarlanabilir olabilir. Bu yüzden DC-DC dönüştürücü teknolojisi birçok farklı türde topoloji kullanır. Elektrik ve elektronik devre ve sistemlerde her durum için ayrı bir uygulama mevcuttur.

DC-DC dönüştürücülerin tasarımı güç elektroniği anahtarlama unsurlarından birinin MOSFET'ler, çift kutup yüzeyli güç transistörleri (BJT) veya IGBT'leri olarak kullanılmasıyla elde edilebilir. Bu anahtarlama unsurları yüksek verim, hızlı dinamik yanıt, sorunsuz kontrol, küçük boyutlar, düşük maliyet ve bakım sağlayabilir. Dönüştürücünün çıkış gerilimi bu güç anahtarlarının Açma-Kapama aralığının ayarlanmasıyla kontrol edilir. Yani bu ana anahtarlama cihazları arzu edilen görev döngüsüyle çalıştırılır.

Görev döngüsü Açık sürenin toplam anahtarlama süresiyle arasındaki orandır. Görev döngü kontrolü konusunda kullanılabilecek iki farklı kontrol stratejisi söz konusudur. Bunlar oran kontrol ve akım sınırlaması kontrolüdür. Süre oranı bir sabit frekans işletimi veya değişken frekans işletimi kullanılarak gerçekleştirilebilir. Aynı zamanda PWM (sinyal genişlik modülasyonu) tekniği olarak da bilinen sabit frekans tekniğinin kullanılması durumunda, AÇIK süre değişiklik gösterir ve anahtarlama frekansı sabit tutulur. Aynı zamanda frekans modülasyon tekniği olarak da bilinen değişken frekans tekniğinin kullanılması durumunda ise, anahtarlama frekansı değişiklik gösterir ve AÇIK frekans sabit tutulur. Frekans modülasyonu, anahtarlama frekansında geniş değişiklikler esnasında doğru kontrol gerektirdiğinden farklı dezavantajlara sahiptir. Bu yüzden tasarımcılar cihazın KAPALI süresinin yüksek bir değere çıkarılması halinde filtre tasarımının gereklilikleri, sinyalde parazit ve kesintili yük akımı gibi sorunlarla karşılaşabilir. Akım sınır kontrolü, yükler gibi enerji depolama unsurları kullanan uygulamalarla kullanılmaktadır. Daha sonra AÇMA-KAPAMA aralık süresi anahtarı yük akımının maksimum ve minimum arzu edilen değerlerine ilişkin olarak kontrol edilir.

Bu dönüştürücülerin görev döngülerinin kontrol edilmesi için bir geribesleme devresine ihtiyaç duyulur. Bu konuda yaygın bir kullanıma sahip meşhur geribesleme devresi referans gerilimi ile çıkış gerilimi arasındaki karşılaştırmadır. Görev döngüsü değişikliklerini kontrol etmek ve ihtiyaç duyulan sabit çıkış gerilimini elde etmek için PWM tekniği kullanılarak anahtarlama cihazı için kontrol sinyali geliştirilir ve uygulanır. Hem çıkış gerilimi hem de endüktör akım geribesleme devresinde kullanılabilir. Bu yüzden iki tür geribesleme devresi mevcuttur: gerilim mod kontrolü ve akım mod kontrolü. Gerilim modu yalnızca çıkış gerilimini kullanırken, akım modu geribesleme için hem çıkış gerilimi ve endüktör akımı kullanılır.

Gerilim modu kontrolünde gerçek gerilim ile arzu edilen referans gerilimi karşılaştırmak için yalnızca bir dış döngü kullanılır. Daha sonra kompenzatöre PWM tekniği vasıtasıyla görev döngüsünü kontrol eden kontrol sinyalini gerçekleştirmek ve uygulamaya sokmak üzere hata sinyali verilir. PWM sinyali birçok farklı şekilde yapılabilir. Örneğin tahrik devresi ile mikrokontrolörden elde edilebileceği gibi, anahtarlama cihazının tahriki için gerekli olan sinyalleri üretmek için bir rampa sinyali ile kontrol sinyalini karşılaştıran karşılaştırma teknikleri kullanılarak da elde edilebilir.

Akım modu kontrolünde iki döngüye ihtiyaç duyulur. Temel dış döngüye bir iç döngü eklenir. Sistemin yanıtının hızlandırılması için iç döngü kullanılır. Bu mod Boost ve Boost-Buck Dönüştürücüler gibi karma evreli dizeler kullanıldığında çok etkilidir. Yukarıda gerilim mod kontrolünde bahsedilen dış döngü burada da iç döngüden daha yavaş bir yanıtla kullanılır. Dış döngüden uygulanan sinyal iç döngü için bir giriş akım referansı sinyalidir. Bu referans değeri gerçek endüktör akım değeriyle karşılaştırılır ve bu yaklaşım dış döngüden çok daha hızlı bir şekilde gerçekleştirilir. Bu yüzden dış döngünün amacı hata gerilim sinyaline göre iç döngüye referans değer sağlamaktır. Böylelikle sistemin yanıt performansı çok daha iyi ve daha hızlı olur. Bu gözetilmeksizin, bu yaklaşım kompensatör tasarımı boyunca alt harmonikler dolayısıyla yüksek frekans istikrarsızlığına neden olabilir.

Bu araştırma kapsamında DC-DC Boost Dönüştürücü gerilm mod kontrolü kapsamında ele alınmıştır. Boost Dönüştürücü (kademeli artırma dönüştürücüsü olarak da bilinir) giriş geriliminden daha büyük olan sabit çıkış gerilimi üreten anahtarlı mod DC-DC dönüştürücü türüdür. AA (alternatif akım) eşdeğeri devre modellemesi için devre ortalaması, akım enjeksiyonlu yaklaşım, ortalama anahtarlama modellemesi ve durum uzayı ortalama yöntemi gibi birçok yöntem mevcuttur. Durum uzayı ortalama modellemesi DC-DC Dönüştürücü'nün modellemesinde yaygın olarak kullanılır.

Küçük sinyal modeli gibi ortalama teknikleri DC-DC dönüştürücülerin modellerini türetmek için kapsamlı olarak kullanılmıştır. Bu devre modeli sistemi uyarmak ve uygun kontrolörü tasarlamak için kullanılır. Küçük sinyal modeli dinamik ve sabit durum çalışma noktasındaki değişiklikler hakkında bir fikir verir. Durum değişkenleri ve kontrol değişkenlerinin sabit durum çalışma noktasının çevresindeki aa değişkenlikler/parazitleri olduğu varsayılır. Fakat küçük sinyal modeli çalıştırma noktalarındaki değişikliklere ilişkili olarak değişir.

Araştırmacılar DC-DC dönüştürücü kontrolleri konusunda büyük çabalar göstererek farklı hususlar kapsamında birçok kontrolör önerilmiştir. Elde edilen araştırmadan durum geribesleme kontrolörler tekniklerinin kararlılık ölçütünü elde ettiği ve çalışma noktası koşulları kapsamında iyi bir dinamik performansa ulaştığı anlaşılmıştır. Daha önceki tasarımlarda çeşitli sorunlarla karşılaşılmıştır. Bunun iki ana nedeni vardır: ilk neden konvertörler için iyi modelleme ve etkili analiz gerekliliğinden gelmektedir. İkinci neden de anahtarlama konvertörlerinin devre topolojilerinin geniş bir aralığa sahip olması ve böylece geleneksel yaklaşımlar kullanan bu konvertörlerin kontrolünün karmaşık bir hal alması ve topolojinin bağımlı kalmasıdır. Bu yüzden sorun kontrolü DC-DC konvertörler karma evreli dizge olarak kabul edildiğinde yani Boost Dönüştürücü gibi sağ yarıdaki düzlem üzerinde kararsız bir sınıf olduğunda daha zor bir hal alır.

Genellikle ortalama modeller doğrusal bir kontrolör türetmek için belli bir kullanım noktasında doğrusal bir hale getirilebilir. Lâkin konvertör doğrusalsızlığı dikkate almayan bir tasarım, kötüleşen çıkış sinyaliyle ve büyük pertübasyonların varlığındaki kararsız davranışla sonuçlanabilir. Doğrusalsızlıkların ve parametre karasızlığının dikkate alınması için, konvertör modellerinin ve dayanıklı kontrol yöntemlerinin araştırılması hala aktif olarak devam etmektedir..

Küçük sinyal modelleri Boost Dönüştürücüler için ayrıntılı olarak elde edilmiştir. İki kontrol stratejisi önerilmiştir; Kök-yer eğrisi teknikleri ve durum geribeslemesi yaklaşımı. Görev döngüsü arzu edilen çıkış gerilimlerini elde etmek için kontrol edilir. Konvertörün doğrusallaştırılmış küçük sinyal modelini temel alan kontrol stratejileri çalışma noktaları çevresinde iyi bir performansa sahiptir. Durum uzayı modeli görev döngüsüne bağlıdır. Fakat Boost Dönüştürücü'nün küçük sinyal modeli çalışma noktası değişiklik gösterdikçe değişir. Kutuplar ve sağ yarı düzlem sıfırının yanında frekans yanıtının büyüklüklerinin hepsi görev döngüsüne bağlıdır. Bu yüzden kontrolör tesis dinamik özelliklerindeki değişikliklere uyum sağlayabilmelidir.

Genelde PID kontrolörü geleneksel bir doğrusal kontrol yöntemidir. Bu yüzden, bu çalışma noktasındaki değişikliklere iyi karşılık vermek için doğrusal PID kontrolörü gibi küçük sinyal doğrusallaştırma tekniklerini kullanan kontrolör için zordur.

Bu araştırmada kök-yer eğrisi tekniklerini kullanarak hem sürekli-zaman alanında hem de ayrık-zaman bir şekilde bir İntegral Kontrolör tasarlanmıştır. Ayrıca durum uzayı teknikleri kullanılarak kutup yerleşim ve LQR yöntemlerini temel alan Boost Dönüştürücüsü için bir durum geribeslemesi kontrolörü tasarlanmıştır.

Kök-yer eğrisi teknikleri veya frekans yanıt teknikleri gibi tasarımın frekans bölgesi yöntemleri bütün kapalı döngü kutuplarını yerleştirmek için yeterli parametreye sahip olmadığından daha yüksek düzen sistemlerinin bütün kapalı döngü kutuplarını tasarlayamaz ve belirleyemez. Durum geri beslemesi kontrolörü gibi durum uzayı yöntemleri bu sorunu sisteme diğer ayarlanabilir parametreler getirerek çözmektedir.

Telafi edilen Boost Dönüştürücü giriş geriliminde ve yükteki ani değişiklikler kapsamında kontrol edilmiştir. Dönüştürücü aynı zamanda referans geriliminin izlenmesi kapsamında da kontrol edilmiştir. Ayrıca performans parametresi ve sabit durum parametreleri gibi simülasyon sonuçları da tartışılmış ve çizelge haline getirilmiştir. Çıkış gerilimini yanıtının taslağı çizilmiş ve farklı kontrolör durumları için kıyaslama yapılmıştır.

Tasarlanan kontrolör türleri tasarım metodolojisi, uygulama problemleri ve performans açısından karşılaştırılmıştır. Tasarlanan kontrol yöntemlerinin birbirine yakın bir performansa sahip olduğu gözlemlenmiştir.

Simülasyon ve sonuçlar temel alınarak doğrusal Integral Kontrolör sistem yüksek yük değişikliklerine tabi tutulduğunda kötü performans sergilemiş ve çalışma noktalarındaki değişikliklere kötü yanıt vermiştir. Diğer yandan kutup yerleşimi ve LQR yöntemleri temel alınarak tasarlanan kontrolörler çalışma noktasındaki değişiklikler gözetilmeksizin gayet iyi çıkış voltaj regülasyonu ve mükemmel dinamik performanslar sergilemiştir.

Doğrusal optimal kuadratik regülatör (LQR) yöntemi, bu kontrolörler iyi bir çözüme sahip olduğundan ve mükemmel statik ve dinamik özellikler, kabul edilir bir dayanıklılık, çıkış regülasyonu, parazit reddi ve bütün çalışma noktalarında daha yüksek verim sağladığından bahsi geçen kontrolörlerin en iyi tekniği olarak sonuç vermiştir.

Bu yüzden LQR baskın kapalı döngü kutuplarının arzu edilen bölgelere yakın olarak atandığı ve kalan kutupların da baskın olmayacağı bir şekilde tasarımcının teknik özelliklerine ilişkin optimal yanıtı sağlamaktadır. Bu yöntem uygun performans endeksini seçerek tesis parametre değişikliklerine duyarsızlığı temin eder. Başka bir deyişle, kontrol sisteminin durumları veya çıktıları kontrol çabasının kabul edilebilir bir tüketimini kullanarak bir referans durumundan kabul edilebilir bir sapma içerisinde tutulur. Ayrıca sistem düzeninin bağımsızlığıyla uygulanabilir ve sistemin küçük sinyal modelinin matrislerinden kolaylıkla hesaplanabilir.

Bu çalışmada, yükseltici tip DC-DC dönüştürücüler için sürekli-hal ve dinamik karakteristik açısından uygun bir performansa sahip kontrolör tasarımı amaçlanmıştır.

Bu araştırma sekiz bölümden oluşmaktadır: ilk bölümde konuya bir giriş sunulmakta, araştırmanın amacı ve tez organizasyonu verilmektedir. Diğer bölümler de aşağıdaki şekilde düzenlenmiştir:

Bölüm 2'de güç kaynaklarının tanımları ve sınıflandırmaları hakkında kapsamlı bir girişe yer verilmiştir. Doğrusal Gerilim ve anahtarlama regülatörlerinin temel fonksiyonları tartışılmıştır. Anahtarlamalı güç kaynağı (SMPS) topolojilerinin temelleri. Bazı güç, enerji ve DC kazanç ilişkilerinden bahsedilmiştir.

3. Bölümde Boost Dönüştürücü devresinin çalışma ilkeleri anlatılmıştır. Boost Dönüştürücü devresinin sürekli iletim modu (CCM) üzerine analiz yapılmıştır. Boost Dönüştürücünün güç dönüştürme tekniği, gerilimleri, akımları ve güç denklemleri ayrıntılı olarak çıkarılmıştır. Boost Dönüştürücü'nün elektrik seçim mekanizmalarının unsurları anlatılmıştır.

4. Bölüm durum uzayı ortalama tekniği kullanılarak durum uzayıyla Boost Dönüştürücü'nün tasarım ve modellenmesini içerir. Burada büyük sinyal, sabit durum ve küçük sinyal durum uzayı modelleri elde edilir ve sürekli zaman alanı kapsamında gerçekleştirilir.

5. Bölüm'de kontrol sistemi kavramları hakkında genel bakışlar anlatılır. Bazı önemli ve temel terminolojiler hakkındaki tanımlar tartışılır. Açık döngüler ile kapalı döngü sistemleri kavramları ile bunların belli avantaj ve dezavantajları sunulmuştur. Kontrol sistem tasarımından, prosedürlerinden ve telafilerden bahsedilmiştir. Bilgisayarla kontrol edilen sistemler ile dijital kontolör tasarım kavramları da sunulmuştur. Bunlara ek olarak, ayrı kök yer eğrisi ve z alanındaki geçici zaman yanıtı verilmiştir.

6. Bölüm'de hem sürekli zaman alanı hem de ayrık zaman alanı kapsamında kök yer eğrisi teknikleri kullanılarak Boost Dönüştürücü'nün bir İntegral kontrolör ile tasarımı, uygulanması ve hesaplamaları ayrıntılı olarak anlatılmıştır. Simulink/MATLAB kullanılarak yapılan simülasyonlar gerçekleştirilmiştir.

Performans parametreleri ve integral kontrolör kullanılarak elde edilen sonuçlar çizelgeler olarak verilmiştir.

7. Bölüm'de tam durum geri besleme kontrolü kavramları ayrıntılı olarak tartışılmıştır. Kutup yerleştirme tekniği ve Doğrusal optimal kuadratik regülatör (LQR) yöntemleri kullanılarak ayrık-zaman alanı kapsamında Boost Dönüştürücü'nün tasarımı, modelleme ve uygulanması gerçekleştirilir. Öncelikle kutup yerleşim yöntemi ile dijital kontrolör tasarımı ayrıntılı olarak tartışılır ve gerçekleştirilir. İkinci olarak LQR yöntemi kullanılan dijital kontrolör tasarımı tartışılır ve kontrolör elde edilir. Bu iki yöntem birbiriyle kıyaslanarak gösterilir. Simulink/MATLAB kullanılarak elde edilen simülasyon sonuçları gösterilir ve her bir kontrolör için aynı zamanda performans parametreleri de çizelgeler halinde verilir.

8. Bölüm'de Sonuçlar verilir ve ardından referanslar gelir.

CHAPTER 1. INTRODUCTION

1.1. Introduction

Power electronics and control system concepts have a basic relation to each other since the beginning. Switch mode power supplies (SMPS) is a variable structure periodic systems which its mechanism is determined by logic signals. A lot of researches are obtained with the analysis of switching DC-DC converters. Generally, a mathematical representation is considered with the related control circuits [1], [2].

Power electronics can be defined as the technology that use means of power semiconductor devices (operate as switches) for the process control and conversion of electrical energy into a form suitable for utilization by electronic equipment, machines and other devices.

“With the advent of Silicon Controlled Rectifiers (SCRs) in 1950s, the application of Power Electronics spread to various fields of Engineering such as in solid state industrial drives, high frequency converters, inverters, uninterruptible power supplies, Electronic tap changers, lighting control, home appliances and in medical instrumentation. Gradually since 1970, various Power Electronic devices were developed and were available commercially. The typical classification of the devices based on the controllability characteristics are, Uncontrolled turn on and turn off devices (eg. Diode), Controlled turn-on and uncontrolled turn-off (eg. SCR) and Controlled turn on and off characteristics (eg. Power Bipolar Junction Transistor (PBJT)), Metal Oxide Semiconductor Field Effect Transistor (MOSFET), Gate Turn Off thyristors (GTOs), Static Induction Thyristors (SITH), Insulated-Gate Bipolar Transistors (IGBTs), Static Induction Thyristors (SITs) and MOS-Controlled Thyristors (MCTs) ”[3].

In the power electronics circuits, some elements like diode can be controlled by power circuit without any control signals, and some elements like SCR needs a control signal to turn ON and can be turned OFF by the power circuit. Also, some elements like MOSFET need a control signal to be turn ON as well as to turn OFF. In addition, some improvements to the current and voltage ratings are continued with the evolution of those power electronics devices [3].

Then, related to the acceleration in the scientific production, the control signal and power devices can be included in the same semiconductor for the design of energy processing circuits. Thus, there is no any distance in the design methodology between the engineers of power electronics and digital integrated circuit designers. Therefore, wide digital solutions to the control problem for power electronic circuits are expected in the next years [1].

Power converters can be classified like: DC-DC Converters, AC-AC Converters, Phased controlled converters (AC-DC Converters), DC-AC converters (Inverters), and AC voltage controllers (regulators). Based on this classification, electric power is transformed from one form to another one in order to increase the efficiency and the production which are needed in several devices and industrial applications [4].

DC-DC converters are widely used in the industrial applications since they convert a DC voltage source to other different voltage levels which is very important in power electronics field. DC-DC converters are extensively used in distributed power supply systems and modern power electronics devices due to their high efficiency, high power density, high power levels, low cost, and small size [5]. They are also used extensively in Uninterruptible power supply, power factor improvement, harmonic elimination, fuel cells applications and in photovoltaic arrays. They are also used in other various applications like battery operated vehicles, trolley cars, traction motor control and DC motors control [3]–[8].

DC-DC Converters (also known as Choppers) can be step-up or step-down converters, and can have multiple output voltages. In case of step-down converters, output voltage is lower than the input voltage, while it is higher than the input voltage in case of step-up converters. Some converters could set to be step-up and step-down converters. In addition DC-DC converters could have a single output or multiple outputs. Also, the output voltage can be fixed or adjustable. Therefore, a wide variety of topologies is employed by DC-DC converters technology [9]. Every case has its application in electrical and electronic circuits and systems.

DC-DC converters design can be obtained using one of the power electronics switching elements as MOSFETs, power BJTs or IGBTs. These switching elements can provide high efficiency, fast dynamic response, smooth control, small size, low cost and maintenance. The output voltage of the converter is controlled by On-Off interval tuning of these power switches. On other words, these main switching devices are driven with the desired duty cycle. Duty cycle is the ratio between the On-time to the total switching period. Two different control strategies can be used in the subject of duty cycle control; time ratio control and current limit control. Time ratio control can be performed using a constant frequency operation or variable frequency operation. In case of constant frequency which is also known as pulse width modulation (PWM) technique, the On-time varies and the switching frequency is kept constant. While in case of variable frequency which is also known as frequency modulation technique, the switching frequency varies and the On-time is kept constant. Frequency modulation has different disadvantages since it needs accurate control during the wide variations in switching frequency. Therefore, some problems could face the designers such as the needs of filter design, interference with signaling and discontinuous load current if the device’s Off-time increased to high value. Current limit control is performed in the case of applications that use energy storage elements as loads. Then, the switch On-Off interval time is controlled related to the maximum and minimum desired values of load current [3], [10].

In order to control the duty cycle of these converters, feedback circuit is required. The famous feedback circuit which is widely used in this subject is the comparison between the output voltage with the reference voltage. Control signal is generated and performed to the switching device using PWM technique to control the duty cycle variations and to obtain the required fixed output voltage. Both of the output voltage and the inductor current can be used in the feedback circuit. Therefore, there are two modes of feedback circuits; voltage mode control and current mode control. The voltage mode control uses the output voltage only while the current mode uses both the output voltage and the inductor current for the feedback.

In voltage mode control, just one outer loop is used to compare the actual output voltage with reference desired voltage. Then, error signal is given to the compensator to be performed and execute the control signal which controls the duty cycle via PWM technique. PWM signal can be performed in much ways; for example, it can obtained directly from the microcontroller via a driver circuit or it can obtained using comparison techniques which compare the control signal with a ramp signal to produce the needed pulses to drive the switching device.

In current mode control, two loops are required. One inner loop is added to the basic outer loop. The inner loop is used to speed up the system's response. This mode is very effective in case of non-minimum phase systems like Boost and Boost-Buck Converters [3], [11]. The outer loop which is discussed above in the voltage mode control is used here but with much slower response than the inner loop. The signal which is executed from the outer loop is an input current reference signal for the inner loop. This reference value is compared with the actual inductor current value and this approach must be performed much faster than the outer loop. Therefore, the outer loop purpose is to provide the inner loop with the reference value according to the error voltage signal. And thus, the response performance of the system can be much better and faster. Regardless of this, this approach can face high frequency instability due to sub-harmonics during the compensator design [3].

In this research work, DC-DC Boost Converter under voltage mode control is considered. Boost Converter, (also known as a step-up converter) is a type of switched-mode DC-DC converter which produces a constant output voltage that is greater than input voltage. A number of methods are appeared for ac equivalent circuit modeling such as circuit averaging, current injected approach, averaged switch modeling and state space averaging method [5]. The state space averaged modeling is widely used in DC-DC Converter's modeling [12].

Averaging techniques such as small signal model has been widely used to derive model of DC-DC converters [2], [13], [14]. This circuit model is used to simulate the system and design the suitable controller. Small signal model gives an idea of the dynamics and variations about steady state operating point. It is considered that state variables and control variable have small ac variations/disturbances around the steady state operating point. But, the small signal model changes related to the variations in the operating points [12].

Researchers have delivered great efforts in the subject of DC-DC converters control and many controllers under various considerations were proposed. In the mid-1960, investigations of basic switching converters, modeling, and analysis are commenced. In the mid-1980’s, advancement in the closed loop control of switching converters were combined with the regulation and dynamic response improvements [15]. In 1990’s until this time, new approaches and control methods are addressed and discussed [3]. Tse et al (2002) have performed analysis of the non-linear phenomena in the power electronics systems. The chaotic behavior of the Boost Converter is discussed. The author paved the way to the power electronic converters to be used in several applications. Gonzalez et al (2005) used passivity based non-linear design to perform an observer controller for the Boost Converter. The obtained output shows undesirable overshoots and undershoots. A new switching cycle compensation algorithm have proposed by Feng et al (2006) to optimize the transient performance of the DC-DC converters under input voltage variations. The controller is implemented used FPGA and sensing resistors to measure the inductor current. The system has an improvement in the dynamic performance. But this type which based on the current measurement is not much effective due to the higher power loss. Bo-Cheng et al (2008) have designed a state feedback controller for the Boost Converter using sampled inductor current. It shows the control ability of chaotic behavior of the Boost Converter and stability criterion is achieved. But, the robustness of the control law was not checked. Chen et al (2008) have identified the stable operating point by making analysis to the Boost Converter. They suggested that the effects of fast and slow scale bifurcations that occur in voltage mode control and current mode control can be eliminated by increasing the feedback gain. Sreekumar and Agarwal (2008) have obtained output voltage regulation for the Boost Converter using new hybrid control algorithm. System was stable under operating conditions, but this control algorithm cannot be applied for higher switching frequency since it causes limitations on driver circuit, device speed, and power loss. On other hands, Carlos et al (2009) have designed a controller using Linear Quadratic Optimal Regulator method. Stability and performance of the converter are achieved. Authors built the controller taking into account more than one plant by using Linear Matrix Inequalities (LMI) approach. Liping Guo, John Y. Hung and R. M. Nelms (2009) have designed a fuzzy controller for the Boost Converter. Experimental results showed fast transient response with stable steady state and voltage regulation under circuit parameter variations. Also, Mariethoz et al (2010) have used state feedback control techniques for the Boost Converter with load estimation based on observer controller. Authors have taken into the account the time response and the capability of disturbance rejection. They used five different methodologies for the controller design. The system response shows performance improvements and disturbance rejection. Mohammed Alia et al (2011) and Mohammed Abuzalata (2012) have used LabVIEW software as a platform to make PID tuning and to generate PWM techniques respectively.

It is understood from the above survey that the state feedback controller techniques have obtained the stability criterion and have achieved a good dynamic performances under operating point conditions. The previous designs suffer from various problems. There are two main reasons: the first reason comes from the requirement of good modeling and effective analysis for the converters. While the second reason is that the circuit topologies of switching converter have a wide range, and thus the control of these converter using conventional approaches become complicated and topology dependent [16]. Therefore, the problem control will be more difficult when the DC-DC converter is considered as non-minimum phase system (has an unstable zero on the right-half-plane) such as the Boost Converter.

Usually, such averaged models are linearized at a certain operation point in order to derive a linear controller. Nevertheless, a design that disregards converter nonlinearities may result in deteriorated output signal or unstable behavior in presence of large perturbations [17]. In order to take into the account nonlinearities and parameter uncertainty, the study of converter models and robust control methods is still an active area of investigation [18]–[22].

PID controller is a traditional linear control method used in many applications [23]. Linear PID controllers for DC-DC converters are usually designed by frequency domain methods applied to the small signal models of the converters. PID controller response could be poor against changed in the operating points [24]–[26].

Frequency domain methods of design such as root locus techniques or frequency response techniques can't design and specify all closed loop poles of the higher order system since those methods don't have sufficient parameters to place all of the closed loop poles. State space methods such as state feedback control solve this problem by introducing into the system other adjustable parameters [27].

State feedback control and the approach of Linear Quadratic Optimal Regulator (LQR) have a good control solution for the systems with good dynamic response, accepted robustness, output regulation, and disturbances rejection.

1.2. Problem Statement

Traditionally, small signal linearization techniques have largely been employed for controller design. Many control strategies have been proposed, and duty cycle is controlled to obtain the desired output voltage. Control strategies that are based on the linearized small signal model of the converter have good performance around the operating point as it will discussed in this research. However, a Boost Converter's small signal model changes when the operating point varies. The poles and a right-half-plane zero, as well as the magnitude of the frequency response, are all dependent on the duty cycle. Therefore, it is difficult for the controller which using small signal linearization techniques such as linear PID controller to respect well to changes in operating point, and they exhibit poor performance when the system is subjected of large load variations.

1.3. Research Objectives

In this research, various methods and control techniques will be applied to the Boost Converter system. It is aimed to check these controllers' effects to the system's performance. Also, it is aimed to select the best controller in which robust stability and performance despite model inaccuracies will be achieved. The designed controller is expected to provide excellent static and dynamic characteristics at all operating points. These objectives are organized as follows:

1. To investigate different topologies currently working for power systems.
2. To investigate different control techniques and their effects.
3. To protect the input source and the load.
4. To maintain a stable regulation of the output voltage.
5. To maximize the bandwidth of the closed-loop system in order to reject disturbances.
6. To satisfy desirable transient characteristics.
7. Development of control strategy with fast response in order to attain stable, quality and fault tolerant power system under static and dynamic conditions.

1.4. Overview of the Research Work

This research consists of eight chapters; the first chapter presents an introduction, research objectives and thesis organization, while the other chapters are organized as follows:

Chapter 2 presents extensive introduction about the definitions and classifications of power supplies. Linear Voltage and switching regulators basic functions have been discussed. The fundamental of switching-mode power supply (SMPS) topologies. Some power, energy and DC gains relations have been covered.

Chapter 3 presents the operating principles of the Boost Converter circuit. Analysis is applied to the Boost Converter's circuit in continuous conduction mode (CCM). Boost Converter's power conversion technique, voltages, currents and power equations are derived in details. The electrical selection mechanisms of Boost Converter's elements have been covered.

Chapter 4 presents the design and modeling of Boost Converter by state space method using state space averaging technique; large signal, steady state and small signal state space models are obtained and satisfied under continuous time domain.

Chapter 5 presents some overviews about control system concepts. Definitions about some important and basic terminologies have been discussed. The concept of the open loop versus closed loop systems, and the major advantages and disadvantages of them have been presented. Control system design; procedures and compensation have been covered. Computer controlled systems and digital controller design concepts are also presented. In addition, discrete root locus, stability and transient time response in z-domain have been covered.

Chapter 6 presents design, implementation and calculations in details of Boost Converter with an Integral controller using root locus techniques under both continuous time domain and discrete time domain. Simulation using Simulink/MATLAB has been carried out. Performance parameters and obtained results using integral controller are tabulated.

Chapter 7 discussed in details the concepts of full state feedback control. Design, modeling, and implementation of Boost Converter with state feedback controller using pole placement technique and Linear Quadratic Optimal Regulator (LQR) methods under discrete time domain are obtained. Firstly, digital controller design with pole placement method is discussed in details and carried out. Secondly, digital controller design with LQR method is discussed and controller has been obtained. The two methods are shown compared against each other. Simulation results using Simulink/MATLAB are shown and the performance parameters are also tabulated for each controller.

Chapter 8 Conclusion are given and references follow this chapter.

CHAPTER 2. AN OVERVIEW OF POWER SUPPLIES

2.1. Classification of Power Supplies

Power supply is a constant voltage source with a maximum current capability, this technology allows us to build and operate systems and electronic circuits. All electronic circuits, both analog and digital, require power supplies. In some cases, electronic systems need more than one dc supply voltage. In our daily life, power supplies are used widely, such as personal computers, communications, instrumentation equipment, medical, and defense electronics. Using a transformer, rectifier and filter, a dc supply voltage can be derived from battery or an ac utility line. This resultant dc supply voltage is not constant and could has ac ripples, *voltage regulator* usually used to attenuate the ac ripples and set the dc voltage more constant, so it will be enough suitable and safe for most applications [9].

Power supplies have two general classes: regulated and unregulated. Despite source line voltage, load and temperature variations, regulated power supplies have fixed output voltage with small change range, 1-2% of the nominal/desired value. Regulated dc power supplies also called *dc voltage regulators*. There are also *dc current regulators*, as battery chargers [9], [10].

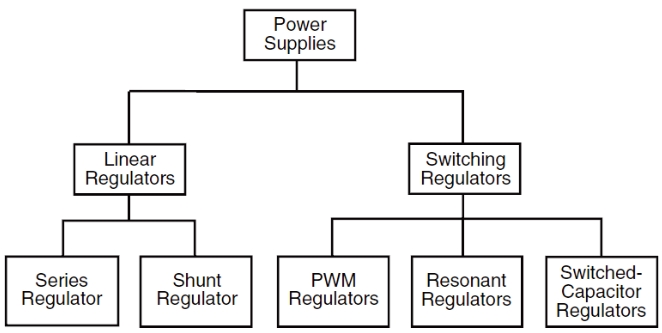


Figure 2.1. Classification of Power Supply Technologies [9]

Voltage regulators or power supplies are classified into two popular categories, *linear regulators* and *switching-mode power supplies* as shown in Figure 2.1. Linear regulator has two basic topologies, series and shunt voltage regulator. On the other hand, switching-mode voltage regulators have three categories, pulse width modulator, resonant and switched-capacitor regulators.

Transistors in the linear regulator circuit work in the active region as dependent current sources, but they work as switches in the switching regulator circuit. In the first case, it is expected to have high voltage drops at high currents, waste large amount of power; which leads to have a low efficiency system. Furthermore, linear regulators are large and heavy, but offer low noise scale. On the contrary, switching regulators show less power dissipation, very low voltage drop at high currents and nearly zero current in the case of high voltage drop across them, resulting high efficiency (approximately 80-90%) related to the low conduction losses. Switching losses and switching frequency have proportional relation, so efficiency will be reduced in the high frequencies.

PWM and resonant regulators have small size, light weight and very good conversion efficiency, thus they are used at high power and voltage levels. Switched-Capacitor and linear regulators can be integrated fully and are used in low power and voltage applications.

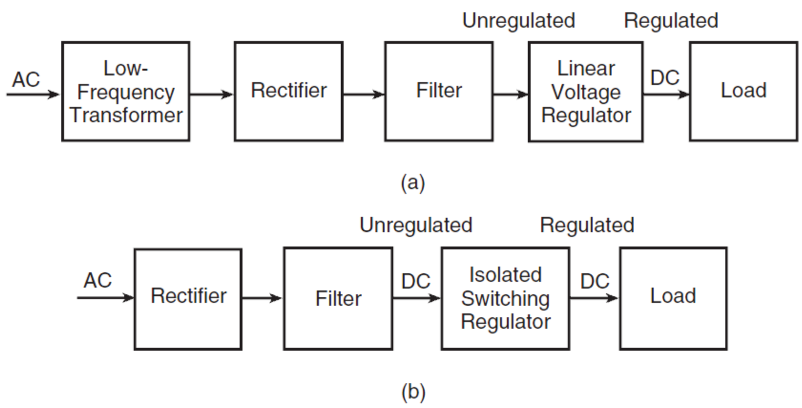


Figure 2.2. Block diagrams of AC-DC power supplies. (a) With a linear regulator. (b) With a switching mode voltage regulator [9]

Two AC-DC power supplies examples are shown in Figure 2.2. The power supply in the Figure 2.2(a) has a linear voltage regulator, while power supply in the Figure 2.2(b) has a switching-mode voltage regulator.

As shown in Figure 2.2(a), diagram contains step-down low-frequency transformer, rectifier, low pass filter, linear voltage regulator and a load. Since, the ac line has a very low frequency (50Hz, 60Hz in Europe and USA respectively), thus the line transformer will be heavy and huge. After filter stage, the output voltage is unregulated which can be varies related to the ac line changes, then a voltage regulator is needed to have a stable and constant dc voltage to the load.

Figure 2.2(b) contains rectifier, low pass filter, switched-mode voltage regulator and a load without using the step-down transformer, the ac voltage is directly rectified from the ac power line. The switching-mode voltage regulator works under high switching frequency, this frequency is much higher than one of ac line power sources, thus the transformer will be small in size and weight, and also the capacitor and inductor values are reduced.

"The switching frequency usually ranges from 25 to 500 kHz. To avoid audio noise, the switching frequency should be above 20 kHz. A PWM switching-mode voltage regulator generates a high-frequency rectangular voltage wave, which is rectified and filtered. The duty cycle (or the pulse width) of the rectangular wave is varied to control the dc output voltage. Therefore, these voltage regulators are called PWM DC–DC converters." [9]

A voltage regulator should offer the desired regulated dc output voltage to the load, which is a stable and constant even if the source line voltage, load current or temperature varies. The output voltage in PWM DC-DC converters can be stepped up or stepped down from the source input voltage. The output voltage is lower than the input voltage in case of step-down converters, while it is higher than the input voltage in case of step-up converters. Some converters play the role as step-up and step-down converters. DC-DC converters could have a single output or multiple outputs. In addition, the output voltage can be fixed or adjustable. Every case has its application in electrical and electronic circuits and systems. Some applications require a fixed output voltage and some applications need adjustable output voltages like those in laboratory tests. In general vision, most of power converters or supplies require high efficiency, high reliability and low costs.

2.2. Voltage Regulators Basic Functions

Zener diode regulator is a good simple example for voltage regulator (shunt regulator) as shown in Figure 2.3. In the view of performance, Zener diode regulator is not suitable for most applications. Subsequently, negative feedback techniques have very good response and used to improve the performance. A block diagram of negative feedback voltage regulator is shown in Figure 2.4. Circuit of negative feedback converter has a control circuit. Control circuit acts as a close loop circuit and compares the actual feedback output voltage with the reference voltage, and then it generates an error voltage which adjusts the transistor base current to keep the output voltage constant and stable.

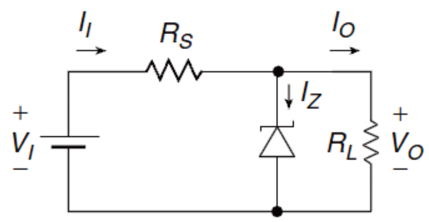


Figure 2.3. Zener diode voltage regulator [9]

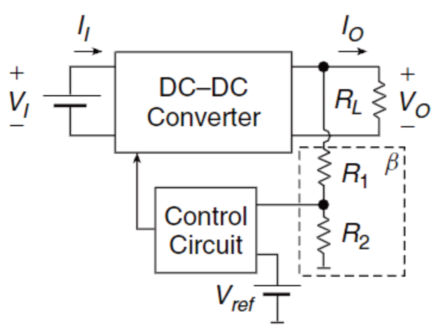


Figure 2.4. Voltage regulator with negative feedback [9]

It is known that the load current can changes over a wide range. Converters must have short circuit or current overload protection circuit to limit the output current, so it leads the power supply and load to safe level.

On the other hand, the DC-DC converters could have some changes in input voltage values, for example:

a. Input voltage can be a battery, and the battery output voltage varies according to battery discharge.
b. The input voltage can be a rectified single phase or three phase ac line voltage, and the ac line voltage normally varies (10-20%) of its nominal peak voltage which directly affects the rectified dc output voltage.
c. Semiconductor and passive elements operating temperature could change, resulting bad effects to the power supply performance.

So DC-DC converters must cover the following points:

a. High performance conversion within a small tolerance range (e.g. ±1%).
b. High performance output voltage regulation against input voltage, load current and the temperature variations.
c. Reduction of output ripples voltage below specific level.
d. Provide fast response against disturbance variations (e.g. input variation).
e. Provide dc isolation and multiple outputs.

2.3. Power in DC Voltage Regulators

Switching-mode converter has pulsating input current i_{in}, the dc component of the converter input current is:

$$I_{in} = \frac{1}{T}\int_0^T i_{in}\, dt \qquad (2.1)$$

The dc input power of a DC-DC converter is:

$$P_{in} = \frac{1}{T}\int_0^T V_{in} i_{in}\, dt = \frac{V_{in}}{T}\int_0^T i_{in}\, dt = V_{in} I_{in} \qquad (2.2)$$

Suppose the ac components of the output voltage and current are very small and neglected, the dc output power of a DC-DC converter:

$$P_o = V_o I_o \quad (2.3)$$

The power loss in the converter is:

$$P_{loss} = P_{in} - P_o \quad (2.4)$$

The efficiency of the DC-DC converter is:

$$\eta = \frac{P_o}{P_{in}} = \frac{P_o}{P_o + P_{loss}} \quad (2.5)$$

The normalized power loss is:

$$\frac{P_{loss}}{P_o} = (\frac{1}{\eta} - 1) \quad (2.6)$$

Which, it decreases as the efficiency of the converter increases.

2.4. DC Voltage Gain of DC Voltage Regulators

The dc voltage conversion ratio can be called also a *dc voltage gain* or a *dc transfer function*. The voltage dc gain is:

$$G_{VDC} = \frac{V_o}{V_{in}} \quad (2.7)$$

The current dc gain is:

$$G_{IDC} = \frac{I_o}{I_{in}} \quad (2.8)$$

Then, the efficiency can be written as:

$$\eta = \frac{P_o}{P_{in}} = \frac{I_o V_o}{I_{in} V_{in}} = G_{IDC} G_{VDC} \quad (2.9)$$

2.5. Static Characteristics of DC Voltage Regulators

The static characteristic of a dc voltage regulator is described by three parameters: line, load, and thermal regulation. In most of regulators, the output voltage V_o increases as input voltage V_{in} increases. Figure 2.5 illustrates line regulation which acts as the ability measurement of converter to save the output voltage at the desired value V_{onom} against input voltage variation.

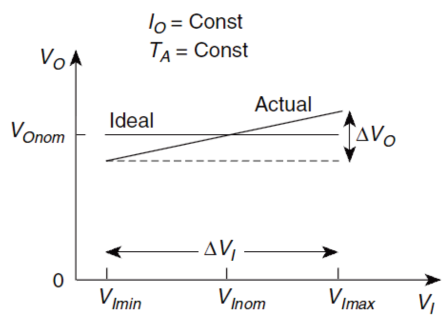


Figure 2.5. Output voltage versus input voltage for voltage regulator [9]

The ratio of change between output voltage and input voltage called *line regulation*, assume the output current I_o and ambient temperature T_A are constant, then the line regulation is [9]:

$$line\ regulation = \frac{\Delta V_o}{\Delta V_{in}} \left(\frac{mV}{V}\right) \quad (2.10)$$

The ratio of change between the percentage change in the output voltage and input voltage called *percentage line regulation*, assume the output current I_o and ambient temperature T_A are constant, then the percentage line regulation is [9]:

$$percentage\ line\ regulation = \frac{\frac{\Delta V_O}{V_{onom}} \times 100\%}{\Delta V_{in}} \quad \left(\frac{\%}{V}\right) \tag{2.11}$$

In order to have ideal system, line regulation must be zero or very small.

Due to the varying in load resistance, the output voltage V_o of regulator increases as the load current I_o decreases. Figure 2.6 illustrates load regulation which acts as the ability measurement of converter to save the output voltage at the desired value V_{onom} against load variation over a certain range of load current.

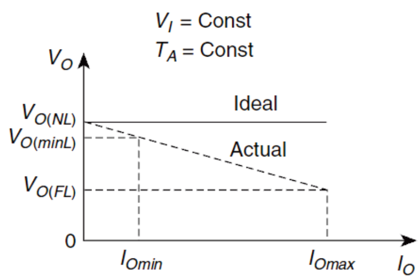


Figure 2.6. Output voltage versus output current for voltage regulator [9]

Assume the input voltage V_{in} and ambient temperature T_A are constant, and then the load regulation is [9]:

$$load\ regulation = \frac{\Delta V_o}{\Delta I_o} \left(\frac{mV}{A}\right) \tag{2.12}$$

$$percentage\ load\ regulation = \frac{V_{o(NL)} - V_{o(FL)}}{V_{o(FL)}} \times 100\% \quad (\%) \tag{2.13}$$

Where: $V_{o(NL)}$ is the no-load output voltage, $V_{o(FL)}$ is the full-load output voltage.

Note: In PWM converters that operate in the continuous conduction mode (CCM), I_{omin} is not zero. Thus, the output voltage at the minimum load current is $V_{o(minL)}$, then the load regulation will be modified to be like [9]:

$$percentage\ load\ regulation = \frac{V_{o(minL)} - V_{o(FL)}}{V_{o(FL)}} \times 100\% \quad (\%) \tag{2.14}$$

For ideal converter systems, load regulation must be zero or very small.

Assume output current I_o and input voltage V_{in} are constant, *thermal regulation* is:

$$percentage\ thermal\ regulation = \frac{\frac{\Delta V_o}{V_{onom}} \times 100\%}{\Delta P_D} \quad \left(\frac{\%}{W}\right) \tag{2.15}$$

where, ΔP_D is the change in power dissipation. For ideal converter systems, thermal regulation must be zero or very small.

At a given operating point, the dc input resistance of a dc voltage regulator is:

$$R_{in(DC)} = \frac{V_{in}}{I_{in}} \tag{2.16}$$

And

$$P_o = \frac{V_o^2}{R_L} \tag{2.17}$$

$$P_{in} = \frac{V_{in}^2}{R_{in(DC)}} \tag{2.18}$$

Then the efficiency of the converter can be written as:

$$\eta = \frac{P_o}{P_{in}} = \frac{\frac{V_o^2}{R_L}}{\frac{V_{in}^2}{R_{in(DC)}}} = \left(\frac{V_o}{V_{in}}\right)^2 \frac{R_{in(DC)}}{R_L} = G_{VDC}^2 \frac{R_{in(DC)}}{R_L} \tag{2.19}$$

2.6. Dynamic Characteristics of DC Voltage Regulators

Ideal voltage regulator system must save the output voltage at desired value against any ripples comes from input, so the system ability to reject any ripples come from input called is *power supply rejection ratio* [28]. This ratio is widely used in voltage regulator datasheets to describe the amount of noise from a power supply that a particular device can reject [29].

$$PSRR = 20 \log \frac{v_o}{v_i} \; dB \qquad (2.20)$$

It is known from control system theory that the *Time Response* is the behavior of the system when some input applied, this response contains much information about the system respect to time response specifications as overshoot, settling time and steady state error. Time response is formed by the *transient response* and *steady state error response*. Transient response describe the behavior of the system at the starting short time until arrives the steady state value. This response will be our study in this section.

Dynamic transient response of DC-DC voltage regulators is tested by applying a step change at one of the input parameters of the voltage regulator; *line transient response* will be derived by applying a step change on the input voltage, and *load transient response* will be derived by applying a step change on the load. Therefore, output voltage must be stable at the desired value.

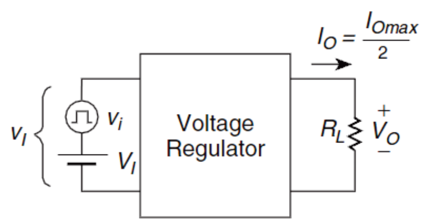


Figure 2.7. Circuit for testing the line transient response of voltage regulators [9]

Figure 2.7 illustrates a test circuit for line transient response at a fixed load current $I_O = \frac{I_{Omax}}{2}$. It is clear from the Figure that a step change is applied to the input voltage and the output behavior of the system is monitored in this case.

Figure 2.8 illustrates the line transient response of the voltage regulator, Figure 2.8(a) contains the applied input voltage waveform which has step changes, and Figure 2.8(b) contains the behavior of the output as a time response against the step input changes.

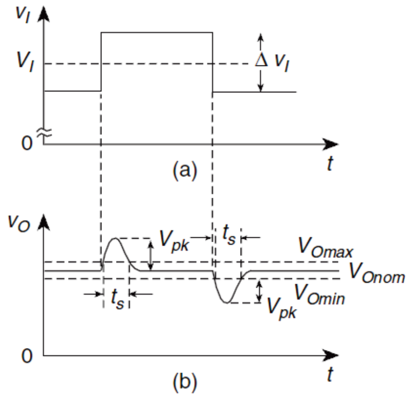


Figure 2.8. Waveforms illustrating line transient response of voltage regulators. (a) Waveform of input voltage. (b) Wave form of the output voltage. [9]

It is seen that the output voltage returns to the steady state value in both cases (first case, when the input voltage increases, other case, when the input voltage decreases).

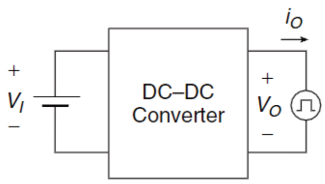


Figure 2.9. Circuit for testing the load transient response using an active current sink [9]

The same idea, Figure 2.9 illustrates a test circuit for load transient response. It is clear from the Figure that a step change is applied to the load, and the output behavior of the system is monitored in this case.

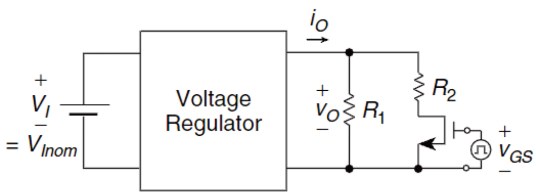


Figure 2.10. Circuit for testing the load transient response using a switched load resistance [9]

Step changes to the load can be done by applying a current sink using an active load as shown in Figure 2.9, or the circuit can be tested using a switched load resistance as shown in Figure 2.10.

Figure 2.11 illustrates the load transient response of the voltage regulator, Figure 2.11(a) contains waveform of the change in load current, and Figure 2.11(b) contains the behavior of the output as a time response against the step load changes.

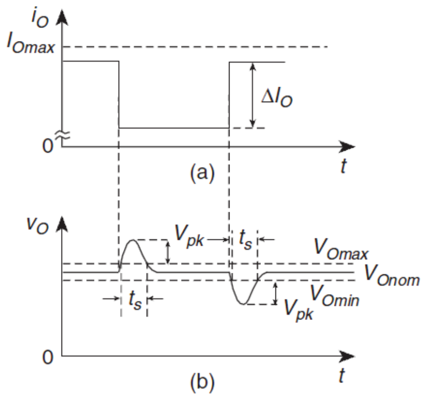


Figure 2.11. Waveforms illustrating load transient response of voltage regulators. (a) Waveform of load current. (b) Wave form of the output voltage. [9]

It is seen that the output voltage returns to the steady state value when the load current increases, also when the load current decreases.

After two tests are applied to check the behavior of output voltage against line and load variations, the transient response is obtained. This response is very important in any system. It opens the door to the stability and un-stability; also the system can have overshoot, over-damped, under-damped or critically-damped transient response. The settling time t_s and peak value V_{pk} should be below specific levels (very small values). Here to avoid any risk, close loop system is applied, in close loop power supplies is expected to have acceptable and very good transient response (non-oscillatory with very small settling time).

2.7. Linear Voltage Regulators

In this section, linear voltage regulator will be briefly discussed with its two basic topologies: series and shunt voltage regulator. In linear voltage regulators, transistors are operated as dependent current sources [9], thus high voltage drop at high currents, large power dissipations and low efficiency.

"The major characteristics of linear voltage regulators are as follows:

1. Simple circuit;
2. Very small size and low weight;
3. Cost-effective;
4. Low noise level;
5. Wide bandwidth and fast step response;
6. Low input and output voltages, usually below 40 V;
7. Low output current, usually below 3A;
8. Low output power, usually below 25 W;
9. Low efficiency (especially for $VI \_ VO$), usually between 20% and 60 %;
10. Only step-down linear voltage regulators are possible;
11. Only non-inverting linear voltage regulators are possible;
12. Large low-frequency (50 or 60 Hz) transformers are required in AC–DC power supplies with linear voltage regulators."[9]

2.7.1. Series Voltage Regulators

In series voltage regulator, the transistor acts as a pass-transistor. Transistor's collector to emitter voltage V_{CE} works as a compensator to the output voltage against input voltage varying. As shown in Figure 2.12,

$$V_{in} = V_{CE} + V_o \qquad (2.21)$$

Since $V_o$is constant, then:

$$\Delta V_{in} = \Delta V_{CE} \qquad (2.22)$$

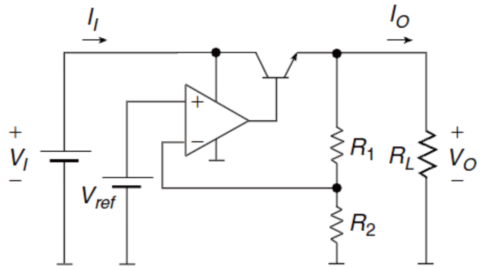


Figure 2.12. Series Voltage Regulator [9]

Since V_{CE} works as a compensator and V_o is constant, then any change in the input voltage will result the same change in the V_{CE} as expressed in Equation (2.22). On other words, pass transistor works like a variable resistor R_T. Therefore, series voltage regulator acts as a voltage divider:

$$V_o = \frac{R_L}{R_L + R_T} V_{in} \tag{2.23}$$

The voltage across the variable resistor R_T is:

$$V_{CE} = V_{in} - V_o = R_T I_o \tag{2.24}$$

From Equation (2.22), and I_o equal to fixed value:

$$\Delta V_{in} = \Delta V_{CE} = \Delta R_T I_o \tag{2.25}$$

$$\Delta R_T = \frac{\Delta V_{in}}{I_o} \tag{2.26}$$

Thus, the efficiency of series voltage regulator is expressed like:

$$\eta = \frac{P_o}{P_{in}} = \frac{I_o V_o}{I_{in} V_{in}} \approx \frac{V_o}{V_{in}} = G_{VDC} \quad \text{(Let } I_o \approx I_{in}\text{)} \tag{2.27}$$

From Equation (2.27), it is clear that the efficiency of a series voltage regulator equal to the voltage dc gain. Thus, low efficiency will occur if V_{in} is much higher than V_o.

For example, let $V_{in} = 18V$ and $V_o = 6V$, then $\eta = 33.3\%$. However, let $V_{in} = 7V$ and $V_o = 4V$, then $\eta = 57.14\%$. Therefore, the power loss in the transistor:

$$P_{loss(T)} \approx I_o(V_{in} - V_o) \tag{2.28}$$

It is seen that the power loss in the pass transistor has a proportional relation with the load current I_o and the voltage across the pass transistor ΔV_{CE}. On other words, while input voltage V_{in} does not drop too low, which derive op-amp to saturation region, the series voltage regulator will operate properly.

As shown in Figure 2.12, op-amp circuit acts like control circuit which work properly while op-amp is in the linear region. *Drop-out voltage (V_{DO})* is that voltage when input voltage V_{in} drop low under $V_{in(\min)}$. Most series voltage regulators have a fixed drop-out voltage (e.g. $V_{DO} = 2\ V$). *LDO Regulators* are those regulators with low drop-out voltage (e.g. $V_{DO} = 0.1V$), in this case the pass transistor is replaced with a pnp transistor or n-channel power pass MOSFET [9].

2.7.2. Shunt Voltage Regulators

In shunt voltage regulator as shown in Figure 2.13, the transistor acts like a shunt-transistor. Transistor's collector current I_C works as a compensator against input voltage or load current changes. Thus, the output voltage will held constant as I_C varying.

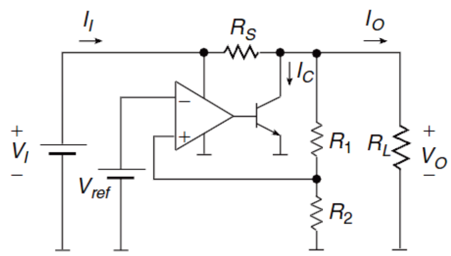


Figure 2.13. Shunt Voltage Regulator [9]

Shunt transistor operate like a variable resistor, any decease in the output voltage will cause a decrease in the op-amp output voltage, the shunt transistor switch less heavily, and then the variable resistor will increase. Then, less current is deviated from the load; causing an increase in load current and the output voltage.

$$I_{in} = I_C + I_o \tag{2.29}$$

At a fixed input voltage V_{in}, input current I_{in} will held constant:

$$I_{in} = \frac{V_{in} - V_o}{R_s} \tag{2.30}$$

Then any change in the load current I_o will cause the same change in I_C,

$$\Delta I_C = -\Delta I_o \tag{2.31}$$

Replace I_{in} in Equation (2.29) by Equation (2.30), then

$$\frac{V_{in} - V_o}{R_s} = I_C + I_o \tag{2.32}$$

Also, if load current I_o is at a fixed value, then any change in the input voltage V_{in} will cause change in I_C,

$$\frac{\Delta V_{in}}{R_s} = \Delta I_C \tag{2.33}$$

Then, the output voltage V_o will be constant while the voltage across R_s varying, which is controlled according to the change in the collector current I_C.

$$\Delta V_{in} = R_s \Delta I_C = \Delta V_{R_s} \tag{2.34}$$

In shunt voltage regulator, power loss will occur in both R_s and shunt-transistor:
The power loss in R_s is:

$$P_{R_s} = V_{R_s} I_{in} = (V_{in} - V_o)(I_C + I_o) \tag{2.33}$$

The power loss in the shunt-transistor is:

$$P_{TS} = V_o I_C = V_o(I_{in} - I_o) \quad (2.34)$$

Then the efficiency is expressed like:

$$\eta = \frac{P_o}{P_{in}} = \frac{I_o V_o}{I_{in} V_{in}} = \frac{V_o}{V_{in}} \frac{I_o}{I_o + I_C} \quad (2.35)$$

Since the power loss in series voltage regulator occurs just in pass-transistor, and then the efficiency in the shunt voltage regulator is less than the efficiency in series voltage regulator.

2.8. PWM DC-DC Converters

Switched-mode converters achieve voltage regulation by transfer energy from input to output using the control input technique of a switching device (power MOSFETs are often used as controllable switches) yielding a regulated output power.

Control circuit in the switched-mode converters is considered as the essential key to obtain a well regulated output voltage under the variations of the input voltage and load current. Therefore, a controller block will be added as an integral part in any power processing system as shown in Figure 2.14.

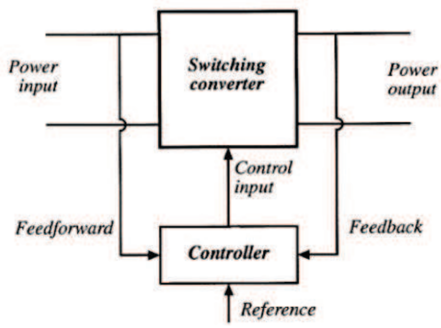


Figure 2.14. The switching converter, a basic power processing block with a control circuit [30]

Switched-mode converters operate at high frequencies, thus a small transformer can be used. In addition, those types of converters have a small size, and this advantage is very important in many applications. In switched-mode converters, the efficiency, power density and power levels are high. Also, they can be step-up or step-down converters and can have multiple output voltages [10], [31].

As a disadvantage of switch-mode converter, a noise is presented due to the switching action of semiconductor elements at both input and output of the supply. Also, control circuit is more complex compared with that used in linear regulation.

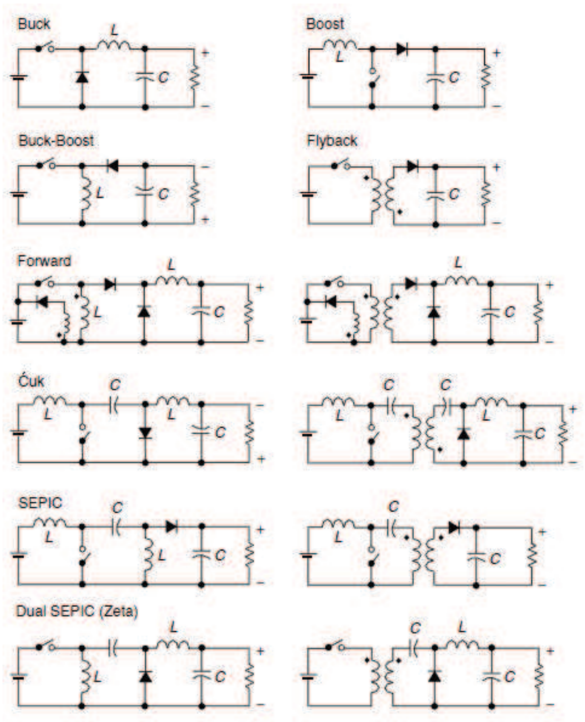


Figure 2.15. Single ended PWM DC-DC non-isolated and isolated converters [9]

A wide variety of topologies is employed by switched-mode converter technology [10], [32]. A family of single-ended and multiple switch PWM DC-DC converters are illustrated in Figure 2.15 and Figure 2.16 respectively. In this research, PWM DC-DC Boost Converter will be considered.

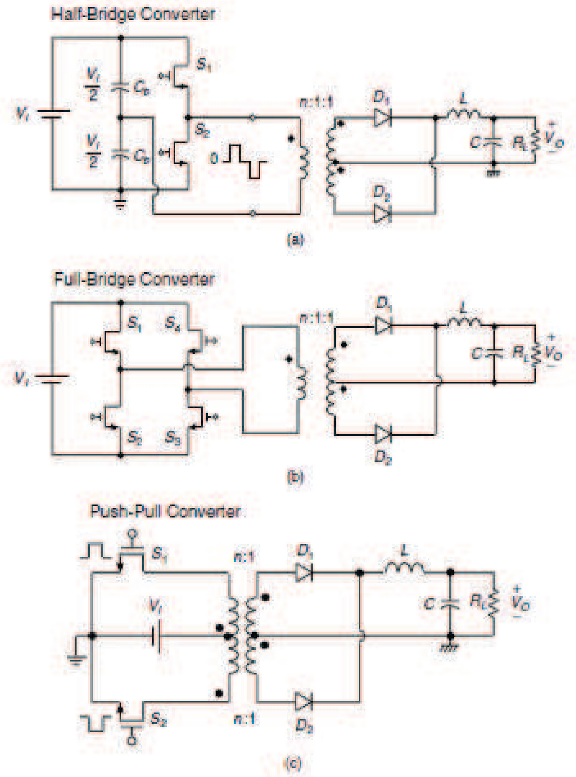


Figure 2.16. Multiple-switch isolated PWM DC-DC converters [9]

2.9. Power and Energy Relationships

The average value of a current $i(t)$ is:

$$I_{AV} = \frac{1}{T}\int_0^T i(t)\,dt \tag{2.36}$$

The rms value of this current is:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} \tag{2.37}$$

The average value of a voltage $v(t)$ is:

$$V_{AV} = \frac{1}{T}\int_0^T v(t)\,dt \tag{2.38}$$

The rms value of this voltage is:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \tag{2.39}$$

The instantaneous power is given by:

$$p(t) = i(t)v(t) \tag{2.40}$$

Over interval of time T the energy dissipation in a component is:

$$W = \int_0^T p(t)\,dt = \int_0^T i(t)v(t)\,dt \tag{2.41}$$

The instantaneous energy stored in a capacitor is:

$$W_C(t) = \frac{1}{2} C v_C^2(t) \tag{2.42}$$

The instantaneous energy stored in an inductor is:

$$W_L(t) = \frac{1}{2} L i_L^2(t) \tag{2.43}$$

The average charge stored in a capacitor over one period is zero. Likewise, the average magnetic flux linkage of an inductor over one period is zero for periodic waveforms in steady state.

CHAPTER 3. BOOST PWM DC-DC CONVERTER

3.1. Introduction

The Boost Converter is one of the fundamental switching-mode power supply (SMPS) topologies [33], containing at least two semiconductors (diode and a transistor) and at least one energy storage element (capacitor, inductor or both in combination) [34].

Boost convert is a DC-DC power converter with an output voltage greater than its input voltage, sometimes called a step-up converter since it steps up the source voltage from one level to high output voltage level. It has its power from any suitable DC sources, such as batteries, solar panels, rectifiers and DC generators. Normally, filters made of capacitors; a filter capacitor is added to the output of the Boost Converter to reduce the output voltage ripple [10], [28], [34], [35].

3.2. Operating Principles & Circuit Analysis

It is very important to understand the operating principles of the Boost Converter circuit, and how it always steps up the input voltage. Analysis will be applied to the Boost Converter circuit which shown in Figure 3.1 during one switching cycle.

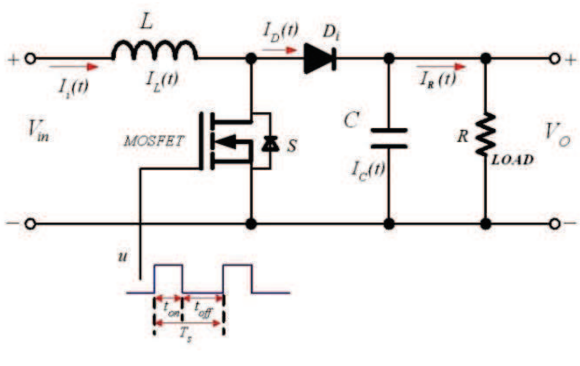


Figure 3.1. Basic circuit of Boost Converter

As shown in Figure 3.1, circuit consists of an inductor (L), a power switching element (e.g. MOSFET) acts as switch (S), a diode (D_i), a filter capacitor (C), and a load resistor (R). The switch S is turned on and off at switching frequency $f_s = {}^{1}/_{T_s}$, where T_s is the switch cycle period. Duty cycle (d) is the ratio of the on-time interval to the switch cycle period (T_s):

$$d = \frac{t_{on}}{T_s} \tag{3.1}$$

The switch cycle period can be defined as:

$$T_s = t_{on} + t_{off} \tag{3.2}$$

There are two operation modes of Boost Converter: Continuous Conduction Mode (CCM), and Discontinuous Conduction Mode (DCM) depending on the behavior of inductor current at the end of the switching cycle [36]. In CCM situation, energy is still left in the inductor when the switch is closed (the inductor current never falls to zero). In DCM situation, all the energy stored in the inductor is transferred to the load before the switch is closed (the inductor current falls to zero). The mode of operation is limited by the duty cycle and the load current for fixed value of inductor (L). This commonly occurs under light loads. As load current decreases, the mode of operation will change from CCM to DCM. Thus, to adjust CCM mode, there is reverse proportional relationship between the value of inductor (L) and the load current. In this research, the CCM is considered.

Figure 3.2, shows equivalent circuits of the Boost Converter. Figure 3.2(b) illustrates the Boost Converter topology when the switch (S) is ON and diode (D) is Off. In this case, polarity of the inductor left side is positive and input current flows through the inductor in clockwise direction. Thus, the inductor stores energy by generating a magnetic field. When the switch (S) is Off and the diode is ON as shown in Figure 3.2(c), the polarity will reversed and the previously created magnetic field will destroyed, then current flows through the load and the capacitor resulting two series sources charging the capacitor through the diode with higher voltage, and then the energy is transferred to the output. When the switch is then closed again, the capacitor provides the voltage and energy to the load. During this operation, diode is off, and then it prevents the capacitor from discharging through the switch.

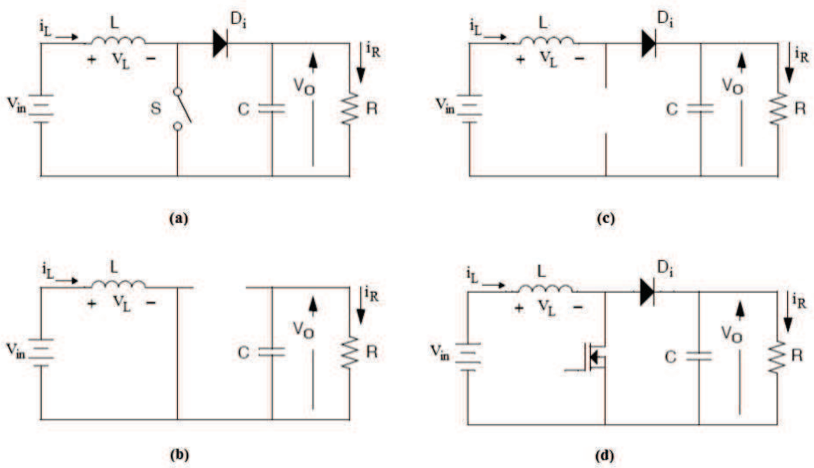


Figure 3.2. Basic Boost Converter circuit topology in CCM [33]

The switch must switching fast enough to prevent the capacitor and inductor from fully discharging. Thus the load will always see a voltage greater than the input source voltage.

In this circuit, a problem can appeared if the input voltage source varied with a high voltage level, then the input voltage of the converter will be higher than its output voltage and the diode will be ON for many switching cycles. Thus, diode could be destroyed because of the large current spike which generated through it. The same problem occurs at the starting time of the converter when the output voltage is initially zero and the input voltage is high. This situation will be continued until the steady state time. The converter must be protected from the previously problems by providing the converter circuit with a diode which acts as a peak rectifier. Diode anode is connected to the input voltage source, and its cathode is connected the output capacitor.

Figure 3.3 illustrates ideal waveforms for the current and voltage behaviors during one switching cycle ($0 < t < dT_s$ and $dT_s < t < T_s$). V_{GS} is the pulse train comes from control circuit to derive the switch, V_{in} is the source input voltage, V_o is the output voltage, d is the duty cycle, I_s is the switch current, V_s is the switch drop voltage, I_D is the diode current and V_D is the voltage across the diode.

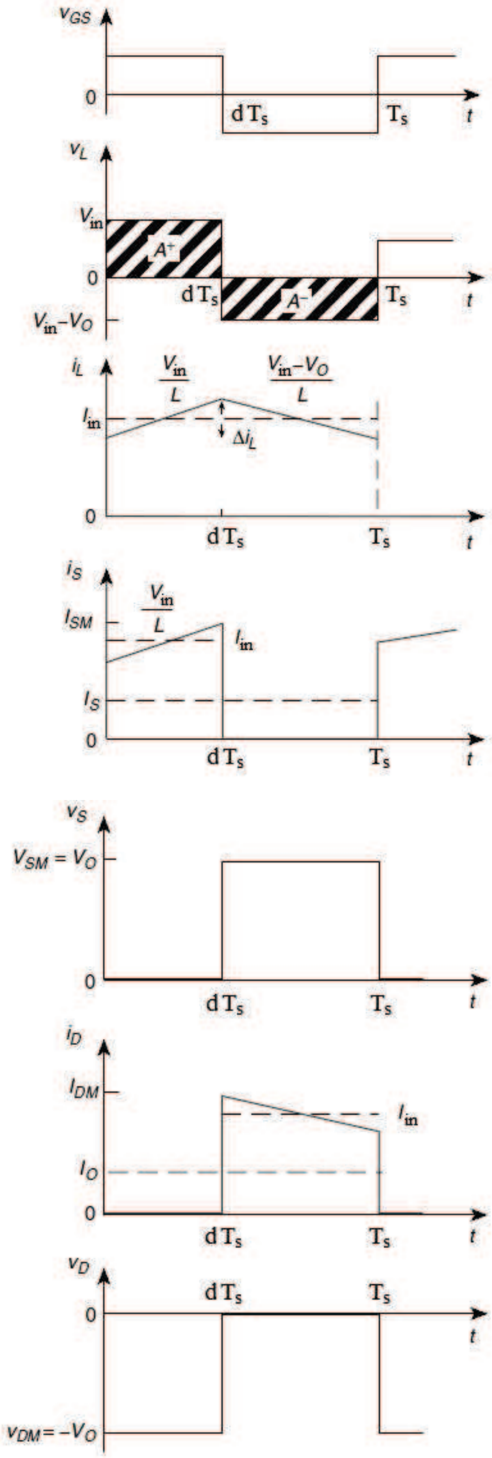


Figure 3.3. Idealized current and voltage waveforms in the PWM Boost Converter for CCM [9]

3.2.1. Assumptions

In our Boost Converter circuit analysis, it will start with some assumptions:

1. Power MOSFET and the diode are ideal switches.
2. Output capacitance of the transistor and diode, lead inductance, and switching losses are all zero.
3. Passive components are linear, time invariant and frequency independent.
4. Zero output impedance of the input voltage source for both dc and ac components [9].

3.2.2. Time interval $0 < t < dT$

The switch (Þ is close and the diode (&) is open. Equivalent circuit is shown in Figure 3.4. The voltage across the diode 8 is approximately equal to F 8 then the diode is reverse biased. The voltage across the switch 8(short circuit) and the diode current (open circuit) is zero. Then:

8- L 8Æ (3.3)

Where,

8- L . ‰ (3.4)

8Æ L . :‰ ?a‰ ; ″ (3.5)

From equations (3.4) and (3.5) and Figure 3.5:

¿E L + F + Æ I ˇ ″ (3.6)

Re-writing the Equation (3.6):

+ L + Æ E ˇ ″ (3.7)

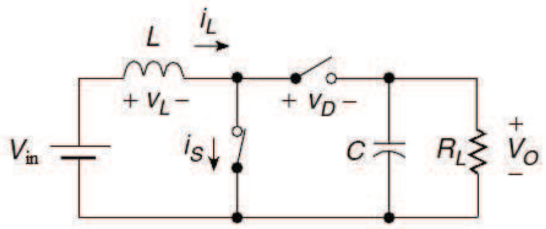


Figure 3.4. Equivalent circuit when the switch is ON and diode is off [9]

The dc input current I_{in} is equal to the average value of the inductor current I_L. The switch current I_s is equal to the inductor current I_L. Then, the switch current I_s can be written in term of dc input current:

$$I_s = I_L = I_{in} + \frac{\Delta i_L}{2} \tag{3.8}$$

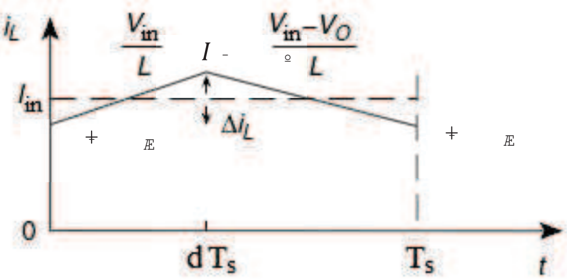


Figure 3.5. Inductor current waveform during one switching cycle for CCM

3.2.3. Time interval $dT_s < t < T_s$

The switch (S) is open and the diode (D_i) is close. In this case, the switch current I_s and the voltage across the diode is zero. Equivalent circuit is shown in Figure 3.6. Then:

$$V_o = V_{in} - V_L \tag{3.9}$$

$$V_o = V_{in} - V_L = V_{in} - L\frac{di_L}{dt} \tag{3.10}$$

From Figure 3.5, in this interval of time, $L\frac{di_L}{dt} < 0$, then:

$$V_o = V_{in} - V_L = V_{in} - L\frac{(I_{Lmin} - I_{Lmax})}{T_s - dT_s} \tag{3.11}$$

By re-writing Equation (3.11):

$$I_{Lmin} - I_{Lmax} = \frac{V_{in} - V_o}{L}(1-d)T_s \tag{3.12}$$

$$I_{Lmax} = I_{Lmin} - \frac{V_{in} - V_o}{L}(1-d)T_s \tag{3.13}$$

Since $V_o > V_{in}$, then:

$$I_{Lmax} = I_{Lmin} + \frac{V_o - V_{in}}{L}(1-d)T_s \tag{3.14}$$

Set Equations (3.7) and (3.14) equal to each other:

$$I_{Lmin} + \frac{V_{in} dT_s}{L} = I_{Lmin} + \frac{V_o - V_{in}}{L}(1-d)T_s \tag{3.15}$$

Then by re-arranging and solving Equation (3.15):

$$\frac{V_o}{V_{in}} = \frac{1}{1-d} \tag{3.16}$$

Where d will never reach 1 ($0 < d < 1, and\ d \neq 1$).

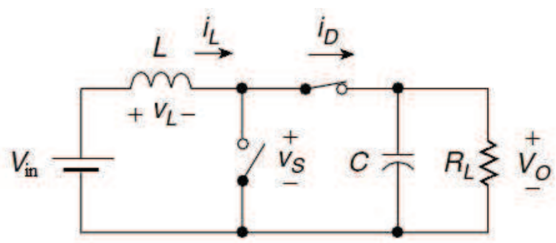


Figure 3.6. Equivalent circuit when the switch is Off and diode is ON [9]

By ignoring the capacitor , inductor and switching losses:

$$P_{in} = P_o \tag{3.17}$$

Then:

$$V_{in}. < i_{in} > = \frac{V_o^2}{R_L} \tag{3.18}$$

Since, the average input current $< i_{in} >$ is equal to the average inductor current $< i_L >$, then:

$$V_{in}. < i_L > = \frac{V_o^2}{R_L} \tag{3.19}$$

By substituting V_o in Equation (3.19) by V_o in Equation (3.16), then:

$$< i_L > = \frac{V_{in}}{(1-d)^2 R_L} \tag{3.20}$$

3.2.4. DC Voltage Gain for CCM

Referring to the Figure 3.3 and Equation (3.16), this gives:

$$G_{VDC} = \frac{V_o}{V_{in}} = \frac{1}{1-d} \tag{3.21}$$

The range of G_{VDC} for lossless converter is ($1 \leq G_{VDC} \leq \infty$). Actually, the maximum value of G_{VDC} is limited by losses.

The dc current gain by assuming lossless converter ($P_{in} = P_o$),

$$G_{IDC} = \frac{I_o}{I_{in}} = \frac{V_{in}}{V_o} = 1 - d \tag{3.22}$$

As d increases from 0 to 1, the dc current gain G_{IDC} decreases from 1 to 0.

3.2.5. Inductor Design & Selection

By definition of the difference between CCM and DCM, and as our research is considered to work under CCM, critically continuous operation occurs when the inductor current reaches zero at the end of switching cycle. Then, our issue is finding the boundary inductor value which guarantees CCM mode.

From Figure 3.5, the minimum inductor current is expressed as:

$$I_{Lmin} = < i_L > - \frac{\Delta i_L}{2} \tag{3.23}$$

By substituting the equations of the average inductor current $< i_L >$ and Δi_L from Equations (3.20) and (3.6) respectively into Equation (3.23), then:

$$I_{Lmin} = \frac{V_{in}}{(1-d)^2 R_L} - \frac{V_{in} d T_s}{2L} \tag{3.24}$$

Since for CCM ($I_{Lmin} > 0$), then to find the boundary or minimum inductor value, Equation (3.24) will be re-expressed as follow:

$$0 = \frac{V_{in}}{(1-d)^2 R_L} - \frac{V_{in} d T_s}{2L} \tag{3.25}$$

By re-arranging and solving Equation (3.25):

$$\boldsymbol{L_{min}} = \frac{\boldsymbol{d(1-d)^2 R_L}}{\boldsymbol{2 f_s}} \tag{3.26}$$

Where R_L is taken as the maximum load value (full load). Also:

$$I_{Lmin} = I_{in} - \frac{\Delta i_L}{2} \tag{3.27}$$

$$I_{Lmax} = I_{in} + \frac{\Delta i_L}{2} \tag{3.28}$$

For inductor selection, an inductor product is selected with ($L > L_{\min}$), and with rating current ($I_L > I_{Lmax}$).

3.2.6. Capacitor Design & Selection

Referring to the converter equivalent circuit during the first interval (Figure 3.4) and its circuit during the second interval (Figure 3.6), the capacitor current is:

+ J P AN :R O=PKO @6 ;Æ E L F+ L F— (3.29)

+ J P AN :@6=OHPO 6 ;Æ E L E F + L E F— (3.30)

Figure 3.7 shown the capacitor current waveform during one switching cycle, then:

¿3 L % ¿8 (3.31)

And, ¿3 L— @6 (3.32)

By equating Equations (3.31) and (3.32), the minimum capacitor value is expressed as:

o L— — (3.33)

Where, ¿8 is the capacitor output ripple voltage, 4 is taken as the maximum load value (full load), and @ as minimum.

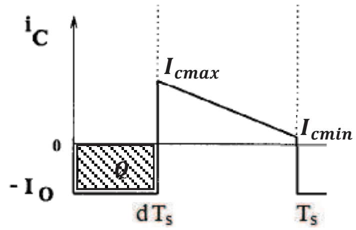


Figure 3.7. Capacitor current waveform during one switching cycle

Let us derive the rms value of the capacitor current which is needed for capacitor selection. From Equation (3.29), Equation (3.30) and Figure 3.7, then:

+ J P AN :R O=PHO @6 ;Æ E ; L + @ L @ (3.34)

+ J P AN :R6=OHPO 6 ;Æ E L E F + (3.35)

E L E F + (3.36)

E ; L . > . / 6 sF @ (3.37)

E L §E6 ; E E6 ; (3.38)

For capacitor selection, a capacitor product is selected with these following supply specifications: (% P % ; rating ripple current :+ P E ;and rated output voltage (8 P 8 ; In addition, the considerations of designers such as cost, permissible temperature and ESR … etc).

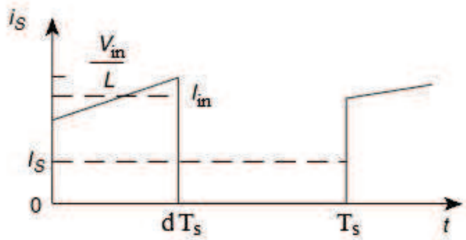


Figure 3.8. Power switch current waveform during one switching cycle [9]

3.2.7. Power Switch Selection

Power switch element can be:

a. Power BJTs: greater capacity, current driven and low conduct loss.
b. Power MOSFETs: fast switching frequency and voltage driven [37].

c. Power IGBTs: they are combined elements (high current handling as BJT and easy to control as MOSFET), powerful and expensive [38].

From Figure 3.8 which shows the waveform of the power switch current, then:

$$Interval\ \ (0 < t < dT_s), \qquad\qquad i_s = i_L \tag{3.39}$$

$$Interval\ \ (dT_s < t < T_s), \qquad\qquad i_s = 0 \tag{3.40}$$

Then, the rms value can be expressed as:

$$I_{s(rms)} = I_{in}\sqrt{d} \tag{3.41}$$

The voltage across the power switch is:

$$V_s = V_o + V_{F(diode)} \tag{3.42}$$

For power switch selection, switch element must has a rating current and voltage greater than the requirements values: +: ,and 8respectively. But, the product can’t be selected based on the current and voltage rating requirements only, because there is more than one product that covers these requirements. For example BJT, IGBT, or MOSFET can handle the same rating requirements [39]. But, some performance differences could be found between them:

1. Base/Gate drive requirements:
   a. BJT is a current driven, and its base current must exceed a fixed value to obtain the needed collector current. This fixed value can be not desirable in high currents values.
   b. MOSFET & IGBT are a voltage driven, and it has desirable gate-source voltage which can be derived easily by TTL (+5V) or CMOS logic.

2. Transient performances: (such as rise, fall, turn-on and turn-off time).

3. Others requirements: as cost, switching frequency, duty cycle and thermal requirements.

Therefore, choosing between these power switch elements is very application-specific. Normally in the applications of switch mode power supply (SMPS), MOSFET is better since it covers the base/gate drive requirements, has good transient response performances, long duty cycle and wide line or load variations [39].

3.2.8. Power Diode Selection

From Figure 3.9 which shows the waveform of the power diode current, then:

$$Interval\ (0 < t < dT_s), \qquad i_D = 0 \tag{3.43}$$

$$Interval\ (dT_s < t < T_s), \qquad i_D = i_L \tag{3.44}$$

The rms value can be expressed as:

$$I_{D(rms)} = I_{in}\sqrt{1-d} \tag{3.45}$$

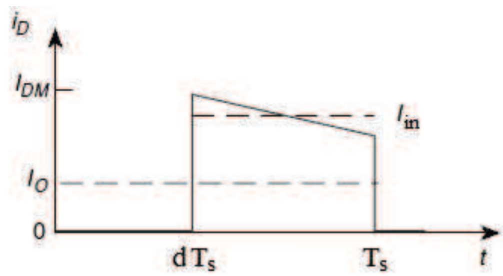


Figure 3.9. Power diode current waveform during one switching cycle [9]

The voltage across the power diode is:

$$Interval\ (0 < t < dT_s), \qquad V_{(diode)} = -V_o \tag{3.46}$$

$$Interval\ (dT_s < t < T_s), \qquad V_{(diode)} = V_{F(diode)} \qquad (3.47)$$

For diode selection, a diode should have a rating current and reverse drop voltage greater than the requirements values: $I_{D(rms)}$ and $V_{(diode)}$ respectively. Also, it is recommended to select a power diode with low forward voltage drop and fast turn-on and turn-off time.

3.2.9. Ripple Voltage for CCM

Figure 3.10 illustrates the output part of Boost Converter circuit. The diode current I_D which consists of ac component and dc component is divided between the capacitor and load resistance R_L. Since, the parallel filter capacitor eliminates the dc component of any voltage signal, then the capacitor current i_C approximately equal to the diode current ac component. Figure 3.11 shows voltage and current waveforms in the converter output circuit.

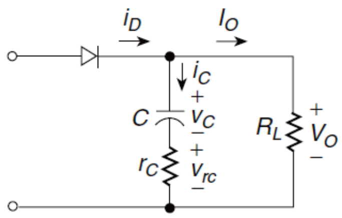


Figure 3.10. Equivalent circuit of the output part of the Boost Converter [9]

As shown in Figure 3.10, the capacitor branch contains capacitance C and its equivalent series resistance r_C, then as much as the equivalent series resistance has much less value, then the voltage across it will be low:

$$V_{rc} = i_C\, r_C \qquad (3.48)$$

In addition, as the capacitor capacitance value increases, the change in the capacitor voltage decreases:

$$\Delta V_C = \frac{\Delta Q}{C} \qquad (3.49)$$

Then, the output ripple voltage is derived from the summation of the capacitor voltage V_C with the voltage across the equivalent series resistance V_{rc}:

$$V_r = V_C + V_{rc} \quad (3.50)$$

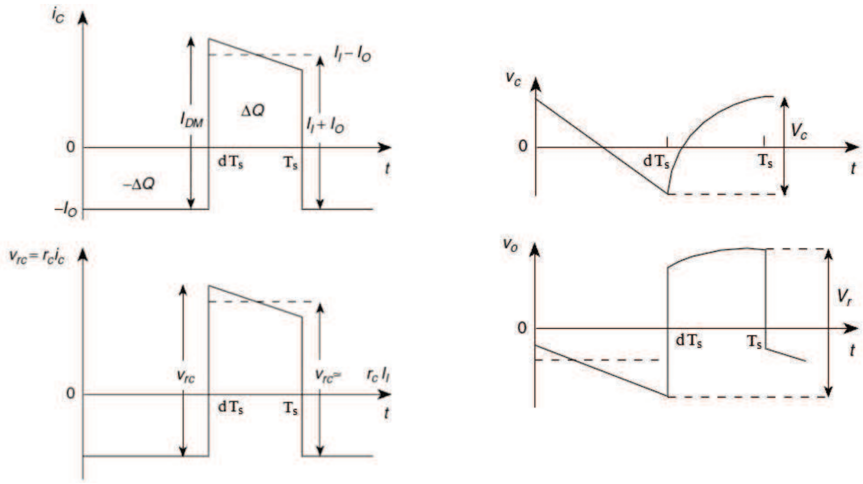


Figure 3.11. Waveforms illustrating the ripple voltage in the PWM Boost Converter [9]

3.2.10. Power Loss & Efficiency for CCM

Parasitic resistances of the Boost Converter circuit elements are shown in Figure 3.12. where r_{DS} is the MOSFET on-resistance, R_F is the diode forward resistance, V_F is the diode forward voltage, r_L is the inductor equivalent series resistance and r_C is the capacitor equivalent series resistance.

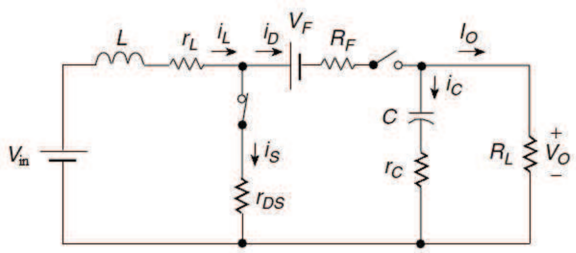


Figure 3.12. Equivalent circuit of the Boost Converter with parasitic resistances [9]

Assume the inductor current i_L is free from ripples and equals to the dc input current I_I. Referring to the Equation (3.41), the MOSFET conduction loss is:

$$P_{rDS}(D) = r_{DS} I_{S}^{2}(D) \qquad (3.51)$$

Where the MOSFET conduction loss increases as the duty cycle D increases at a fixed load current I_O.

Refereeing to the Equation (3.42) and by assuming that the MOSFET output capacitance C_o is linear, then the switching loss is:

$$P_{sw}(D) = f_s C_o V^2 \quad f_s C_o V^2 = f_s C_o R_L P \qquad (3.52)$$

Then, the total power loss in the MOSFET is:

$$P(D) = P_{rDS}(D) + P_{sw}(D) \qquad (3.53)$$

Referring to the Equation (3.45), the diode power loss due to R_F is:

$$P_{RF}(D) = R_F I_{D}^{2}(D) \qquad (3.54)$$

Likewise, the diode power loss due to R_F increases as the duty cycle D increases at a fixed load current I_O.

It is known that the dc component of the diode current flows through the load resistor, then the average value of the diode current approximately equals to the output current, and then the diode power loss due to the voltage V_F is:

$$P_{VF}(D) = V_F I_D = V_F I_O \qquad (3.55)$$

Then, the total diode conduction loss is:

$$P_D(D) = P_{RF}(D) + P_{VF}(D) \qquad (3.56)$$

The rms value of the inductor current is:

$$I_{Lrms} = I_{in} = \frac{I_o}{1-d} \tag{3.57}$$

Then, the inductor power loss is:

$$P_{L(loss)} = r_L I_{Lrms}^2 \tag{3.58}$$

Likewise, the inductor power loss increases as the duty cycle d increases at a fixed load current I_o. By referring to the Equation (3.38), the capacitor power loss is:

$$P_{C(loss)} = r_C I_{C_{rms}}^2 \tag{3.59}$$

The overall power loss of the Boost Converter can be obtained from:

$$P_{T(loss)} = P_{s(loss)} + P_{D(loss)} + P_{L(loss)} + P_{C(loss)} \tag{3.60}$$

Thus, the efficiency of the Boost Converter is:

$$\eta = \frac{P_o}{P_o + P_{T(loss)}} \tag{3.61}$$

CHAPTER 4. MODELING OF BOOST CONVERTER

4.1. Introduction

A system can be defined as any organized input which is processed to give a specific output. For control system, it is consist of subsystems and process (plant) that translate the input signal to the output signal. Description or formulation of the system is formed from the differential equations and their coefficients by applying the fundamental physical lows of science and engineering. System can be described using *Transfer Function Method* or *State Space Method*. First method is obtained in frequency domain and the other in time domain. The state space method is an integrated technique for modeling, analyzing and designing a wide range of systems.

4.2. State Space Modeling of Boost Converter

In this section, the state space modeling of the Boost Converter will be obtained and discussed in details. Basic circuit for the Boost Converter is shown in figure 4.1.

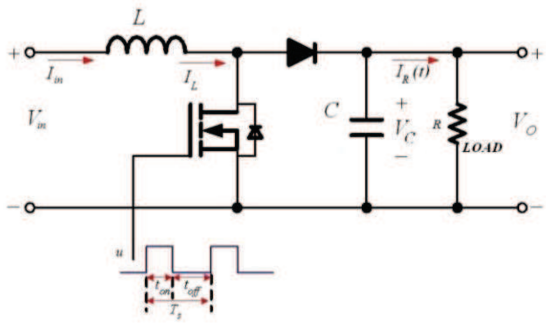


Figure 4.1. Basic circuit of Boost Converter

where, $i_L(t)$ is the inductor current and $v_C(t)$ is the capacitor voltage which are considered as the Boost Converter's state variables. As shown in Figure 1, the circuit is driven using a pulse train that comes from the control signal u.

4.2.1. ON-State Interval

Figure 4.2 illustrates equivalent circuit for the Boost Converter during the ON-state interval (when the control signal $u = 1$).

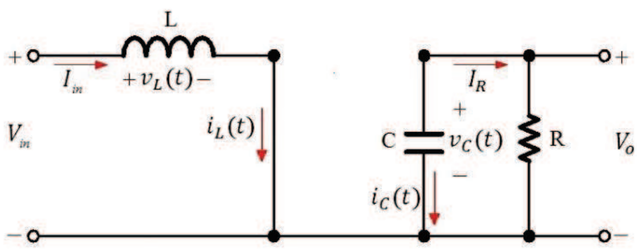


Figure 4.2. Equivalent circuit of Boost Converter during the ON-State interval

From the first loop circuit, the voltage across the inductor is:

$$v_{in} = v_L(t) = L\frac{di_L(t)}{dt} \tag{4.1}$$

$$\frac{di_L(t)}{dt} = \frac{1}{L}v_{in} \tag{4.2}$$

Also,

$$i_{in} = i_L(t) \tag{4.3}$$

From the second loop circuit, the capacitor voltage is considered as:

$$v_c(t) = v_o \tag{4.4}$$

And, the capacitor current is:

$$i_C(t) = -i_R = -\frac{v_o}{R} = -\frac{v_C(t)}{R} \tag{4.5}$$

And,

$$i_C(t) = C\frac{dv_C(t)}{dt} \tag{4.6}$$

Re-writing Equation (4.6), then:

$$\frac{dv_C(t)}{dt} = \frac{1}{C}\, i_C(t) \tag{4.7}$$

By substituting Equation (4.5) into Equation (4.7), then:

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC} v_C(t) \qquad (4.8)$$

Then state space of the system during the ON-State interval can be described as:

$$\frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} \qquad (4.9a)$$

$$v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} \qquad (4.9b)$$

4.2.2. OFF-State Interval

Figure 4.3 illustrates equivalent circuit for the Boost Converter during the OFF-state interval (when the control signal $u = 0$).

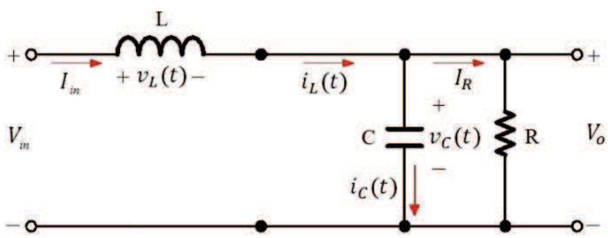


Figure 4.3. Equivalent circuit of Boost Converter during the OFF-State interval

From the first loop circuit, then:

$$i_{in} = i_L(t) \qquad (4.10)$$

And, F R‗Æ E ‰:; E R… :P; L r (4.11)

By re-arrange the Equation (4.11), then:

$$\frac{di_L(t)}{dt} = \frac{1}{L}v_{in} - \frac{1}{L}v_C(t) \tag{4.12}$$

From the second loop circuit, the capacitor voltage is considered as:

$$v_c(t) = v_o \tag{4.13}$$

The capacitor current can be expressed as:

$$i_C(t) = i_L(t) - i_R = i_L(t) - \frac{v_C(t)}{R} \tag{4.14}$$

Also,

$$i_C(t) = C\,\frac{dv_C(t)}{dt} \tag{4.15}$$

From Equations (4.14) and (4.15), then:

$$\frac{dv_C(t)}{dt} = \frac{1}{C}\,i_C(t) = \frac{1}{C}i_L(t) - \frac{1}{RC}v_C(t) \tag{4.16}$$

Then state space of the system during the OFF-State interval can be described as:

$$\frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} \tag{4.17a}$$

$$v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} \tag{4.17b}$$

4.3. State Space Averaging Method

A number of methods are appeared for ac equivalent circuit modeling such as circuit averaging, current injected approach, averaged switch modeling and state space averaging method [5]. The state space averaged modeling is widely used in DC-DC Converter's modeling since it achieves a certain performance objective and provides an accurate model [12], [32]. Three dependent models can be considered from the averaged state space model as follow:

1. ***Large Signal Model:*** it represents the actual system more closely.

$$\dot{x} = Ax + Bu \tag{4.18}$$

2. ***Steady State Model:*** this model represents the actual system during the equilibrium conditions since the system is going in the steady state.

$$0 = AX + BU \tag{4.19}$$

3. ***Small Signal Model:*** it is considered that the state and control variables have small ac variations or disturbances around the steady state operating point. Therefore, this model gives an idea of the dynamics and variations about operating point.

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} \tag{4.20}$$

Small signal model is used for control purposes and controller design. The controller job is to see these variations are made zero.

4.3.1. Averaged Large Signal Model of Boost Converter

In order to obtain the averaged large signal model of the Boost Converter, the two operating modes during the ON-State (section 4.2.1) and OFF-State (section 4.2.2) intervals will be combined together. Therefore, by averaging the state space models which illustrated in Equations (4.9) and (4.17) using the duty cycle d (control signal, $d = {}^{t_{on}}/_{T_s}$), as follows:

$$A = A_{ON}d + A_{OFF}(1-d) \tag{4.21a}$$

$$B = B_{ON}d + B_{OFF}(1-d) \tag{4.21b}$$

$$C = C_{ON}d + C_{OFF}(1-d) \tag{4.21c}$$

$$D = D_{ON}d + D_{OFF}(1-d) \tag{4.21d}$$

By substituting the above equations with their values, then:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} d + \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} (1-d) = \begin{bmatrix} 0 & -\frac{(1-d)}{L} \\ \frac{(1-d)}{C} & -\frac{1}{RC} \end{bmatrix} \tag{4.22a}$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} d + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} (1-d) = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \tag{4.22b}$$

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} d + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} (1-d) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4.22c}$$

$$D = 0.d + 0.(1-d) = 0 \tag{4.22d}$$

Then, the ***averaged large signal model*** of the Boost Converter is considered as:

$$\frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-d)}{L} \\ \frac{(1-d)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{in} \tag{4.23a}$$

$$v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} \tag{4.23b}$$

The averaged large signal model represents the actual system, and then it can be described as follow:

$$\underset{(Actual\ System)}{\boldsymbol{Averaged\ Large\ Signal\ Model}} = \underset{(Equilibrium)}{\boldsymbol{Steady\ State\ Model}} + \underset{(Control)}{\boldsymbol{Small\ Signal\ Model}}$$

Therefore, every variable in the Boost Converter system has small variation around the steady state operating point (steady state "dc" term + small signal "ac" term), small signal terms are represented by (^), and the steady state terms are represented by capital letters as follow:

$$d = D + \hat{d}$$

$$v_o = V_o + \hat{v}_o$$

$$v_{in} = V_{in} + \hat{v}_{in}$$
$$v_C = V_C + \hat{v}_C$$

E L + E -,Æ OKKJÆÆÆÆ (4.24)

Wherein, the small signal deviation term with respect to the steady state term is very much less than 1, which mean that the deviation is very small (e.g. − ′)Ê

4.3.2. Steady State Model of Boost Converter

In the steady state model, every variable will meet its desired value. In this time, every variable acts like a constant variable (equilibrium variable) and it doesn't have a slope since it is a dc term. Therefore, the derivatives of the equilibrium variables are equal zero (T6L)r

In order to obtain the steady state of every variable, the term of small signal will be cancelled from the averaged large term which is illustrated in Equation (4.24), thus the steady state term will be remained and no variation is considered to the operating point as follow:

@ L &
R L 8
RÆ L 8Æ
R… L 8…
E L + OKKJÆÆÆÆ (4.25)

Therefore, by substituting the previous considerations of the steady state conditions (Equation 4.25) into the averaged large signal model (Equation 4.23), the ***steady state model*** of the Boost Converter is considered as follow:

B CL N :5?‰; r F :5?‰; - F 5 ¸… Od 8… + :P; :P; hE H 5 r I8Æ (4.26a)

$$V_o = [0 \quad 1] \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} \tag{4.26b}$$

4.3.2.1. Output DC Value Derivation

In this section, it will discuss how to obtain the output dc value (equilibrium value) when any fixed input value is applied to the system. This derivation is obtained from the steady state model, since it represents the actual behavior of the system during the equilibrium conditions. Since the equilibrium condition is:

$$0 = AX + BU \tag{4.27}$$

Then,

$$X = -A^{-1}BU \tag{4.28}$$

The state space's output vector is:

$$Y = CX \tag{4.29}$$

By substituting Equation (4.28) into Equation (4.29), then:

$$Y = CX = -CA^{-1}BU \tag{4.30}$$

Therefore, the output dc value when any fixed input voltage value is applied:

8 L F%#?5 $7 L F>r s?N :5?%; ... r F :5?%; - F 5 ¸... ?5 5 O H I8Æ r (4.31)

By substituting every variable in Equation (4.31) with its equal fixed value, you can derive the steady state (equilibrium) value of the output.

4.3.3. Small Signal Model of Boost Converter

Small signal model is very important in the control sector and controller design since it contains good information about the deviations around the operating point. A model of averaged large signal is derived as shown in Equation (4.23). This model represents the actual system and contains both of the steady state term and the small signal term. The next step will show how to derive the small signal model from the averaged large signal model.

By substituting the parameter values of Equation (4.24) in Equation (4.23), then the model will be like:

$$\frac{d}{dt}\begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [V_{in}+\hat{v}_{in}] \quad (4.32a)$$

$$V_o+\hat{v}_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix} \quad (4.32b)$$

As shown from the above model (Equation 4.32), every parameter consists of a steady state term and a small signal term. If the steady state term is removed, which that means ($\dot{x}=0$), then the small signal term can be derived as follow:

$$\frac{d}{dt}\begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix}}^{"Ax" \ system \ term} + \overbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [V_{in}+\hat{v}_{in}]}^{"Bu" \ input \ term} \quad (4.33a)$$

$$V_o+\hat{v}_o = \overbrace{\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix}}^{"Cx" \ output \ term} \quad (4.33b)$$

By splitting the first term “Ax” (system matrix) of the above model:

$$\begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} 0 & \frac{\hat{d}}{L} \\ -\frac{\hat{d}}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + \begin{bmatrix} 0 & \frac{\hat{d}}{L} \\ -\frac{\hat{d}}{C} & 0 \end{bmatrix} \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} \tag{4.34}$$

$$\begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix}}^{zero\ (steady\ state\ part)} + \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} 0 & \frac{\hat{d}}{L} \\ -\frac{\hat{d}}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + \overbrace{\begin{bmatrix} 0 & \frac{\hat{d}}{L} \\ -\frac{\hat{d}}{C} & 0 \end{bmatrix} \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix}}^{very\ small\ (neglected)} \tag{4.35}$$

$$\begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L(t)+\hat{\imath}_L \\ V_C(t)+\hat{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} 0 & \frac{\hat{d}}{L} \\ -\frac{\hat{d}}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} \frac{V_C}{L} \\ -\frac{I_L}{C} \end{bmatrix} \hat{d} \tag{4.36}$$

By applying the same splitting operation to the term "Bu" (input vector) of the model:

$$\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [V_{in} + \hat{v}_{in}] = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \hat{v}_{in} \tag{4.37}$$

$$\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [V_{in} + \hat{v}_{in}] = \overbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in}}^{zero} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \hat{v}_{in} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \hat{v}_{in} \tag{4.38}$$

Likewise, by applying the same steps to the output term, then:

$$\hat{v}_o = [0 \quad 1] \begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} \tag{4.39}$$

Note: in the previous split and simplifying process, some constraints are applied:

a. Some terms equaled to zero. These terms are parts of the steady state model since (0 = AX+BU). Then, the terms of steady state is cancelled.
b. The products of small signal terms like $(\hat{d}.\hat{x})$ is very small, not significant and can be neglected.

Referring to the simplified terms (Equations 4.36, 4.38 and 4.39) of the model illustrated in Equation (4.33). Then the small signal model is extracted and obtained from the averaged large signal model and it can be considered as follow:

$$\frac{d}{dt}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} = \overbrace{\begin{bmatrix}0 & -\frac{(1-D)}{L}\\ \frac{(1-D)}{C} & -\frac{1}{RC}\end{bmatrix}}^{A}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} + \overbrace{\begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}}^{B_1}\hat{v}_{in} + \overbrace{\begin{bmatrix}\frac{V_C}{L}\\ -\frac{I_L}{C}\end{bmatrix}}^{B_2}\hat{d} \qquad (4.40a)$$

$$\hat{v}_o = \overbrace{[0 \quad 1]}^{C}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} \qquad (4.40b)$$

If the two input vectors in Equation (4.40a) are combined in one input vector, then:

$$\frac{d}{dt}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} = \begin{bmatrix}0 & -\frac{(1-D)}{L}\\ \frac{(1-D)}{C} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} + \begin{bmatrix}\frac{1}{L} & \frac{V_C}{L}\\ 0 & -\frac{I_L}{C}\end{bmatrix}\begin{bmatrix}\hat{v}_{in}\\ \hat{d}\end{bmatrix} \qquad (4.41a)$$

$$\hat{v}_o = [0 \quad 1]\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} \qquad (4.41b)$$

As illustrated in the small signal model of the Boost Converter in Equation (4.41), the system has one output signal $\hat{v}_o$, and one considered control input (duty cycle $\hat{d}$ or input voltage $\hat{v}_{in}$). In this study, the duty cycle $\hat{d}$ will be considered as the control input. Therefore, the ***small signal model*** is:

$$\frac{d}{dt}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} = \begin{bmatrix}0 & -\frac{(1-D)}{L}\\ \frac{(1-D)}{C} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} + \begin{bmatrix}\frac{V_C}{L}\\ -\frac{I_L}{C}\end{bmatrix}\hat{d} \qquad (4.42a)$$

$$\hat{v}_o = [0 \quad 1]\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} \tag{4.42b}$$

I_L and V_C can be considered as the nominal inductor current and the nominal capacitor voltage respectively, which are calculated as follow:

$$V_C = V_0 \tag{4.43}$$

And:

$$I_L = I_{in} = \frac{I_o}{1-D} = \frac{V_o/_R}{1-D} \tag{4.44}$$

The nominal duty cycle D can be derived as:

$$D = 1 - \frac{V_{in}}{V_o} \tag{4.45}$$

4.4. Transfer Function Derivation from State Space

Transfer function is a representation method for the system in frequency domain. Transfer function can be represented as a block diagram, with the input on the left and the output on the right as shown in Figure 4.4. Transfer function allows us to define a function that algebraically relates a system's output to its input. This function will provide system separation to three distinct parts (input, plant and the output).

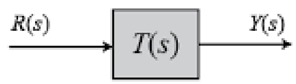


Figure 4.4. Block diagram of a transfer function

Unlike the state space modeling which is related to the differential equations of the system, transfer function is obtained related to the definition of the Laplace transform and the idea of partial fraction which applied to the solution of the differential equations.

Therefore, transfer function is obtained by taking the Laplace transform for the *n*-th order linear time-invariant differential equations of the system. In this section, this technique will not be used. Another derivation technique of the transfer function will be showed directly from the state space model. Then,

$$\dot{x} = Ax + Bu \tag{4.46}$$

By taking the Laplace transform for the above equation, then

$$sX(s) = AX(s) + BU(s) \tag{4.47}$$

By re-writing Equation (4.47), then:

$$X(s) = (sI - A)^{-1}BU(s) \tag{4.48}$$

Likewise, the output is expressed as:

$$Y(s) = CX(s) \tag{4.49}$$

By substituting Equation (4.48) into Equation (4.49), then:

$$Y(s) = CX(s) = C(sI - A)^{-1}BU(s) \tag{4.50}$$

Therefore, the transfer function can be obtained as follow:

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B \tag{4.51}$$

By using the expression which is derived in Equation (4.51), transfer function of any output signal respect to any selected input signal can be obtained.

Referring to the model Equations (4.40) and (4.41), it is noticed that our Boost Converter system has two possible inputs (one of them will be considered as the control input); every input has its related column in B matrix. Then, in order to obtain the transfer function which is expressed in Equation (4.51), the column which is related to the needed input should be selected.

For example, in order to obtain the transfer function with $\hat{v}_o$ as output variable and $\hat{d}$ as input variable, the second column of B matrix should be selected (Refer to Equations 4.40 and 4.41), then:

$$\frac{\overbrace{Y(s)}^{\hat{v}_o}}{\underbrace{U(s)}_{\hat{d}}} = C(sI - A)^{-1}B_2 \tag{4.53}$$

By substituting with the system matrices, then the transfer function $\frac{\hat{v}_o}{\hat{d}}$ is:

$$\frac{\hat{v}_o}{\hat{d}} = C(sI - A)^{-1}B_2$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{V_C}{L} \\ -\frac{I_L}{C} \end{bmatrix} \tag{4.54}$$

Likewise, the transfer function of $\frac{\hat{v}_o}{\hat{v}_{in}}$, then:

$$\frac{\hat{v}_o}{\hat{v}_{in}} = C(sI - A)^{-1}B_1 = C(sI - A)^{-1} \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \tag{4.55}$$

In this way, transfer function representation of any output to any needed input could be directly derived and calculated from the state space model. Thus, this transfer function can be used instead of the state space model in the needed test cases or controller design, as it will be described in the next chapters.

CHAPTER 5. CONTROL SYSTEMS

5.1. Introduction

Control systems are an integral part of modern society [40]. Today, a lot of control theories commonly used; classical control theory, modern control theory and robust control theory [41]. Automatic control acts as a base role in any field of engineering and science. It is an important part of space-vehicle systems, robotics systems, modern manufacturing systems, and any industrial operations containing control of pressure, temperature, flow, humidity, etc. Most of engineers and scientists are familiar with the theory and practice of automatic control [42].

“We are not the only creators of automatically controlled systems. These systems also exist in nature. Within our own bodies there are numerous control systems, such as the pancreas, which regulates our blood sugar. In time of fight or flight, our adrenaline increases along with our heart rate, causing more oxygen to be delivered to our cells. Our eyes follow a moving object to keep it in view, our hands grasp the object and place it precisely at a predetermined location” [27].

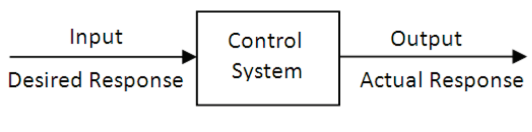


Figure 5.1. Simplified Description of Control System [27]

5.2. Control System Definition

A control system involves subsystem and processes which serves the purpose of controlling the outputs of the processes respect to the desired input. For example, any industrial device produces its output as a result of the input flow. In these processes, sensors, actuators and controllers are used for process regulation. Sensors act as an observer for the actual value of the output in order to meet the desired value of input. Actuators are used to regulate signal flow to the system output related to the controller orders. Controller receives the error signal from the sensor, thus gives the orders to the actuator to allow and give the required signal flow. Figure 5.1 illustrates a simplified description of the control system which shows that the system processes should meets its target and work with actual response equal to the desired one.

As an introduction to control systems definition, it is preferable to make a discussion about some important and basic terminologies:

a. *Plant:* it is a set of machine with inputs and outputs functioning together to perform a particular operation. On other words, it is the physical system to be controlled such as a mechanical device or converters.

b. *Process:* it is an operation to be controlled which is consisting of a series of controlled actions systematically directed to obtain a desired results.

c. *System:* it is a combination of components that operate together and obtain a certain objective. System is not shall to be physical. The concept of system can be called to some virtual preformation, ways or rules such as the applications in economics.

d. *Controlled Variable:* it is the quantity which is measured to be controlled by the controller through the control signal respect and related to some conditions and constraints. Normally, the controller variable is the system output.

e. *Control Signal:* it is the quantity which is varied related to the controller orders in order to affect and correct the controlled variable value. Traditionally, the value of controlled variable changes during the system processes. Therefore, control signal is applied to the value of controlled variable to meet the desired value.

f. *Disturbance:* it is a signal which affects the value of the system output. Disturbance signal can be generated internally (within the system) or externally (outside the system).

g. *Feedback Control:* it is an operation which measures the difference between the actual output value and the reference value from the system input. In case of disturbances, the output of the system will change, and then feedback control signal tends to reduce and eliminate this error via the controller. Therefore, Feedback control systems are those systems which use the idea of feedback control operation and use the difference as a means of control.

h. *Input & Output:* plant or system to be controlled consists of inputs and outputs to be connected with outer environment. Then, the system response for given input is called the output. The input acts as a desired response which is needed from the system output to meet. For example: when the user on the ground floor gives the order to the elevator to go to the second floor, then the elevator rises to the second floor with a response and a speed which designed to cover the user comfort.

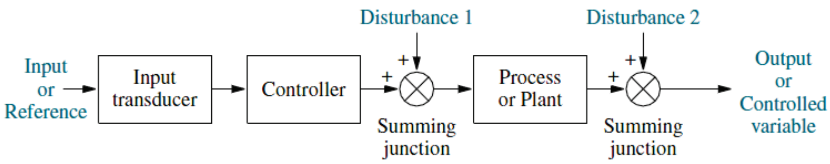


Figure 5.2. Block diagram of open loop Control System [27]

i. *Open Loop Systems:* system input is called the reference value, while the output can be called the controlled variable. In the open loop system, the input is transferred to the output through the controller with the absence of any connection or relationship between the reference input value and the output variable as shown in Figure 5.2. Therefore, these kinds of systems cannot discover any error signal appeared in the actual output variable. Also, they cannot compensate for any disturbances added to the controller or to the system output. On the words, there is no comparison between the input and output since the output is not measured or fed back. Then, the output has no effect to the control action.

j. *Close Loop Systems:* in close loop systems, system's input is converted to the output via the controller as presented in the open loop system. But, in these systems, sensors are introduced as a feedback path to measure the output's actual response. This measured value is compared with the reference input to produce error signal which drives the controller to provide the plant with the needed control signal to meet the desired output value as shown in Figure 5.3. Therefore, the closed loop system provides discovering disturbances by measuring always the output response and comparing that with the input reference value. If there is no difference, then the system is already at the desired response. Then, closed loop systems are expensive, complex and have more accuracy than open loop systems. Also, transient response and steady state can be controlled more conveniently.

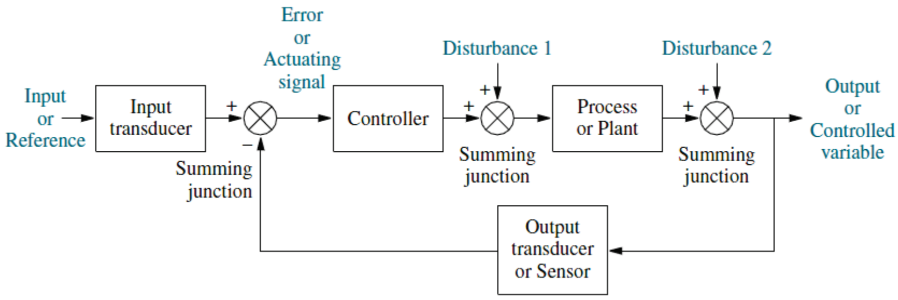


Figure 5.3. Block diagram of closed loop Control System [27]

5.3. Closed-loop & Open-loop Control systems

In the previous section, a brief definition is described about the difference between the two modes of control operation. An advantage of closed-loop system is the existence of the feedback path which guarantees the system response against external disturbances and internal parameter variations. In this case, the inaccuracies in parameters can be limited and can be covered since closed-loop systems obtain accurate control of a given plant. While in open-loop control system cases, this property is impossible to be covered.

Stability of any controlled system is very important to its response. In open-loop control systems, stability is easy to be obtained, since it is not a major problem. On the other hand, stability in the closed-loop systems is a major problem. Since in these control systems, controller face some disturbances during operating. Then controller tends to correct these errors and that can cause system oscillations or changing in the amplitude. Thus, designer must be careful to avoid oscillation, un-stability or any changing problems. Therefore, in case of systems with known inputs ahead of time and has no disturbances; it is preferable to use open-loop control system. It is a very valuable advantage to use closed-loop control systems when unpredictable disturbances and/or unpredictable variations in system parameters are introduced [40].

In the view of the cost, closed-loop control systems is generally higher in cost and power since the number of components in a closed-loop systems is more than that number of components which is used in open-loop control systems.

Then, the major advantages of open-loop control systems are:

1. Construction and maintenance is simple and easy.
2. Less expensive than closed-loop control systems.
3. Suitable in case of the difficulty to measure the output.
4. No stability problem.

On the other hand, the major disadvantages of open-loop control systems are:

1. There is no disturbance detection, thus error will be introduced and the output will have a different value with the desired one.
2. Then to eliminate the error and to save the required quality, recalibration is needed from time to time.

5.4. Advantages of Control Systems

Control systems make our life very interesting like a game. Everything around us is operated under our constraints and control. The existence of control systems makes the elevators to carry us quickly to our destination. The cars start to be driven a lone and automatically stopping in case of obstacles. Control systems can produce the needed power amplification, or power gain, regulate the speed and the position, provide appropriateness of input form, and compensation for disturbances.

5.5. Control Systems Design & Compensation

Compensation is the modification in system dynamics to meet specific properties. In control systems design and compensation, there are a lot of approaches and methods such as design via root locus, design via state space and design via frequency response. Every approach has its cases and techniques. The first and second approaches will be covered and discussed in the next chapters. In addition, these techniques will be applied to our Boost Converter system application.

5.5.1. Performance Specifications

Control systems are performed and designed to obtain specific objectives. These specific objectives or tasks which act as the requirements should performed with appropriate performance specifications. Performance specification or time response is the behavior of a system which contains much information about it. Therefore, performance specification consists of the transient response requirements (such as the settling time, peak time, rise time and overshooting) and of the steady state requirements (such as the steady state error). These specifications should be given before the design of controller.

Transient response (or called natural response) is the behavior of the system in its first short time until arrives the steady state value as shown in Figure 5.4, and this response is the most important in every controller design to meet the performance specifications.

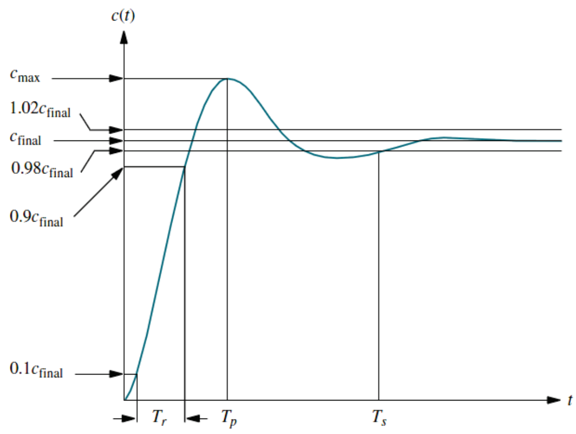


Figure 5.4. Second order under-damped response specification [27]

If the input is a step function, then the output or the response is called step time response and if the input is ramp, then the response is called ramp time response. It is known that the system can be represented by a transfer function which has poles (values make the dominator equal to zero). Depending on these poles, the step response is divided into four cases as shown in Figure 5.5:

1. *Under-damped Response:* in this case, the response has an overshooting with a small oscillation which results from *complex poles* in the transfer function of the system.

2. *Critically Response:* in this case, the response has no overshooting and reaches the steady state value in the fastest time and it resulted from *real and repeated poles* in the transfer function of the system.

3. *Over-damped Response:* in this case, no overshooting will appear and reach the final value in a time larger than the critically response case. This response is resulted from the existence of *real and distinct poles* in the transfer function of the system.

4. *Un-damped Response:* in this case a large oscillation will appear at the output and will not reach a final value and this because of the existence of imaginary

poles in the transfer function of the system and the system in this case is called "Marginally stable".

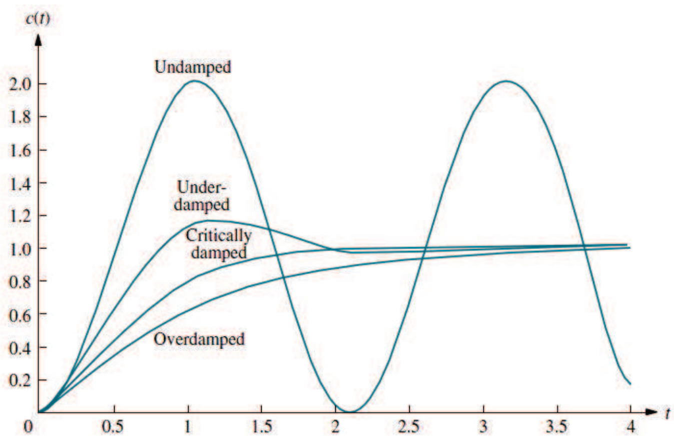


Figure 5.5. Step response for second order system damping cases [27]

5.5.2. System Compensation

A compensator is a device which is inserted into the system to modify the dynamic response and satisfy the performance specifications. The simplest compensator is the gain, so setting the gain and check the modified response of the system is the first step for controller design. Increasing in the gain leads the system to good steady state response but resulting poor stability or high overshooting. In many cases, gain controller or proportional controller alone is not sufficient and cannot meet and improve the required specifications. In this case, redesign the system is needed by adding addition controller or components to affect the overall behavior of the system, so the system is derived to satisfy the desired specifications. Therefore, compensation is the process of adding a device or component to the system structure to affect the overall behavior of the system.

5.5.3. Design Procedures

Any practical system should be modeled by mathematical equations to obtain a transfer function or state space model representation for this system. This representation model acts as the system plant in the control process and related to this model, the required controller is designed and checked. In every controller design process, this controller must be tested; test operations can be done a lot of time in order to reach to the most suitable parameters for the controller or compensator. Then, designers use some programs to cover this task such as MATLAB or other equivalent program to avoid spending too much time for this checking.

In the subject of controller design, the designers test the open-loop system and they monitor the behavior of the system without any feedback, after that the designer test the system with closed-loop using negative unity feedback from the output. In this process of modeling and checking the behavior, a lot of things can be neglected and not taken in the first theoretical consideration such as nonlinearities, distributed parameters and so on. Therefore, difference will appear between the actual performance and the theoretical predictions. Thus, the first design may not cover the requirements on the actual performance. The designers must re-adjust the parameter of the controller until it meets the performance requirements.

5.6. Computer Controlled Systems

Digital computer acts as a controller in many modern systems. Then, same computer can be used to control many loops through time shifting. Furthermore, testing the compensator or controller becomes easy and you can modify the parameters required to satisfy a desired response. These changes can be done in a software rather than hardware.

There are two approaches to introduce the digital computer or a microprocessor into the control loop. Figure 5.6 illustrates the first approach. The first approach involves an analog plant with digital controller; the error signal which resulted from the difference between the actual output value and reference input value is needed to be converted from analog time to discrete time by analog to digital converter "ADC" at a fixed sampling time. After the ADC stage, the computer or the microprocessor receives the error signal in digital form and then computer performs the control algorithm, and then it generates a new sequence of numbers representing the control signal in digital form. The plant input is analog, then the control signal should converted to analog by digital to analog converter “DAC” which is maintained constant between the sampling instants by zero order hold “ZOH”. ADC and DAC blocks must operate with same clock synchronization.

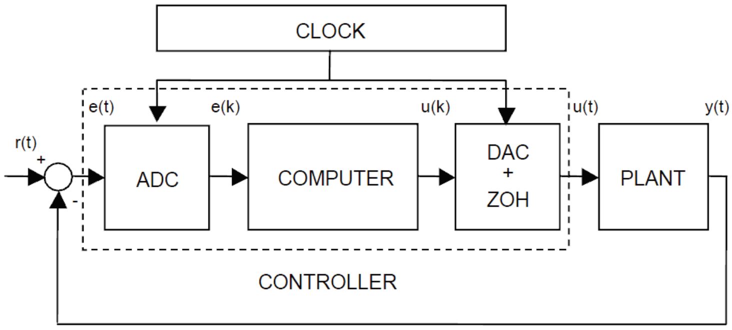


Figure 5.6. Digital realization of an analog type controller [43]

The second approach is shown in Figure 5.7. This approach is more interesting which it involves a discretized plant. Here in this approach, all control algorithm and the required calculations are performed inside the computer or a microprocessor. As seen from the Figure 5.7 that the difference comparator is moved also to be performed inside the computer, then the reference is now specified in a digital form as a sequence provided by a computer.

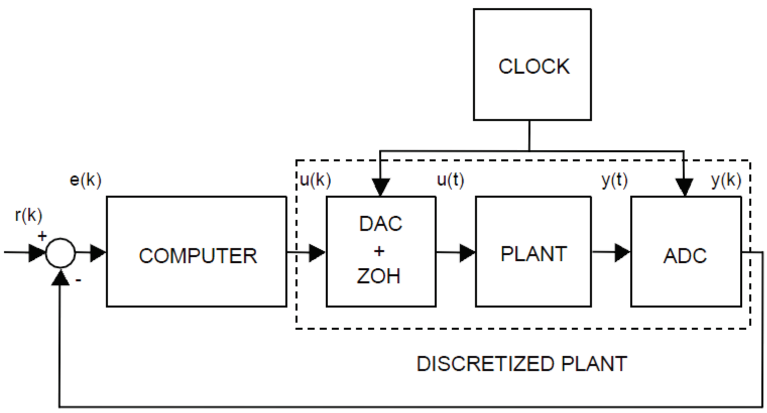


Figure 5.7. Digital Control System [43]

The control signal which performed from the controller is digital and our practical plant is analog, then DAC converter with ZOH block are required after the controller stage to provide analog input to the plant and ADC is also required after the system plant to provide digital output value to the difference comparator. This discretized plant is characterized by “Discrete Time Model” which describes the relationship between the plant’s input and output in discrete time [43].

Analog to digital converter “ADC” includes two stage functions:

a. *Analog signal sampling*: in this operation a sequence of values with equal space between each other in time domain are introduced to the analog signal as shown in Figure 5.8. Therefore, the analog signal is replaced with this sequence, where the temporal distance between the values is the sampling period. Then, these values correspond to the continuous signal amplitude at sampling instants.

b. *Quantization*: in this operation, the amplitude of the sequence values or the samples is coded with a binary sequence. As much as the resolution of the ADC is higher, the accuracy will be increased and it will be more expensive.

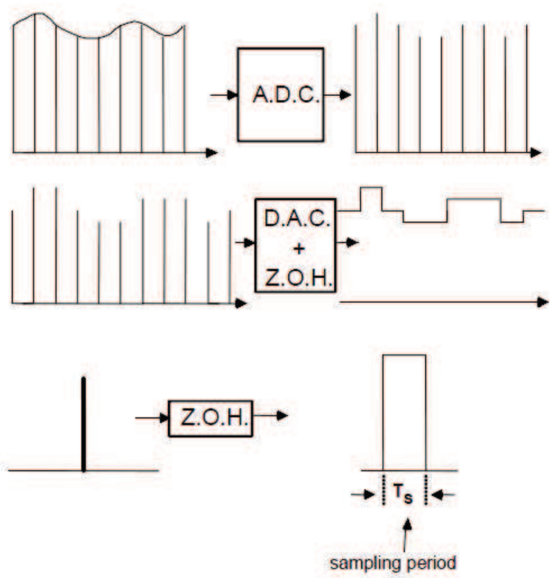


Figure 5.8. Operation of ADC, DAC and ZOH [43]

The digital to analog converter "DAC" converts the discrete signal which is digitally coded to a continuous signal with the same sampling frequency to a void information and data missing.

The zero order hold "ZOH" convert sampled signals to a continuous time signals by holding each sample value constant over one sample period.

5.7. Digital Controller Design

In this section, converting continuous time models into discrete time (or difference equation) models will be discussed. Also the z-transform and how to use it to analyze and design controllers for discrete time systems will be introduced.

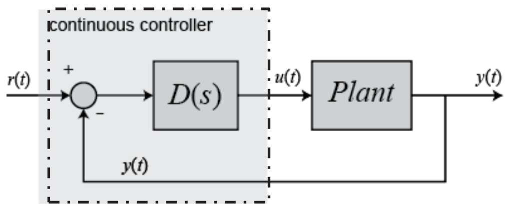


Figure 5.9. A typical Continuous Feedback System [44]

Figure 5.9 shows a typical continuous feedback system. All of the continuous controllers can be built using analog electronics. The continuous controller which enclosed in the dashed square can be replaced by a digital controller which performs the same control task as the continuous controller as shown in Figure 5.10. The basic difference between these controllers is that the digital system operates on discrete signals while the other controller on continuous signals.

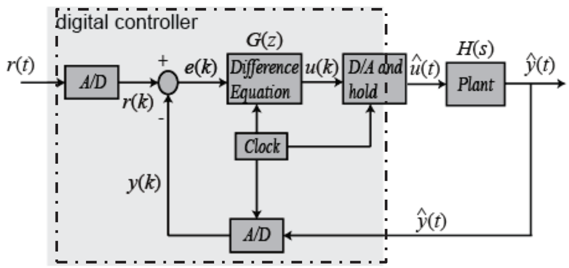


Figure 5.10. A typical Discrete Feedback System [44]

As shown in Figure 5.11, the above digital schematic contains different types of signals, which can be represented by the following signal plots:

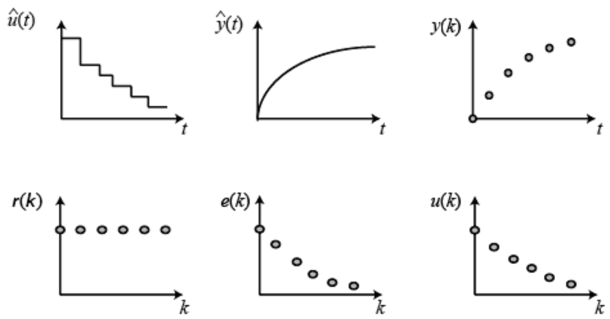


Figure 5.11. Different types of signals in a digital schematic [44]

As it is seen in the schematic diagram of the digital control system (Figure 5.10), the digital control system contains both discrete and the continuous signals (Figure 5.11).

When the designers aim to design a digital control system, it is required to find the discrete equivalent of the continuous signals, since it is only needed to deal with discrete functions.

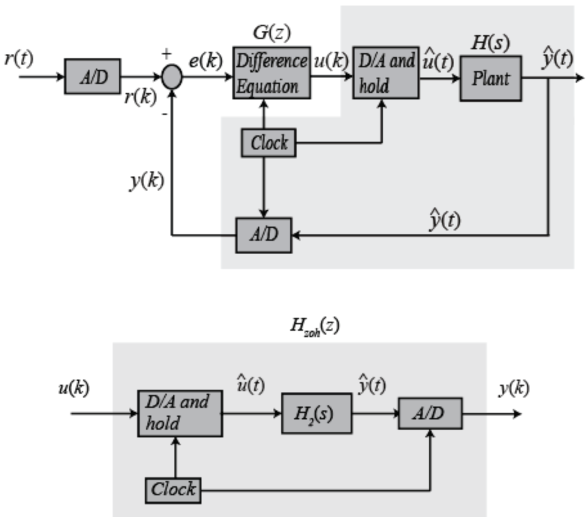


Figure 5.12. Zero-Hold equivalence for the system plant [44]

Referring again to the Figure 5.10, a clock is connected to the D/A and A/D converters which supplies a pulse every T seconds, and then D/A and A/D converters send a signal only when the pulse arrives. By generation this pulse in the system, $H_{zoh}(z)$ has only samples of input u(k) to work on and produce only samples of output y(k). Therefore, $H_{zoh}(z)$ can be realized and considered as a discrete function as shown in Figure 5.12.

As shown in Figure 5.12, the plant is a continuous system which deals with continuous input and output signals. The output $y(t)$ of the continuous system is sampled via the A/D converter to produce the discrete output $y(k)$. The discrete signal $u(k)$ which represents a sample of the input signal is required to be hold in order to produce a continuous signal $\hat{u}(t)$. Therefore, $\hat{u}(t)$ is held constant at $u(k)$ over the interval $[kT\ to\ (k+1)T]$. This technique of holding $\hat{u}(t)$ constant over the sampling time is called *Zero-Order Hold.*

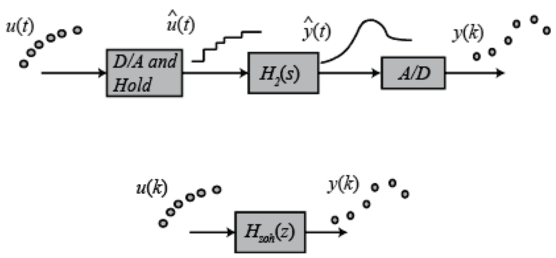


Figure 5.13. Zero-Hold Order principle [44]

Based to the above discussion, $H_{zoh}(z)$ is obtained and only discrete functions are considered. And then a discrete input signal $u(k)$ is applied to the system and goes through $H_{zoh}(z)$ to produce a discrete output signal $y(k)$ as shown in Figure 5.13.

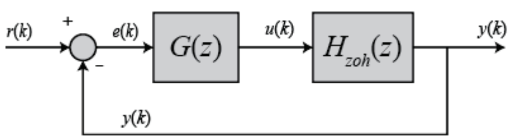


Figure 5.14. Full discrete feedback system [44]

Therefore, the schematic diagram in Figure 5.10 which represents a typical discrete feedback system can be re-expressed as shown in Figure 5.14. Then, by placing $H_{zoh}(z)$, digital control systems can be dealing with only discrete functions.

5.8. Stability and Transient Response in z-domain

For continuous systems, it is known that the certain behavior results from different pole locations in the s-plane. For example, a system is unstable when any pole is located to the right of the imaginary axis. For discrete systems, the system behaviors from different pole locations are analyzed in the z-plane. The characteristics in the z-plane can be related to those in the s-plane by the expression:

$$z = e^{sT} \tag{5.1}$$

Where: "T" is the sampling time (sec/sample), "s" is the location in s-plane and "z" is the location in z-plane. Figure 5.15 shows the mapping of lines of constant damping ratio (ζ) and natural frequency (ω_n) from s-plane to z-plane using the previous expression.

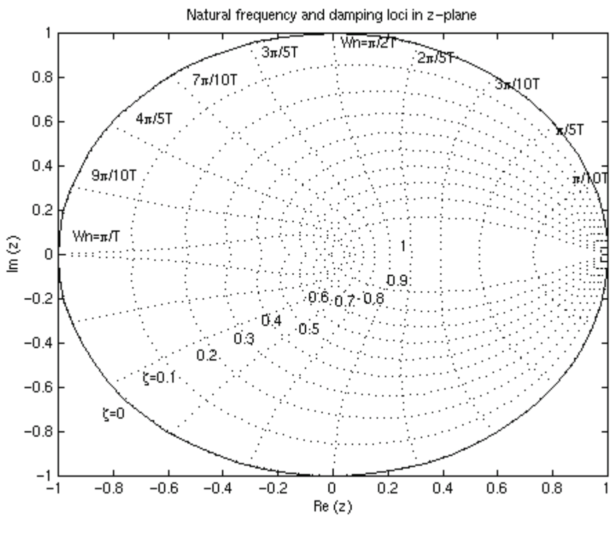


Figure 5.15. Natural frequency and damping ratio in z-plane [44]

The natural frequency (ω_n) in z-plane has the unit of rad/sample, while is s-plane has the unit of rad/sec. In the z-plane, the stability boundary is the unit circle (z=1). The system is stable when all poles are located inside the unit circle and unstable when any pole is located outside the unit circle.

The equations which used in continuous system transient response design are still applicable for the transient response analysis in the z-plane,

$$\zeta\,\omega_n \geq \frac{4}{t_s} \tag{5.2}$$

$$\omega_n \geq \frac{1.8}{t_r} \tag{5.3}$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln(\%OS/100)^2}} \tag{5.4}$$

Where: “ζ” is the damping ratio, “ω_n” is the natural frequency, “t_s” is the settling time, “t_r” is the rise time and “%OS” is the percentage overshoot.

5.9. Discrete Root Locus

The root-locus is the location of points where roots of characteristic equation can be found as a single gain. As the gain varies from zero to infinity, a plot for all expected closed loop poles location is obtained. The mechanism of drawing the root locus in the z-plane is exactly the same as in the s-plane [45]. The characteristic equation of a unity feedback system is:

: E G) :\ ,* :V; L r (5.5)

Where:) :V; is the compensator acts as a digital controller and * :V; is the plant transfer function in z-domain.

The required transient response design can be covered by assuming a sampling time, and using the Equations (5.2) to (5.5) which shown above. A specific settling time, rise time and maximum overshoot will be considered. The root locus tells the designer about the poles that can be achieved by some gain to meet the needed design requirements. Upon the completion of the design parameters, step response is obtained to check the system behavior.

CHAPTER 6. DESIGN VIA ROOT LOCUS

6.1 Introduction

It is known that the root locus is a graphical presentation of the closed loop poles as system parameters are varied. It is a powerful method of analysis and design for stability and transient response [40]. Then, root locus is a graphical technique which gives the required information and gains about the needed control system's performance.

A close loop system diagram is shown in Figure 6.1. Since the root locus is actually the locations of all possible closed-loop poles, then a K-gain can be selected from the root locus such that the closed loop system will perform like the wanted and required way. If any of the selected poles are at the right-half-plane, then the closed loop system will be unstable. The poles that are closest to the imaginary axis have great effects on the closed loop response, so even though the system has three or four poles, it may still act like a second or even first order system depending on the location(s) of the dominant pole(s) [41], [46], [47].

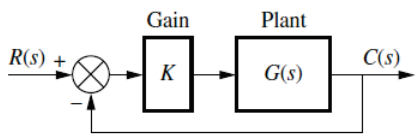


Figure 6.1. Close loop system with K controller [27]

6.2. Improving Transient Response

Root locus can be drawn quickly to get a general view of the changes in the transient response as the K-gain changes. Therefore, dominant poles or the specific points which cover the needed transient response can be found accurately to give the required design information. But, not in the all cases of root locus plot can be met with the required transient response since it is limited to those responses which are lie on the root locus plot lines.

Figure 6.2 illustrates the concept of design a desired transient response which comes from a dominant poles are not exist along the root locus. Assume that the desired transient response described by percentage overshoot and settling time is represented by point B. By moving the K-gain, the closed loop poles start to move along the loot locus plot line. It is clear from the Figure 6.2(a), that point A can only meet the overshoot condition after a simple gain adjustment without obtaining the settling time condition which makes the system response faster as shown in Figure 6.2(b). On other words, the faster response has the same overshoot as the slower response.

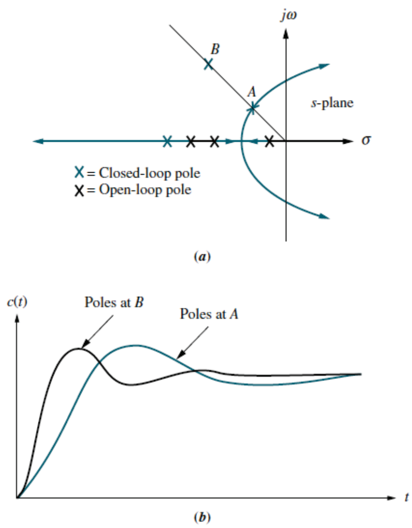


Figure 6.2. (a) Root locus sample plot. (b) Transient responses from poles A and B. [27]

Since the K-gain adjustment is limited by the current root locus plot. Then, K-gain only cannot meet any desired transient response out the range of the root locus plot. This problem can be solved if the current system is replaced by another system whose root locus intersects the desired point B.

In order to change the current system, it is required to add some additional poles and zeros, thus the new system or the compensated system has a new root locus which goes through the desired pole location for some K-gain value. Those additional poles and zeros cab be generated with a passive or an active network and can be added before the system plant without affecting the power output requirements, present additional load, or the design problems.

The order of the system with additional poles and zeros can be increased with an effect on the desired response. Thus, designer should simulate the transient response after the completion of the design to be sure that the requirements have been met. This method of compensating the transient response introduces the derivative controller.

6.3. Improving Steady State Error

In the previous section, it is introduced a compensator which improves the transient response with defined overshoot and settling time. This compensator is not only used to improve the transient response of a system, but also it can be used to improve the steady state error.

It is learned that the steady state error can be eliminated by adding an open loop pole at the origin, thus the system type will be increased and leads the steady state error to be zero. This additional pole at the origin introduces the integral controller.

6.4. Controller Design via Root Locus

Controller design can be obtained by tuning techniques directly to the system like trial and error technique, something similar to *Zeigler-Nichols Method*. Also, both of root locus and bode plot are popular in controller design. But in this section a controller design method via root locus techniques will be introduced for the DC-DC Boost Converter. Some converters have a zero on RHP like the Boost Converter system case, because of that, the bode plot will not work, bode plot can work for minimum phase system only [48].

“For continuous system (exclusively), non-minimum phase systems have one or more unstable zeros. The main effect of the unstable zero is the appearance of a negative overshoot at the beginning of the step response. The effect of the unstable zeros cannot be offset by the controller (one should use an unstable controller)” [43].

6.4.1. Boost Converter under Continuous Time Domain

Let us take the system as general as shown in Figure 6.3. A model description for the system is needed as a first step to design a controller via root locus technique. This point had been discussed in chapter 4. By using the open loop transfer function ($\frac{V_o}{d}$) or the state space model of the Boost Converter system, root locus will be sketched and that will give us the open loop pole locations.

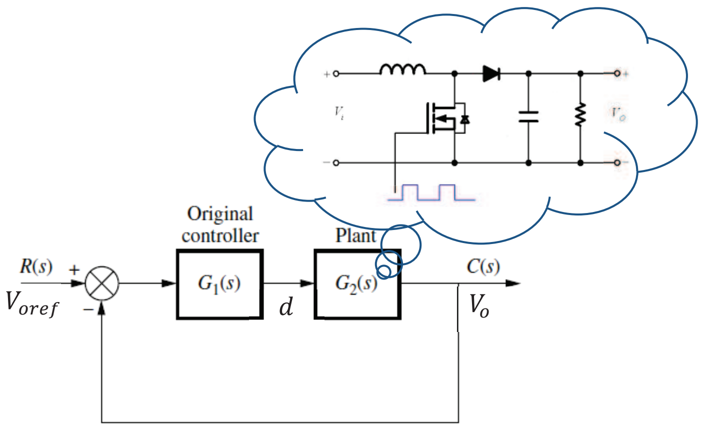


Figure 6.3. Block diagram for the closed loop of the Boost Converter system

The idea is to design the location of the desired pole which obtains a specific transient response, and then the gain will be selected to meet those update of location. If that didn’t meet the required specification, then controller design must be changed and try again [40], [48].

By referring to chapter 4, the small signal state space model of the Boost Converter is:

$$\frac{d}{dt}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} = \overbrace{\begin{bmatrix}0 & -\frac{(1-D)}{L}\\ \frac{(1-D)}{C} & -\frac{1}{RC}\end{bmatrix}}^{A}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} + \overbrace{\begin{bmatrix}\frac{V_C}{L}\\ -\frac{I_L}{C}\end{bmatrix}}^{B}\hat{d} \tag{6.1a}$$

$$\hat{v}_o = \overbrace{\begin{bmatrix}0 & 1\end{bmatrix}}^{C}\begin{bmatrix}\hat{\imath}_L\\ \hat{v}_C\end{bmatrix} \tag{6.1b}$$

where, $\hat{\imath}_L$ and $\hat{v}_o$ is the small signal ac variation of the inductor current and the output voltage respectively which are also considered as the state variables. I_L and V_C can be considered as the nominal inductor current and the nominal capacitor voltage respectively, which are calculated as follow:

$$V_C = V_0 \tag{6.2}$$

And:

$$I_L = I_{in} = \frac{I_o}{1-D} = \frac{V_o/_R}{1-D} \tag{6.3}$$

The nominal duty cycle D can be derived as:

$$D = 1 - \frac{V_{in}}{V_o} \tag{6.4}$$

The transfer function of the system can be derived as follow:

$$\frac{\hat{v}_o}{\hat{d}} = C(sI - A)^{-1}B \tag{6.5}$$

Table 6.1. Design values of the Boost Converter

No	Parameters	Design Values
1.	Input Voltage, V_{in}	24V
2.	Output Voltage, V_o	50V
3.	Inductance, L	72µH
4.	Capacitance, C	50µF
5.	Load Resistance, R	23Ω
6.	Switching Frequency, f_s	100KHz

Both of Equations (6.1) and (6.5) which represent the open loop state space and transfer function of the system respectively can be used to plot the root locus.

Based on the above discussion and the design parameters for Boost Converter which showed in Table 6.1, then:

The open loop state space of the Boost Converter is:

$$\frac{d}{dt}\begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} = \begin{bmatrix} 0 & -6666.67 \\ 9600 & -869.57 \end{bmatrix}\begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} + \begin{bmatrix} 694444.4 \\ -90580 \end{bmatrix}\hat{d} \tag{6.6a}$$

$$\hat{v}_o = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} \hat{\imath}_L \\ \hat{v}_C \end{bmatrix} \tag{6.6b}$$

And the open loop transfer function of the Boost Converter is:

$$\frac{\hat{v}_o}{\hat{d}} = \frac{-9.058x10^4\ s + 6.667x10^9}{s^2+869.6\ s+6.4x10^7} \tag{6.7}$$

Based on the system's transfer function, there is one unstable zero at :OL yuxTsr8 ; and two complex stable poles at :Q3,4 L Fvu wG Fy{{Tsr7 ; Step response for the open loop system is shown in Figure 6.4.

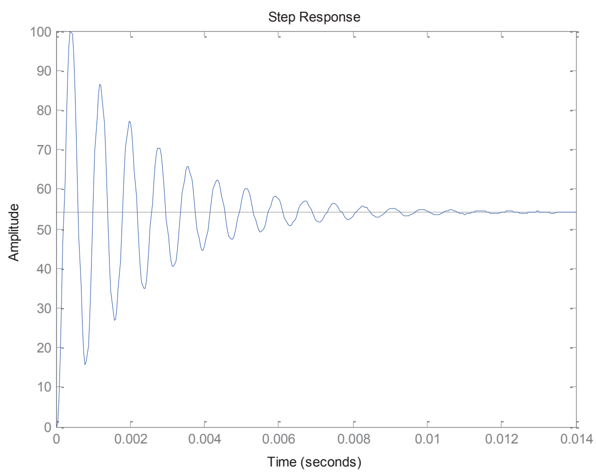


Figure 6.4. Open loop step response of the system in continuous time domain

After the open loop state space model and transfer function are obtained and calculated, the root locus for the open loop system is shown in Figure 6.5. Most of the root locus is located in the RHP since one unstable zero is exist in the system plant. The unstable zero makes limitations on the obtainable closed loop bandwidth of the controlled converter. These limitations come from a fundamental nature and not causes from a particular design criterion [49], [50].

Therefore, the range of the K-gain in the LHP region is very small and can't obtain a suitable transient response for the system. It can obtain good settling time but very high overshoot. In this case, additional poles and zeros should be added to the system in order to fix the problem and try to meet the desired transient response.

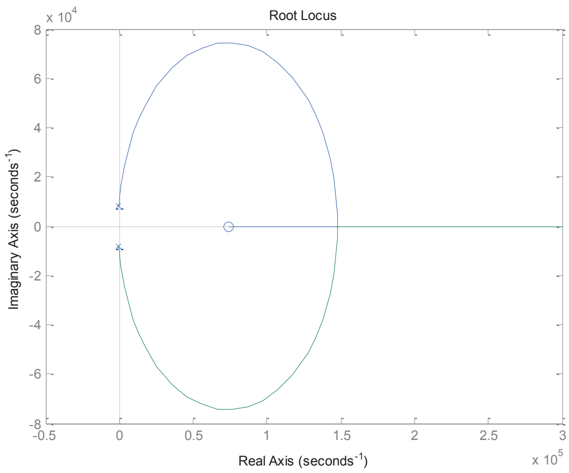


Figure 6.5. Root locus plot for the Boost Converter system in s-domain

It is clear from the Figure 6.5 that the real axis in the LHP region is not lie on the root locus. This region is very important since it opens the door to have additional parameter to adjust the transient response.

Then to activate the real axis in LHP region, an additional pole should be added at the origin. This pole acts like an integrator action and guarantees that no steady state error will appear since it increases the system type by one.

Based on the above discussion, integral compensator is introduced to the system as shown in Figure 6.6. Then, again a root locus is sketched for the open loop compensated system as shown in Figure 6.7. It is clear from the Figure that an additional pole is added at the origin, which leads the real axis at the LHP to lie on the root locus.

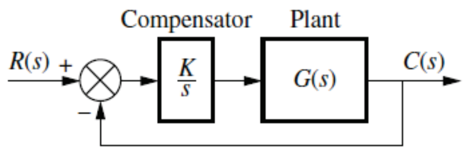


Figure 6.6. Block diagram of the system with integral compensator [27]

As the concept of the root locus technique, K-gain will start to increase from zero (starts from open loop pole) to infinity (end at open loop zero) [48]. Therefore, as K gain varies, both of the complex poles will start to move right towards the unstable zero on the RHP. The new added third real pole which lies at origin will start to move left toward the infinity since there is no any other open loop zero. On other words, as K gain varies, new closed loop poles are evaluated; each one is related to a fixed value of K gain which achieves this closed loop pole.

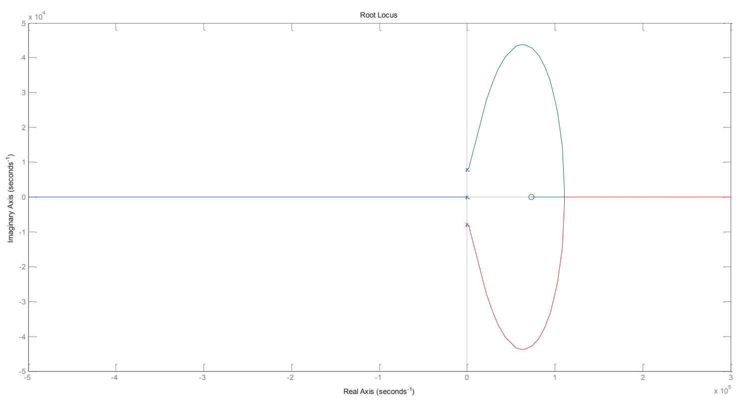


Figure 6.7. Root locus plot for the compensated Boost Converter system in s-domain

In this subject, designers should specify the required closed loop pole locations which meet the needed transient response. In this application case, the updated system has three open loop poles, two complex poles (system poles) and one real pole (controller pole).

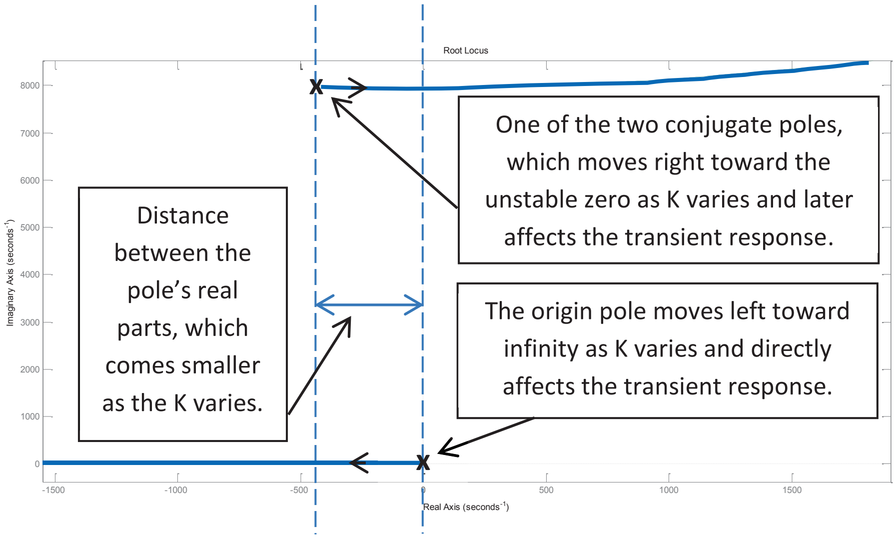


Figure 6.8. Root locus plot for the compensated Boost Converter system with a zoom to the open loop poles

The real part of the two complex poles ($s_{1,2} = \mathbf{-435} \pm j\ 7.99x10^3$) is located at (-435), and the third real pole is located at zero. The distance between the third real pole and the real part of the complex poles is two much times. Therefore, the effect to the system response will come firstly from the third real pole which will adjust the pole location to meet the required transient response. The third real pole will continue affecting the system as the distance with the real part of the complex poles is high, and when they start to be close to each other, the effect to the system response will start to come from the complex poles as shown on the Figure 6.8.

As the effect to the transient response comes firstly from the third real pole, then the required transient response will be adjusted and considered using this real pole [51].

It is known that the overshoot is zero along the real axis since the damping ratio is unity (ζ =1). The transient response is required to be with no overshoot and a settling time P L sw I, then the natural frequency æ$_{Æ}$ will be:

$$t_s = \frac{4}{\zeta\,\omega_n} \tag{6.8}$$

Then, $$\omega_n = \frac{4}{\zeta\,t_s} = 266.67\;rad/sec \tag{6.9}$$

And then the real dominant pole will be:

$$s = -\zeta\,\omega_n = -\omega_n = -266.67 \tag{6.10}$$

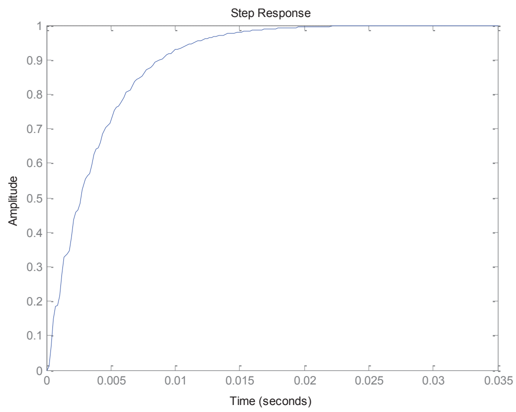


Figure 6.9. Step response for the Boost Converter system with integral controller in continuous time domain

By referring to the root locus plot and check which gain will achieve this dominant pole, then ($K_i = 2.5$) will be found.

Figure 6.9 illustrates a step response for the closed loop system of the Boost Converter with an integral compensator. It is clear from the Figure that it is a good response with no overshoot, small settling time and zero steady state error.

6.4.2. Boost Converter under Discrete Time Domain

A similar case can be derived for the Boost Converter under discrete time domain. The sampling frequency is assumed to be sr r- *(V Æ6 L sr)Ç, then the state space matrices for the Boost Converter under the discrete time domain are given by:

$$G = \begin{bmatrix} 0.9968 & -0.0663 \\ 0.0955 & 0.9882 \end{bmatrix} \tag{6.11a}$$

$$H = \begin{bmatrix} 6.9671 \\ -0.5687 \end{bmatrix} \tag{6.11b}$$

$$C_d = [0 \quad 1] \tag{6.11c}$$

$$D_d = [0] \tag{6.11d}$$

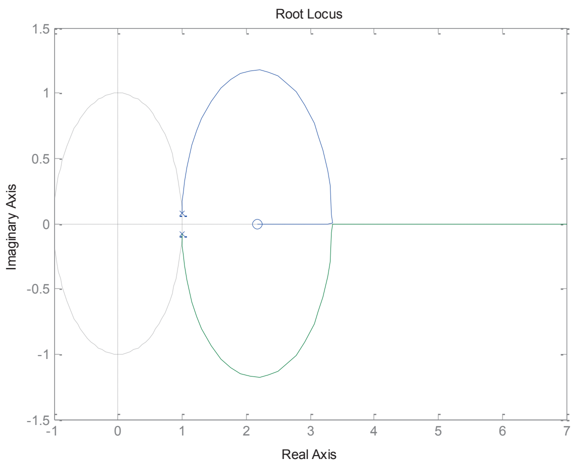


Figure 6.10. Root locus plot for the Boost Converter system in z-domain

The mechanism of drawing the root-locus is exactly the same in the z-plane as in the s-plane. As shown from the root locus plot in Figure 6.10, that most of the root locus is located out the unit cycle since one unstable zero is exist in the system plant.

Likewise the system under continuous time, it is clear from the Figure 6.10 that the region inside the unit cycle is not lie on the root locus. This region is very important for stability and since it opens the door to have additional parameter to adjust the required transient response.

Then to activate the region inside the unit cycle, additional pole and zeros should be added. As a first step of PID controller design, an integral compensator will be introduced to the system, and the system's behavior will be again checked.

The integral compensator in z-domain is expressed like $\frac{z}{z-1}$, which equal to $\frac{1}{s}$ in s-domain [52]. Then, in order to introduce an integrator to the system, a pole at 1 and a zero at origin should be added. And then, a root locus again is sketched for the open loop compensated system as shown in Figure 6.11.

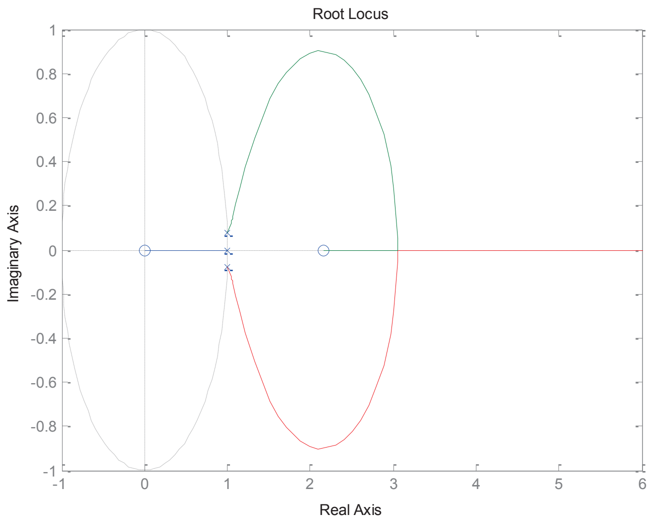


Figure 6.11. Root locus plot for the compensated Boost Converter system in z-domain

It is clear from the Figure 6.11 that an integrator is applied to the uncompensated system, and then a new root locus line is introduced to the system inside the unit cycle. This line will open the door for designers to adjust the K gain related to the desired transient response.

In this subject, designers should specify the required closed loop pole locations which meet the needed transient response. In this case, the updated system has two zeros at (z = 0 and z = 2.17) and has three open loop poles; two complex poles at ($z = e^{sT} = z_{1,2} = 0.992 \pm j\ 0.079$) and one real pole at 1 ($z = 1$), where “s” is the open loop complex poles ($s_{1,2} = -435 \pm j\ 7.99x10^3$)of the system in continuous time domain and “T” is the sampling time ($T = {}^{1}/_{f_s} = 10\mu s$).

The transient response is required to be with no overshoot and a settling time $t_s = 1.5\,ms$. It is known that the overshoot is zero along the real axis since the damping ratio is unity (ζ =1), then the natural frequency ω_n will be:

In s-domain: $$\omega_n = \frac{4}{t_s} = 2667\ rad/sec \tag{6.12}$$

Referring to the Digital Control Systems, the natural frequency needs to be in units of rad/sample, then:

$$\omega_n = 2667 \times T = 2667 \times 10^{-6} = 0.00267\ rad/sample \tag{6.13}$$

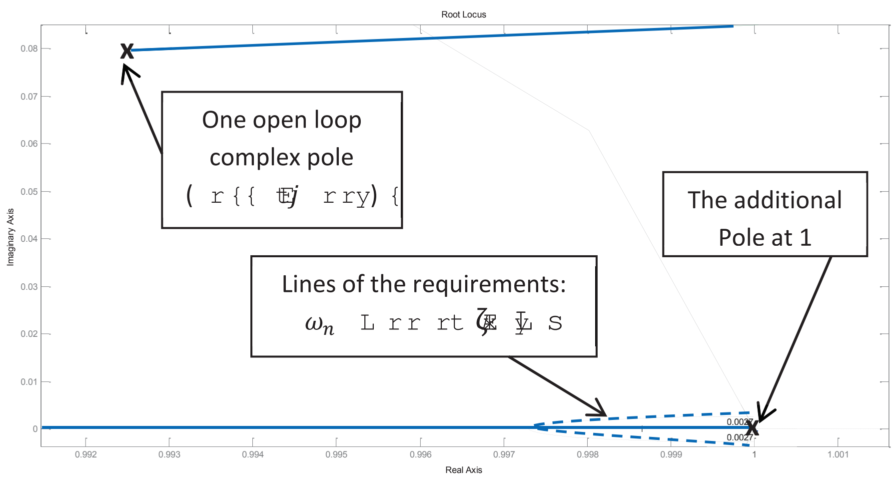


Figure 6.12. Root locus plot for the compensated Boost Converter system with a zoom to the lines of damping ratio and natural frequency

The requirements are a damping ratio equal to 1 and a natural frequency less than 0.00267 rad/sample. A new plot is sketched for the root-locus with the lines of the required constant damping ratio and natural frequency as shown in Figure 6.12.

The system is stable since all poles are located inside the unit circle as shown in Figure 6.12. Also, two dotted lines of constant damping ratio and natural frequency are shown. The natural frequency is 0.00267 rad/sample, and the damping ratio is 1 related to the desired transient response. In this case, since the real pole (z = 1) is lie at the desired region, then the effect to the system response firstly will come from this pole, which will adjust the pole location to meet the required transient response. Therefore, a K-gain should be chosen to satisfy the design requirements. By referring to the root locus plot and check which K-gain will achieve the dominant pole, then ($K_i = 3$) will be found which satisfies a settling time $t_s = 12.3\,ms$, and no overshoot.

Figure 6.13 illustrates a step response for the closed loop system of the Boost Converter with an integral compensator. It is clear from the Figure that it is a good response with no overshoot, small settling time and zero steady state error.

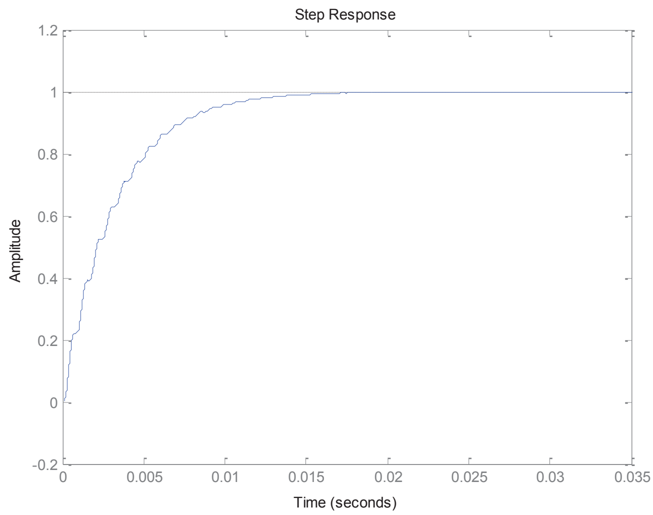


Figure 6.13. Step response for the Boost Converter system with integral controller in discrete time domain

In the previous controller design for the Boost Converter system, just an integral controller (I) is used without the need for proportional controller (P) or derivative controller (D). In PID controller design, designer starts with choosing K_i parameter, then K_p, if not met the required specifications, then K_d parameter is taken. It is preferable to not use D portion if the performance specifications are met with the integrator and proportional portions. Most of cases, P and I portions are sufficient and D can be used in the necessary cases [23], [48], [50], [53].

6.5. Results and Discussion

The previous sections discussed the design of an Integral Controller for the Boost Converter system under both continuous and discrete time domain. In this section, the simulation results will be discussed in details. The design and the performance of Boost Converter are executed in continuous conduction mode (CCM) and simulated using MATLAB/ Simulink. Figure 6.14 illustrates a MATLAB/Simulink test model for the Boost Converter with a discrete time integral controller.

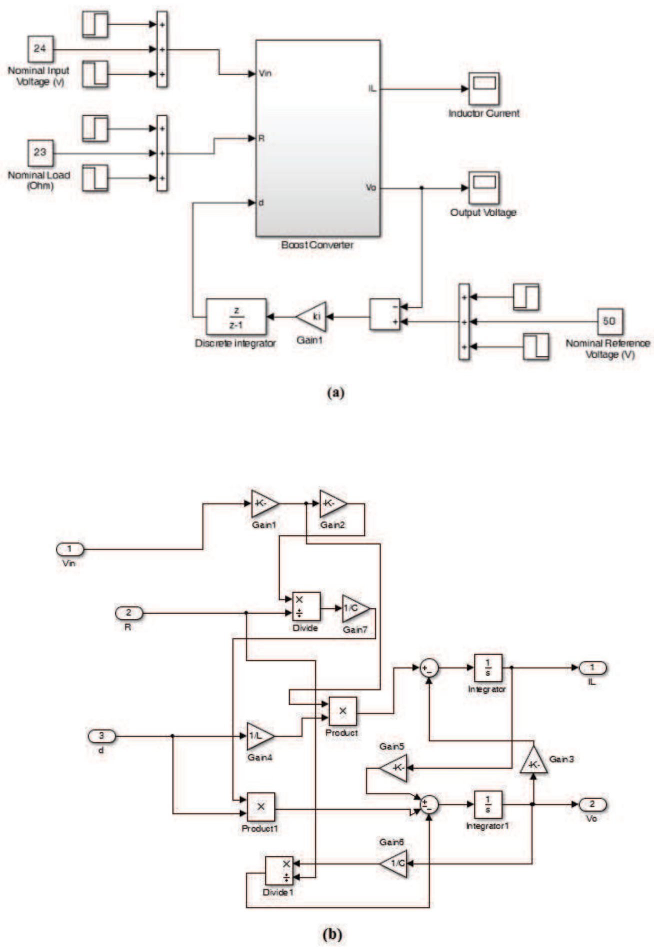


Figure 6.14. (a) Simulink Model of the I-controlled Boost Converter. (b) The constructed Boost Converter block in Simulink Model.

The performance parameters of the Boost Converter with integral controller such as; settling time, peak overshoot, steady state error, rise time, and output ripple voltage are simulated and tabulated in Table 6.2.

Table 6.2. Performance Parameters of Boost Converter with integral controller in discrete time domain

No	Performance Parameters	Values
1.	Settling Time (ms)	15
2.	Peak Overshoot (%)	0
3.	Steady State Error (V)	0
4.	Rise Time (ms)	8.7
5.	Output Ripple Voltage (V)	0

In addition, the performance of the I-Controlled Boost Converter is checked under the effects of sudden changes in the input voltage and load as shown in Figures 6.15 and 6.16 respectively. Also, it is checked under tracking the reference voltage as shown in Figure 6.17. The nominal input voltage and reference voltage for the Boost Converter are adjusted to 24V and 50V respectively, where the nominal load is 23Ω as considered before in Table 6.1.

The first test is performed by changing the input voltage in this sequence: 24V, 22V, 28V and 20V respectively with a fixed load at the nominal value. The output voltage response of the Boost Converter shows fixed output voltage regulation irrespective of the input voltage variations as shown in Figure 6.15.

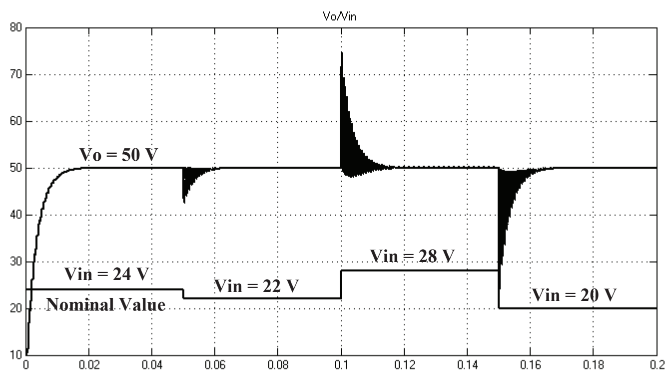


Figure 6.15. Simulation results for I-Controlled Boost Converter's output response under input voltage variations

The second test is performed by changing the load in this sequence: 23Ω, 15Ω and 8Ω respectively with fixed input voltage at the nominal value. The output voltage response of the Boost Converter shows fixed output voltage regulation irrespective of the load variations as shown in Figure 6.16.

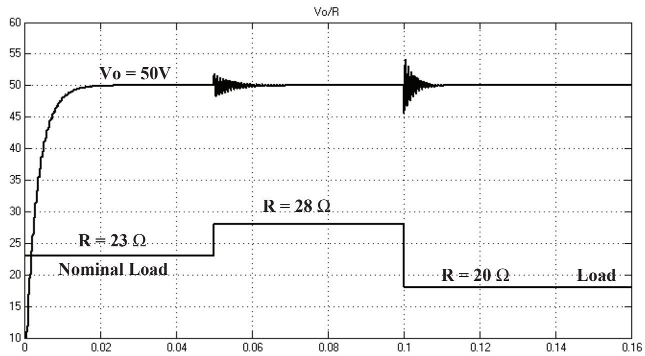


Figure 6.16. Simulation results for I-Controlled Boost Converter's output response under load variations

The third test is performed by changing the reference voltage in this sequence: 50V, 60V and 40V respectively with fixed input voltage and load at their nominal values. The output response of the Boost Converter tracks the reference voltage with no steady state error and a fixed output voltage regulation as shown in Figure 6.17.

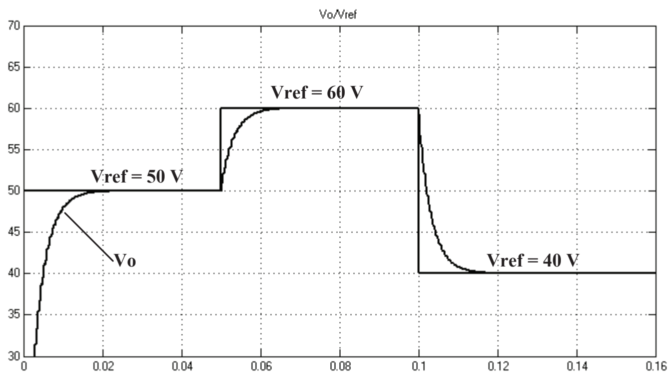


Figure 6.17. Simulation results for I-Controlled Boost Converter's output response under reference voltage variations

In Conclusion, an integral controller has been designed and simulated for the Boost Converter system in discrete time domain. The simulation analysis illustrates that the controller which is designed by the root locus technique enable the designers to locate the dominant poles at the desired locations with a suitable K-gain, but not all the closed loop poles of the system can be located. In our case system, the desired pole locations were limited since the system has an unstable open loop zero which makes a limitation on the bandwidth of the closed loop system. In addition, the system's ability to reject the external disturbances will be also limited.

The acquired results show zero steady state error and good transient time response. Also, the simulation results show that the output is not much robust against parameters variations.

CHAPTER 7. DESIGN VIA STATE SPACE

7.1. Introduction

In chapter 4 of this research, state space analysis and modeling are discussed, and the concepts are introduced. Like a transform methods, a state space is a method for designing, modeling and analyzing a feedback control systems. And unlike the transform methods since state space techniques can be applied to a wider class of systems. For example: non-linear systems and multiple input-multiple output systems (MIMO) [27], [41], [54].

Generally, designers took the decision to design a compensator when the K-gain only can't satisfy the desired transient response or when the dominant poles are not lie on the system's root locus. In this case, compensator in cascade with plant or in the feedback path should be introduced which adds additional poles, zeros or both to the system, thus the compensated system will meet the designer's desired requirements; transient response and steady state error specifications [40], [54].

In frequency domain methods of design such as root locus technique or frequency response techniques, designers still worried after designing and specifying the location of the closed loop poles to meet the desired requirements. This new update introduces a higher order poles to the system which may affects the second order approximation. The frequency domain methods can't design and specify all closed loop poles of the higher order system since those methods don't have sufficient parameters. Just one gain adjustment or a compensator selection is not enough to produce sufficient parameters to place all the closed loop poles at the desired locations. Designers need n-adjustable parameters to place n-quantities of system's poles. Therefore, an applicable and a new method should be introduced to this problem design [27], [40], [41].

State space methods have covered and solved this problem by introducing into the system other adjustable parameters, and the technique to find these parameters values, thus all closed loop poles can be placed at the desired locations.

Unlike the frequency domain methods, one disadvantage can be considered to the state space methods that they not allow the placement of the closed loop zeros since the zero's locations affect the system response. Frequency domain methods allow that through the placement of lead compensator or derivative controller [27].

A state space design methods such as pole placement and Linear Quadratic Regulator (LQR) will be discussed and applied to the Boost Converter system in this chapter. A controller will be developed using optimization techniques. It is aimed to check the updated controller's ability to provide excellent static and dynamic characteristics at all operating point.

7.2. Stability

Firstly, one of the most basic and important things is to check the system's stability. Some analysis should be applied to the open loop system without any control to check the system's stability. The eigenvalues of the system matrix A should be calculated which indicate the open loop pole locations. In s-domain systems; a system is considered stable if all pole locations are in the left-half plane (LHP). A system is unstable if any pole is located in the right-half plane (RHP). Likewise, in z-domain; a system is considered stable if all pole locations are inside the unit circle. A system is unstable if any pole is located outside the unit circle.

7.3. Controllability & Observability

A system is considered to be a *controllable system;* if there is a control input u ?; which can control the behavior of each state variable. On other words, this control input) ?takes every state variable from an initial state to a desired final state. If any one of the state variables cannot be derived and controlled by the control input

$u(t)$, then the poles cannot be located as desired, and the system is considered *uncontrollable system* [27], [40].

An LTI discrete system is considered a *controllable system*, if and only if its controllability matrix C_M has a full rank (i.e. $rank(C_M) = n$, where n is the number of variable states). For an n-th order plant whose state equation is:

$$x[k] = Gx[k] + Hu[k] \tag{7.1}$$

The controllability matrix can be derived as follow:

$$C_M = [H \quad GH \quad G^2H \quad \cdots \quad G^{n-1}H] \tag{7.2}$$

The rank of the controllability matrix can be determined by MATLAB.

Likewise, a system is considered an *observable system*; if the system's output can conclude all the state variables. If any state variable has no effect to the output behavior, then this state variable cannot be evaluated by observing the output. On other words, the initial state x_o should be determined from the system's output over a finite time. Some state variables of a system may not be directly measurable, for example; if the component is in an inaccessible location. In these cases, estimation for the unknown internal state variables should be introduced using only the available system outputs.

For LTI discrete system, the system is an *observable system,* if and only if the observability matrix O_M has a full rank (i.e. $rank(O_M) = n$, where n is the number of variable states) [27]. The observability matrix can be derived as follow:

$$O_M = \begin{bmatrix} C_d \\ C_d G \\ \vdots \\ C_d G^{n-1} \end{bmatrix} \tag{7.3}$$

Both of controllability and observability can be checked by MATLAB.

7.4. Full State Feedback Control

In full-state feedback control, all state variables are known to the controller at all times. State variable feedback controller cannot be designed if any state variable is uncontrollable [55]. In some applications, some state variables may not be available or it costs too much to be measured. In this case, it is possible to estimate the states and then send them to the controller. An observer or estimator is used to measure and calculate the state variables which are not accessible from the plant.

In the next sections, state feedback controller for the Boost Converter system is designed and discussed using pole placement technique and Linear Quadratic Optimal Regulator (LQR) methods. Simulations results are shown and the performance parameters are tabulated.

7.5. Controller Design Using Pole Placement Technique

A controller for the Boost Converter system will be built using pole placement technique. Figure 7.1 illustrates the schematic diagram of a full-state feedback system.

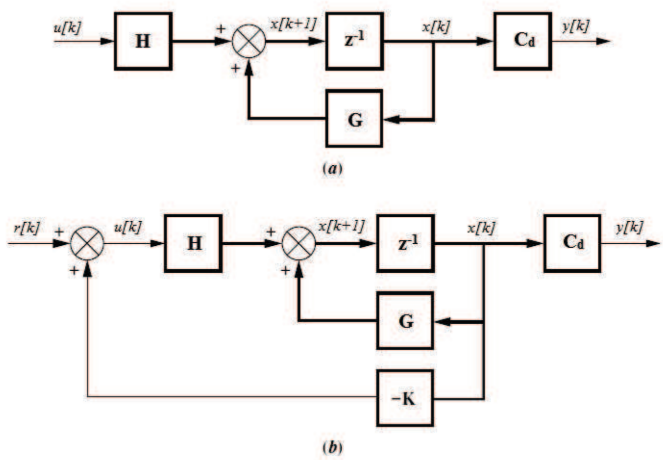


Figure 7.1. (a) State space of a plant, (b) Plant with full-state feedback.

Consider a plant is represented in discrete-time state space as follow:

$$x[k+1] = Gx[k] + Hu[k] \tag{7.4a}$$

$$y[k] = C_d x[k] \tag{7.4b}$$

Normally, the output “y” in any feedback control system is fed back to the summing unit. The topology in state variable feedback control is that all the state variables are fed back; each state variable “x_n” is fed back to the control input “u” through a gain “K instead of feeding back the output “y” as shown in Figure 7.1(b). Therefore, there are n-gains will be adjusted to meet all the desired closed loop pole locations of the system [27].

Referring to Figure 7.1(b), the control input “u” is considered as follow:

$$u[k] = r[k] - Kx[k] \tag{7.5}$$

Where $r[k]$ is the reference input and K is considered as the gain matrix which consists of n-gains for n-state variables:

$$K = [k_1 \quad k_2 \quad \cdots \quad k_n] \tag{7.6}$$

By substituting Equation (7.5) into Equation (7.4), the state equations for the closed loop system can be represented by:

$$x[k+1] = Gx[k] + Hu[k] = Gx[k] + H(r[k] - Kx[k])$$

$$= Gx[k] + Hr[k] - HKx[k] = (\boldsymbol{G} - \boldsymbol{HK})\boldsymbol{x}[\boldsymbol{k}] + \boldsymbol{Hr}[\boldsymbol{k}] \tag{7.7a}$$

$$\boldsymbol{y}[\boldsymbol{k}] = \boldsymbol{Cx}[\boldsymbol{k}] \tag{7.7b}$$

After the state equation for the closed loop system is obtained as shown in Equation (7.7a), characteristic equation ($|zI - (G - HK)|$) for the closed loop system will be found. The other characteristic equation can be determined by deciding the desired closed loop pole locations. By equating the same coefficients of the characteristic equations, and then K can be solved.

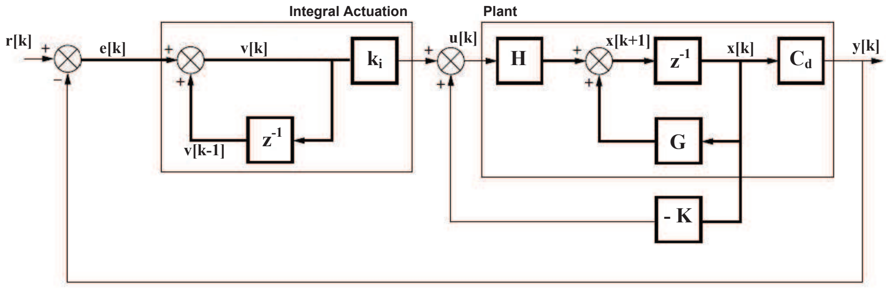


Figure 7.2. Basic State-feedback with integral actuation

The previous step is designed to meet the transient requirements, and then the design of steady state error characteristic should be addressed since an error signal will be introduced to the closed loop system [56].

A feedback path from the output “$y[k]$” is added to be compared with a reference input. And then an error “$e[k]$” is identified, which is fed forward to the controlled plant though an integrator as shown in Figure 7.2. The integrator increases the system’s type and eliminates the steady state error.

Referring to Figure 7.2, additional state variable “$v[k]$” has been added to the system state variables during the identification of the integrator block, then:

$$\begin{aligned} v[k] &= v[k-1] + e[k] \\ &= v[k-1] + r[k] - y[k] \\ &= v[k-1] + r[k] - C_d x[k] \end{aligned} \tag{7.8}$$

Equation (7.8) can be re-written as follow:

$$v[k+1] = v[k] + r[k+1] - C_d x[k+1] \tag{7.9}$$

By substituting Equation (7.4) into Equation (7.9), then:

$$e(k+1) = e(k) + r(k+1) - C[Ax(k) + Bu(k)]$$

$$= e(k) + r(k+1) - CAx(k) - CBu(k) \qquad (7.10)$$

where $e(k)$ is the actuating error vector, and $r(k)$ is the command input vector [57].

Therefore, by referring to Equations (7.4) and (7.10), the updated state space model will be expressed as follow:

$$\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -CA & I \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} B \\ -CB \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(k+1) \qquad (7.11a)$$

$$y(k) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \qquad (7.11b)$$

And the updated control vector $u(k)$ will be expressed as follow:

$$u(k) = -Kx(k) + K_I e(k) = \begin{bmatrix} -K & K_I \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \qquad (7.12)$$

By substituting Equation (7.12) into Equation (7.11), then the closed loop state space is:

$$\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} A - BK & BK_I \\ -CA + CBK & I - CBK_I \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} r(k+1) \qquad (7.13a)$$

$$y(k) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \qquad (7.13b)$$

where, $\hat{A}$ will be considered as the system matrix of the state space's Equation (7.13a). Therefore, the characteristic equation associated with Equation (7.13) can be used to design K and K_I to meet the desired transient response, and do not depend on the command input $r(k)$.

7.5.1. Controller Design for the Boost Converter

By referring to chapter 4, the state space analysis is carried out and the small signal state space model of the Boost Converter is obtained as:

$$\frac{d}{dt}\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} = \begin{bmatrix}0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} + \begin{bmatrix}\frac{V_C}{L} \\ -\frac{I_L}{C}\end{bmatrix}\hat{d} \quad (7.14a)$$

$$\hat{v}_o = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} \quad (7.14b)$$

Where, $\hat{\imath}_L$ and $\hat{v}_o$ is the small signal ac variation of the inductor current and the output voltage respectively which are also considered as the state variables. I_L and V_C can be considered as the nominal inductor current and the nominal capacitor voltage respectively, which are calculated as follow:

$$V_C = V_0 \quad (7.15)$$

And:

$$I_L = I_{in} = \frac{I_o}{1-D} = \frac{V_o/R}{1-D} \quad (7.16)$$

The nominal duty cycle D can be derived as:

$$D = 1 - \frac{V_{in}}{V_o} \quad (7.17)$$

Based on the considered design parameters for the Boost Converter which are shown in Table 7.1, then the open loop state space of the Boost Converter is:

$$\frac{d}{dt}\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} = \begin{bmatrix}0 & -6666.67 \\ 9600 & -869.57\end{bmatrix}\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} + \begin{bmatrix}694444.4 \\ -90580\end{bmatrix}\hat{d} \quad (7.18a)$$

$$\hat{v}_o = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\hat{\imath}_L \\ \hat{v}_C\end{bmatrix} \quad (7.18b)$$

Table 7.1. Design values of the Boost Converter

No	Parameters	Design Values
1.	Input Voltage, V_{in}	24V
2.	Output Voltage, V_o	50V
3.	Inductance, L	72μH
4.	Capacitance, C	50μF
5.	Load Resistance, R	23Ω
6.	Switching Frequency, f_s	100KHz

Since the discrete-time state space will be used for the discrete-time state variable feedback controller design. A similar case can be derived for the Boost Converter under discrete time domain. The sampling frequency is assumed to be 100 KHz, and then the state equations for the Boost Converter under discrete time domain are:

$$\begin{bmatrix} i_L[k+1] \\ v_C[k+1] \end{bmatrix} = \begin{bmatrix} 0.9968 & -0.0663 \\ 0.0955 & 0.9882 \end{bmatrix} \begin{bmatrix} i_L[k] \\ v_C[k] \end{bmatrix} + \begin{bmatrix} 6.9671 \\ -0.5687 \end{bmatrix} \hat{d} \tag{7.19a}$$

$$y[k] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L[k] \\ v_C[k] \end{bmatrix} \tag{7.19b}$$

Thus,

$$G = \begin{bmatrix} 0.9968 & -0.0663 \\ 0.0955 & 0.9882 \end{bmatrix} \tag{7.20a}$$

$$H = \begin{bmatrix} 6.9671 \\ -0.5687 \end{bmatrix} \tag{7.20b}$$

$$C_d = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{7.20c}$$

As a first step of the controller design, the main necessary condition for pole placement is that the system should be completely state controllable. The controllability of a control system ensures the existence of a complete solution of the system. The concept of controllability is expressed and defined before in section 7.3.

Referring to Equation (7.2), the rank of the controllability matrix C_M should be calculated; if $rank(C_M) = n$, where n is the number of variable states, then the system is full controllable. The Boost Converter has two state variables (n = 2), Therefore:

$$C_M = [H \quad GH] = \begin{bmatrix} 6.9671 & 6.9826 \\ -0.5687 & 0.1033 \end{bmatrix} \tag{7.21}$$

As it is known that the rank of C_M equals to the number of linearly independent rows or colunms. The rank can be found by finding the highest order square submatrix that is nonsingular. The determinant of $C_M = 4.6905$. Since the determinant in not zero, the 2x2 matrix is nonsingular, and the rank of C_M is 2. Then, the system is controllable since the rank of C_M equals the system order.

By substituting Equation (7.20) into Equation (7.13), the closed loop state space will be given as follow:

$$\begin{bmatrix} i_l[k+1] \\ v_c[k+1] \\ v[k+1] \end{bmatrix} = \overbrace{\begin{bmatrix} 0.9968 - 6.9671k_1 & -0.0663 - 6.9671k_2 & 6.9671k_i \\ 0.0955 + 0.5687k_1 & 0.9882 + 0.5687k_2 & -0.5687k_i \\ -0.0955 - 0.5687k_1 & -0.9882 - 0.5687k_2 & 1 + 0.5687k_i \end{bmatrix}}^{A_d} \begin{bmatrix} i_l[k] \\ v_c[k] \\ v[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r[k] \tag{7.22a}$$

$$y[k] = [0 \quad 1 \quad 0] \begin{bmatrix} i_l[k] \\ v_c[k] \\ v[k] \end{bmatrix} \tag{7.22b}$$

where $K = [k_1 \quad k_2]$ since the Boost Converter system contains two original state variables (i_l *and* v_c). Referring to Equation (7.22a), the characteristic equation to find the unknown values is formed as:

$$\begin{aligned} |zI - A_d| = z^3 + (-2.985 + 6.9671k_1 - 0.5687k_2 - 0.5687k_i)\, z^2 + \\ (2.9764 - 13.8143k_1 + 1.8k_2 + 1.2322k_i)\, z + \\ (6.8472k_1 - 1.2322k_2 - 0.9914) = 0 \end{aligned} \tag{7.23}$$

Since the updated state space which is mentioned in Equation (7.22) has three state variables, then the system has three closed loop poles which are needed to be adjusted as follow:

a. Two dominant complex poles are formed with a damping ratio (ζ = 0.95), a settling time ($t_s = 1ms$), and with a sampling frequency assumed to be $100\ KHz$ (sampling time, $T = 10\mu s$),

$$z_{1,2} = e^{sT} = e^{\left(-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}\right)T} = e^{(-4000\pm j1314.74)T} = 0.9607\pm j0.0126 \tag{7.24}$$

b. The third real pole ($s + 100{,}000$) is adjusted near to the unstable zero and it has a real part many times greater than the desired dominant second order poles in order to not affect the transient response:

$$z_3 = e^{sT} = e^{(-\zeta\omega_n)T} = e^{(-100{,}000)T} = 0.3679 \tag{7.25}$$

Therefore, the desired third order closed loop system characteristic polynomial is:

$$(z - 0.9607 \pm j0.0126)(z - 0.3679) = z^3 - 2.2893\, z^2 + 1.63\, z - 0.339 \tag{7.26}$$

By matching coefficients from Equations (7.23) and (7.26), then:

$$k_1 = 0.104 \tag{7.27a}$$

$$k_2 = 0.049 \tag{7.27b}$$

$$k_i = 1.724x10^{-3} = 172\, T \tag{7.27c}$$

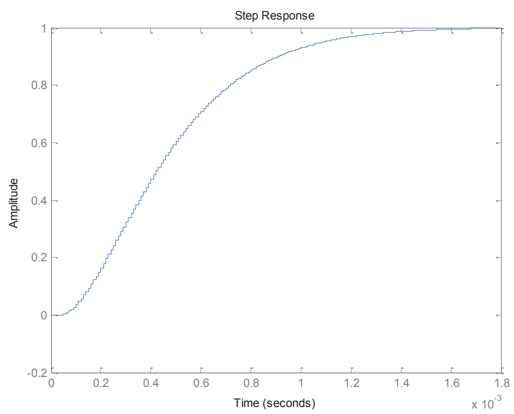


Figure 7.3. Step response for the boost converter system with pole placement method

Substituting these values (Equation 7.27) into Equation (7.22) yields the closed loop of the full state feedback controlled system. In order to check the system under the control law, a step input is used and the output response is shown in Figure 7.3. It is clear from the Figure that the system is response very fast with low a settling time ($t_s = 1.27\ ms$), no overshoot and zero steady state error.

7.6. Controller Design Using LQR Technique

State feedback techniques give the ability to locate the closed loop poles at any needed position. In practice, big external disturbances could occur in input voltage, load or any other factor resulting variations in the system parameters which cause modeling errors. Therefore, pole placement techniques could lead to unsatisfactory results in system performance. In view of this problem, controller should have insensitive response against the external disturbances or the system's parameter variations [6], [16], [58].

Linear Quadratic Optimal Regulator (LQR) is a method to choose state feedback K-gain and it is still a powerful tool in term of tuning the state feedback K-gain to obtain the optimal control law given by $u[k] = -kx[k]$. Then, LQR obtains an optimal response related to the designer's specifications in such a way that the dominant closed loop poles are assigned close to the desired locations and the remaining poles are non-dominant. This method assures insensitivity to plant parameter variations by choosing appropriate performance index [59]–[62]. On other words, states or outputs of the control system are kept within an acceptable deviation from a reference condition using acceptable expenditure of control effort. In addition, it can be applied with the independence of system's order and can be easily calculated from the matrices of the system's small signal model [63].

Referring to Equation (7.13) and Figure 7.2 which illustrates the basic schematic diagram of the state feedback system, the closed loop system can be considered as:

$$\begin{bmatrix} x[k+1] \\ v[k+1] \end{bmatrix} = \begin{bmatrix} (G - HK) & Hk_i \\ -C_d G + C_d HK & 1 - C_d H k_i \end{bmatrix} \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} \tag{7.28}$$

With a control law described by:

$$u[k] = -Kx[k] + k_i v[k] = -[K \quad -k_i]\begin{bmatrix} x[k] \\ v[k] \end{bmatrix} = -K_e \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} \tag{7.29}$$

Where:

$$K_e = [K \quad -k_i] = [k_1 \quad k_2 \quad -k_i] \tag{7.30}$$

As it is seen in Equation (7.28), the input $r[k]$ is removed and there is no any input introduced. Input $r[k]$ can be introduced, and then the system will be given like:

$$\begin{bmatrix} x[k+1] \\ v[k+1] \end{bmatrix} = \begin{bmatrix} (G - HK) & Hk_i \\ -C_d G + C_d HK & 1 - C_d H k_i \end{bmatrix}\begin{bmatrix} x[k] \\ v[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r[k] \tag{7.31}$$

Here in this method description, no input term will be introduced to the system state equation since this method can work well without the need of input term. This is why this method is called quadratic regulator as opposed to tracker. The basic problem is to drive the desired state to zero from any initial state. Therefore, a new autonomous system which tries to drive its state vector to zero will be created. If this target is covered well, then all tracking problem can be obtained very well [64].

In addition, the system's eigenvalues that make the regulator work well are the same system's eigenvalues that make the tracker work when you give a non-zero reference input, so this regulator problem can be considered without any additional input [64].

Extremely powerful results can be obtained, if the system is controllable, then the eigenvalues of the system can be placed at the desired locations to obtain the required transient response.

In practice systems, the designers face two problems; the first problem is that the designers don't have the dynamic response associated with the certain eigenvalues. If the system is a very simple second order system with no zeros, then characteristic equation and transfer function is obtained easily, and the system response can be modeled with a suitable damping ratio and natural frequency. In this case, designers will have a good sense about the system dynamic response [64].

But the first majority of systems in the world don't look like that even something as simple as adding a zero which is enough to affect the designer's sense about the system dynamic response, and then approximated damping ratio and natural frequency will be considered. Thus, instead of having a critically damped response, some overshoot could be introduced even through the dominator parameters seem like it would be a critically damped response. Therefore, designers start to lose the sense of what it will be in the time response as a function of the system poles and eigenvalues. And imagine if the system is a third or fourth order system, then designers are going to lose their intuition for how this system will work. So, for general systems, it will not going to have a perfect sense at the time response.

The other problem is although the designers can place the system's eigenvalues wherever they want, they don't have a great sense of the input that is required to accomplish those goals. By looking at the real and imaginary plane, designers think about the time constant and getting fast system, and then why should they place the eigenvalues at -1 when they can place them at -10 which is tends much faster, or place the eigenvalues at -100. Since the designers can place the eigenvalues wherever they want, it seems like why not doing this and makes the system superfast.

If the designers try to make the system superfast, it requires huge inputs which the real physical system will not be able to achieve this input. Then designers have to backing the system down in order to get actual results that are achievable [64].

So just placing the system's eigenvalues from those two fundamental reasons, sometimes it is not completely intuitive, and that is what the LQR method is coming to obtain and makes the choice of the eigenvalues and K-gains a little bit more intuitive to the designers.

The idea of LQR method is to minimize the performance index as follow:

$$J = \frac{1}{2} \sum_{k=0}^{N-1} [X^T[k]\, Q\, X[k] + U^T[k]\, R\, U[k]] \tag{7.32}$$

Where, $X[k]$ and $U[k]$ are the state and control vectors respectively, and Q and R are real and positive definite symmetric constant matrices. These Q and R matrices act as a scalar which is selected by the designer depending on the importance of state versus another. On other word, they are considered as weighting matrices.

If any simulation for the system states are carried out to watch the behavior for every state as a signal in time. One state may do some behavior, and the other state is doing another behavior [3]. The states and input signals are function in time, then a cost function "J" could be somehow implemented in a single scalar which boils down the size of all system states and drive them to zero as time goes to infinity.

It is required to have Q and R matrices with a some number that drive the states to zero during the summation process, thus minimize the cost function "J". Therefore, by choosing the values of Q and R, the relative weighting of one state versus another can be changed and adjusted [64].

Let, $Q = \begin{bmatrix} q_1 & q_3 \\ q_3 & q_2 \end{bmatrix}$ $and\ X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, then\text{:}\ X^T Q\, X = \ q_1 x_1^2 + q_2 x_2^2 + 2q_3 x_1 x_2$ (7.33)

Where q_1 penalizes the first state and q_2 penalizes the second state individually. If you want to penalize the combinations of the states via q_3, you can, but you need to be careful with their weighting which need to be relatively small compared to the diagonal elements [64].

It is assumed that the K_e gain matrix which is the solution of the closed loop control law in Equation (7.29) is determined and solved optimally which is given by:

$$K_e = \left({H_d}^T P H_d + R\right)^{-1} {H_d}^T P G_d \tag{7.34}$$

Where, G_d & H_d are the system matrices which are considered in Equation (7.11), "P" is the solution to the *discrete-time Algebraic Riccati Equation* (ARE) [65], [66].

There is an equation called *Algebraic Riccati Equation* (ARE) which is given by:

$$G_d{}^T P G_d - P - G_d{}^T P H_d \left(H_d{}^T P H_d + R\right)^{-1} H_d{}^T P G_d + Q = 0 \tag{7.35}$$

For the appropriate “P” value, system should be asymptotically stable in the steady state condition [3], [63]. By substituting of the “P” value into Equation (7.34), the value of optimal “K_e” gain matrix can be obtained. All the values required for the LQR control is included in this gain matrix as given in Equation (7.30).

So, this is the basic philosophy of the LQR controller method which is considered as a topic like an optimal control problem. This control technique minimizes a function that contains all the state space variables, and then a system with output regulation and excellent transient time performance is expected at all operating points [59]-[62].

7.6.1. Controller Design for the Boost Converter

In this technique method, system also must be controllable and the controllability of the system must be verified. Satisfaction of this property is carried out in the previous controller design with pole placement method and the Boost Converter system is full controllable.

Based on the same Boost Converter parameters which are tabulated in table 7.1 and by substituting Equation (7.20) into Equation (7.11), the closed loop state space will be given as follow:

$$\begin{bmatrix} i_l[k+1] \\ v_c[k+1] \\ v[k+1] \end{bmatrix} = \overbrace{\begin{bmatrix} 0.9968 & -0.0663 & 0 \\ 0.0955 & 0.9882 & 0 \\ -0.0955 & -0.9882 & 1 \end{bmatrix}}^{G_d} \begin{bmatrix} i_l[k+1] \\ v_c[k+1] \\ v[k+1] \end{bmatrix} + \overbrace{\begin{bmatrix} 6.9671 \\ -0.5687 \\ 0.5687 \end{bmatrix}}^{H_d} u[k] \tag{7.36}$$

In the cost function that you are trying to optimize, the simplest case is to assume $R = 1$, and $Q = C'C$. The element in the (1,1,1) position of Q represents the weight on the first state variable (inductor current), and the element in the (2,2,2) position represents the weight on the second state variable (capacitor voltage). The input weighting R will remain at 1, and then K matrix that will give us an optimal controller will be checked by trying different trials of Q to reach a smooth result.

After some trial and error and numerous simulations, the positive definite Q and R for this converter is assumed as:

$$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1.7 \end{bmatrix} \quad (7.37)$$

$$R = 1 \quad (7.38)$$

By using the system matrices in Equation (7.36) and solving the *discrete-time Algebraic Riccati Equation* (ARE) in Equation (7.35), then (See Annex C):

$$P = \begin{bmatrix} 273.8 & 965.4 & -37.26 \\ 965.4 & 6365 & -207 \\ -37.26 & -207.0 & 50.18 \end{bmatrix} \quad (7.39)$$

The "K_e" gain matrix of this converter can be obtained by substituting the above matrices in Equation (7.34). Therefore:

$$k_1 = 0.2157 \quad (7.40a)$$

$$k_2 = 0.3942 \quad (7.40b)$$

$$k_i = 0.015 = 1500\,T \quad (7.40c)$$

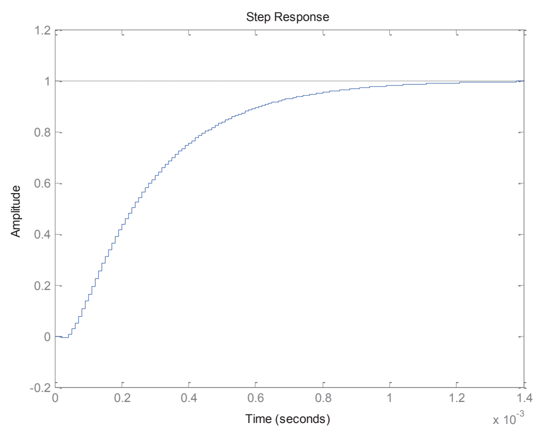


Figure 7.4. Step response for the boost converter system with LQR method

Substituting the values of Equation (7.40) into Equation (7.22) yields the closed loop of the full state feedback controlled system.

Therefore, in order to check the system under the optimal control law, a step input is used and the output response is shown in Figure 7.4. It is clear from the Figure that the system is response very fast with low a settling time ($t_s = 1\ ms$), no overshoot and zero steady state error.

It is noted that if the values of the elements of "Q" is increased even higher, the response can be improved even more. This weighting was chosen, because it just satisfies the transient design requirements. Increasing the magnitude of "Q" more would make the tracking error smaller, but would require greater control force "u". Generally more control effort corresponds to greater cost (more energy, larger actuator, etc.).

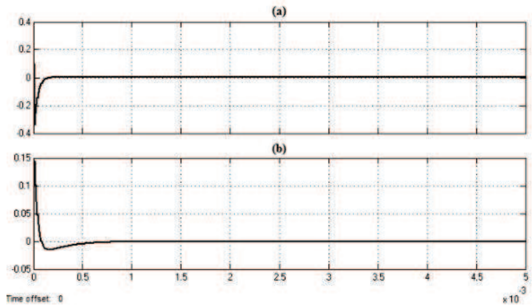
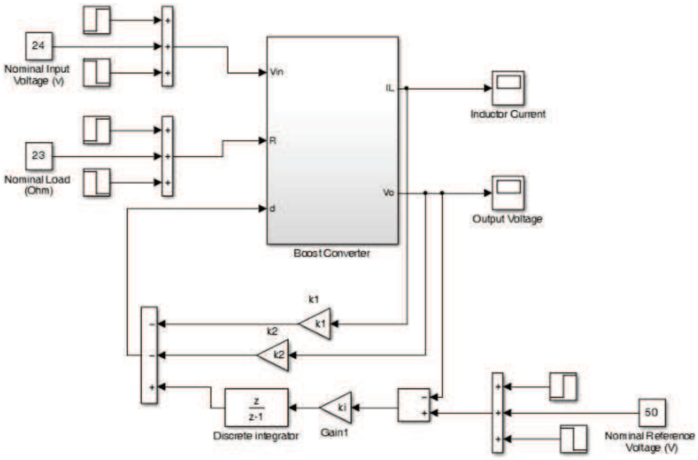


Figure 7.5. Estimations of the system's state variables: (a) State variable 1. (b) State variable 2.

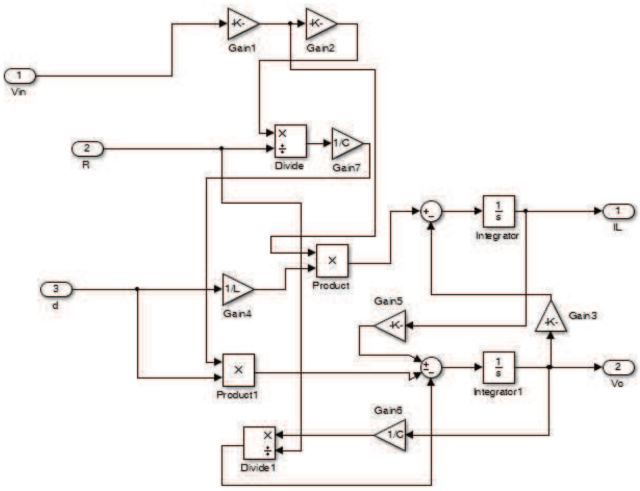
The estimated state variables of the Boost Converter under this controller should be driven to zero from any initial state value. This step should be checked to ensure the robustness of the control law and the system stability under consideration with the desired pole locations. Figure 7.5 illustrates all state variables of the Boost Converter such as x_1 and x_2 which attain zero value from any non-zero value. Therefore, the state variable feedback control is very strong to obtain the stability of the Boost Converter.

7.7. Results and Discussion

The previous sections discussed the derivation of the full state feedback controller for the Boost Converter under discrete time domain using pole placement technique and Linear Quadratic Optimal Regulator methods. The simulation results will be discussed in details. The design and the performance of Boost Converter are executed in Continuous Conduction Mode (CCM) and simulated using MATLAB/ Simulink. Figure 7.6 illustrates a MATLAB/Simulink test model for the Boost Converter with a discrete time state feedback controller.



(a)



(b)

Figure 7.6. (a) Simulink Model of the state feedback controlled Boost Converter. (b) The constructed Boost Converter block in Simulink Model.

7.7.1. Controller Using Pole Placement

The performance parameters of the state feedback controlled Boost Converter such as; settling time, peak overshoot, steady state error, rise time, and output ripple voltage are simulated and tabulated in Table 7.2.

Table 7.2. Performance Parameters of Boost Converter with pole placement method

No	Performance Parameters	Values
1.	Settling Time (ms)	1.28
2.	Peak Overshoot (%)	0
3.	Steady State Error (V)	0
4.	Rise Time (ms)	0.74
5.	Output Ripple Voltage (V)	0

The performance of the state feedback controlled Boost Converter with pole placement technique method is checked under the effects of sudden changes in the input voltage and load as shown in Figures 7.7 and 7.8 respectively. Also, it is checked under tracking the reference voltage as shown in Figure 7.9. The nominal input voltage and reference voltage for the Boost Converter are adjusted to 24V and 50V respectively, where the nominal load is 23Ω as tabulated before in Table 7.1.

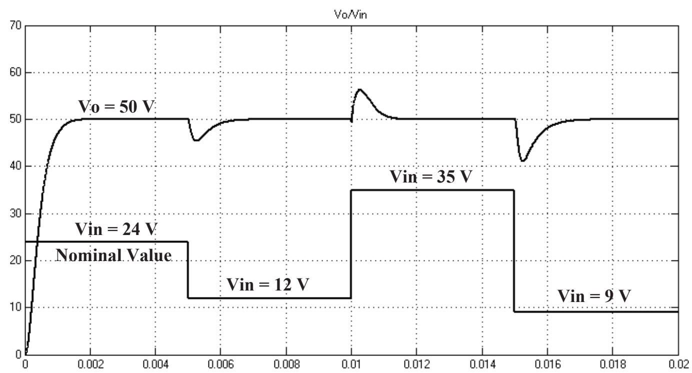


Figure 7.7. Simulation results for the Boost Converter with pole placement method under input voltage variations.

The first test is performed by changing the input voltage in this sequence: 24V, 12V, 35V and 9V respectively with a fixed load at the nominal value. The output voltage response of the Boost Converter shows fixed output voltage regulation irrespective of the input voltage variations as shown in Figure 7.7.

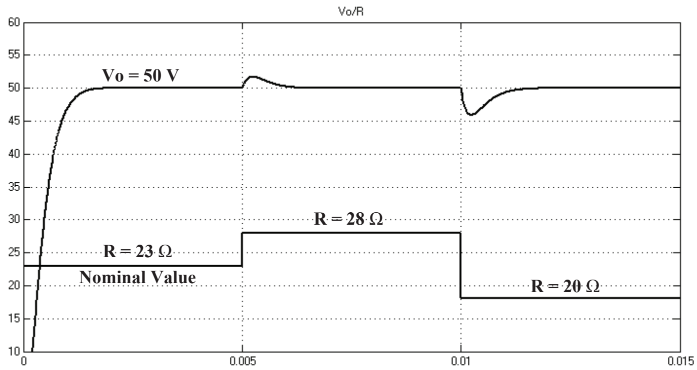


Figure 7.8. Simulation results for the Boost Converter with pole placement method under load variations.

The second test is performed by changing the load in this sequence: 23Ω, 15Ω and 8Ω respectively with a fixed input voltage at the nominal value. The output voltage response of the Boost Converter also shows fixed output voltage regulation irrespective of the load variations as shown in Figure 7.8.

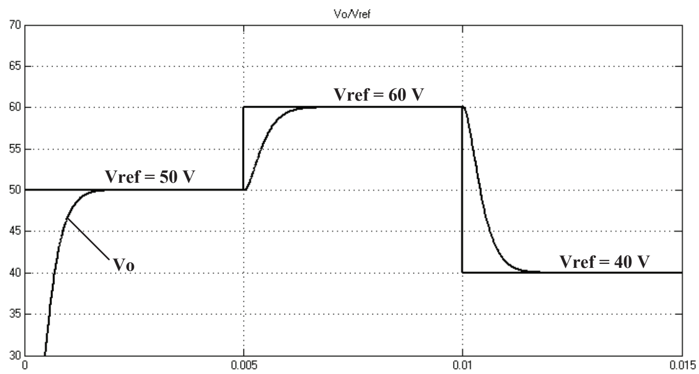


Figure 7.9. Simulation results for the Boost Converter with pole placement method under reference voltage variations

The third test is performed by changing the reference voltage in this sequence: 50V, 60V and 40V respectively with fixed input voltage and load at the nominal values. The output response of the Boost Converter tracks the reference voltage with no steady state error and a fixed output voltage regulation as shown in Figure 7.9.

State feedback controller using pole placement technique has been designed for the Boost Converter in discrete time domain. Simulation results show zero steady state error and very good transient time response. Also the simulation results show that the compensated Boost Converter obtains high dynamic performances and output voltage regulation.

7.7.2. Controller Using LQR

The performance parameters of the state feedback controlled Boost Converter with LQR method are also simulated and shown in the Table 7.3.

Table 7.3. Performance Parameters of Boost Converter with LQR Controller

No	Performance Parameters	Values
1.	Settling Time (ms)	1
2.	Peak Overshoot (%)	0
3.	Steady State Error (V)	0
4.	Rise Time (ms)	0.54
5.	Output Ripple Voltage (V)	0

The performance of the controlled Boost Converter with LQR technique method is checked under the effects of sudden changes in the input voltage and load as shown in Figures 7.10 and 7.11 respectively. Also, it is checked under tracking the reference voltage as shown in Figure 7.12. The nominal input voltage and reference voltage for the Boost Converter are adjusted to 24V and 50V respectively, where the nominal load is 23Ω as tabulated before in Table 7.1.

The first test is performed by changing the input voltage in this sequence: 24V, 12V, 35V and 9V respectively with fixed load at the nominal value. The output voltage response of the Boost Converter shows fixed output voltage regulation irrespective of the input voltage variations as shown in Figure 7.10.

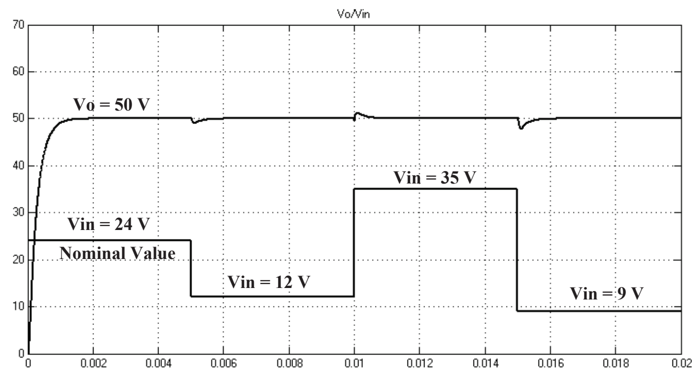


Figure 7.10. Simulation results for the Boost Converter with LQR method under input voltage variations

The second test is performed by changing the load in this sequence: 23Ω, 15Ω and 8Ω respectively with fixed input voltage at the nominal value. The output voltage response of the Boost Converter shows fixed output voltage regulation irrespective of the load variations as shown in Figure 7.11.

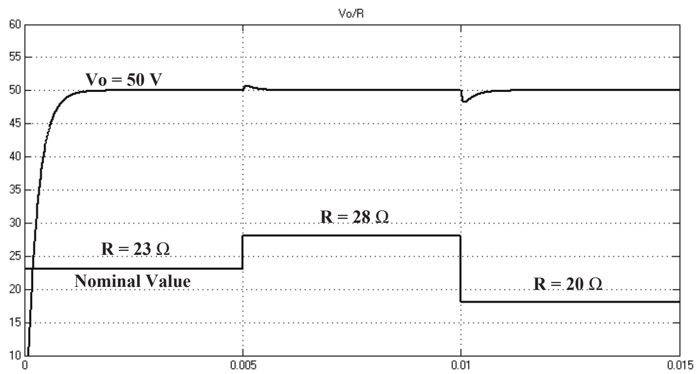


Figure 7.11. Simulation results for the Boost Converter with LQR method under load variations

The third test is performed by changing the reference voltage in this sequence: 50V, 60V and 40V respectively with fixed input voltage and load at the nominal values.

The output response of the Boost Converter tracks the reference voltage with no steady state error and a fixed output voltage regulation as shown in Figure 7.12.

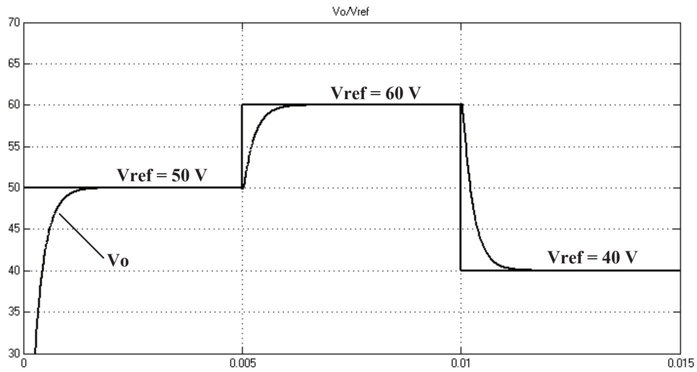


Figure 7.12. Simulation results for the Boost Converter with LQR method under reference voltage variations

State feedback controller using LQR technique has been designed for the Boost Converter in discrete time domain. Simulation results show zero steady state error and excellent transient time response. Also, the simulation results show that the compensated Boost Converter obtains very high dynamic performances and output voltage regulation.

Therefore, a state feedback controller using pole placement technique and Linear Quadratic Optimal Regulator (LQR) methods has been designed and discussed in details. The simulation analysis shows that the two methods obtain robust output voltage regulation, excellent dynamic performances and higher efficiency.

The acquired results are compared against each other and it is noticed that the Linear Quadratic Optimal Regulator (LQR) method certifies higher performance specifications and satisfactory performance in term of static, dynamic and steady state characteristics than the other methods irrespective of the circuit parameter variations.

CHAPTER 8. CONCLUSIONS AND FUTURE WORK

8.1. Conclusions

Usually DC-DC Boost Converter, when operated under open loop condition, it exhibits poor voltage regulation and unsatisfactory dynamic response, and hence, this converter is generally provided with closed loop control for output voltage regulation. Traditionally, small signal linearization techniques have largely been employed for controller design.

Small signal model has been obtained in details for the Boost Converter. Two control strategies have been proposed; root locus techniques and the state feedback approach. The duty cycle is controlled to obtain the desired output voltage. Control strategies that are based on the linearized small signal model of the converter have good performance around the operating point. The state space model is dependent on the duty cycle. However, a Boost Converter’s small signal model changes when the operating point varies. The poles and a right-half-plane zero, as well as the magnitude of the frequency response, are all dependent on the duty cycle. Therefore, the controller should be able to adapt the changes of plant dynamic characteristics.

Generally, PID controller is a traditional linear control method. Therefore, it is difficult for the controller which using small signal linearization techniques such as linear PID controller to respect well to changes in operating point.

In this study, an Integral Controller has been designed in both continuous time domain and discrete time domain using root locus techniques. Besides, a state feedback controller has been designed for the Boost Converter based on pole placement and LQR methods using state space techniques.

Frequency domain methods of design such as root locus techniques or frequency response techniques can't design and specify all closed loop poles of the higher order system since those methods don't have sufficient parameters to place all of the closed loop poles. State space methods such as state feedback controller solve this problem by introducing into the system other adjustable parameters.

The compensated Boost Converter has been checked under sudden changes in the input voltage and the load. Also it has been checked under tracking the reference voltage. In addition, the simulation results such as performance parameter and steady state parameters have been discussed and tabulated. The response of the output voltage has been sketched and compared for the different controller cases.

Based on simulation and results, the linear Integral Controller exhibited poor performance when the system is subjected of large load variations and showed poor response against changed in the operating points. On other hand, the designed controllers based on pole placement and LQR methods show very good output voltage regulation and excellent dynamic performances irrespective of the operating point variations.

The Linear Quadratic Optimal Regulator (LQR) method exhibits as the best technique of the mentioned controllers since this controller has a good control solution and provides excellent static and dynamic characteristics, accepted robustness, output regulation, disturbances rejection and higher efficiency at all operating points.

8.2. Future Work

Averaged models are linearized at a certain operation point in order to derive a linear controller. Nevertheless, a design that disregards converter nonlinearities may result in deteriorated output signal or unstable behavior in presence of large perturbations. The study of converter models and robust control methods related to nonlinearities and parameter uncertainty is still an active area of investigation.

Systems with conventional LQR controllers present good stability properties and are optimal with respect to a certain performance index. However, LQR control does not assure robust stability when the system is highly uncertain. In next research and works, a convex model of converter's dynamics should obtained taking into account high uncertainty of parameters. The performance index is proposed to be optimized. Thus, a new robust control method for dc-dc converter will derived.

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ANNEX

ANNEX A

Digital I-Controller MATLAB Code

```
clc
clear all

% The values of the plant parameters
Vin=24;
D=0.52;
R=23;
L=72e-6;
C=50e-6;

%Steady state model of the Boost Converter
As=[0 (-(1-D)/L);(1-D)/C -1/(R*C)];
Bs=[1/L 0 0;0 -1/C 0];
Cs=[0 1;1 0];
Ds=[0 0 0;0 0 0];

Vo=-Cs(1,:)*inv(As)*Bs(:,1)*Vin;
Iin=-Cs(2,:)*inv(As)*Bs(:,1)*Vin;

%Small signal model of the Boost Converter
a=[0 (-(1-D)/L);(1-D)/C -1/(R*C)];
b=[Vo/L;-Iin/C];
c=[0 1];
d=[0];

%Transfer function Vo/d
syss=ss(a,b,c,d);
[num,den]=ss2tf(a,b,c,d);
sys=tf(num,den);

%Discrete time system
T=10e-6;   %Sampling Time
sysd=c2d(sys,T);

%Check the system root locus with controller
sisotool(sysd)    %discrete integrator z/z-1 with gain K=3 is added

K=3*T;
TF=sysd*tf([K 0],[1 -1],T); %open loop plant with controller
TFF=feedback(TF,1); %close loop system
step(TFF)   %step response for the close loop system
```

ANNEX B

Digital State Feedback Controller with Pole Placement Method MATLAB Code

```
clc
clear all

% The values of the plant parameters
Vin=24;
D=0.52;
R=23;
L=72e-6;
C=50e-6;

%Steady state model of the Boost Converter
As=[0 (-(1-D)/L);(1-D)/C -1/(R*C)];
Bs=[1/L 0 0;0 -1/C 0];
Cs=[0 1;1 0];
Ds=[0 0 0;0 0 0];

Vo=-Cs(1,:)*inv(As)*Bs(:,1)*Vin;
Iin=-Cs(2,:)*inv(As)*Bs(:,1)*Vin;

%Small signal model of the Boost Converter
a=[0 (-(1-D)/L);(1-D)/C -1/(R*C)];
b=[Vo/L;-Iin/C];
c=[0 1];
d=[0];

%Transfer function Vo/d
syss=ss(a,b,c,d);
[num,den]=ss2tf(a,b,c,d);
sys=tf(num,den);

%Discrete time system
T=10e-6;   %Sampling Time
sysd=c2d(syss,T);
[Ad,Bd,Cd,Dd] = ssdata(sysd);

Ax=[Ad,zeros(length(Ad),1);-Cd*Ad,1];
Bx=[Bd;-Cd*Bd];
Cx=[Cd 0];

%Pole Placement Controller Design
pos=0.01;   %desired overshoot percentage
Ts=0.001;   %desired settling time

z=-log(pos/100)/sqrt(pi^2+[log(pos/100)]^2);   %damping ratio
wn=4/(z*Ts);   %natural frequency
```

```
[numx,denx]=ord2(wn,z);
r=roots(denx);      %roots of the desired charc. equation

rd1=exp((r(1)*T));   %first desired pole in z-domain
rd2=exp((r(2)*T));   %second desired pole in z-domain
rd3=exp((-100000*T));   %third desired pole in z-domain which is
adjusted near to the system's unstable zero.

poles=[rd1 rd2 rd3];
Ke = acker(Ax,Bx,poles);  %pole placement

ki = -Ke(3)
k1=Ke(1)
k2=Ke(2)

K=[k1 k2];
Ka=ki;

%Check the system with pole placement controller
sysx=ss([Ad-Bd*K,Bd*Ka;-Cd*Ad+Cd*Bd*K,1-
Cd*Bd*Ka],[0;0;1],[0,1,0],[0],T);
step(sysx)
```

ANNEX C

Digital State Feedback Controller with LQR Method MATLAB Code

```
clc
clear all

% The values of the plant parameters
Vin=24;
D=0.52;
R=23;
L=72e-6;
C=50e-6;

%Steady state model of the Boost Converter
As=[0 (-(1-D)/L);(1-D)/C -1/(R*C)];
Bs=[1/L 0 0;0 -1/C 0];
Cs=[0 1;1 0];
Ds=[0 0 0;0 0 0];

Vo=-Cs(1,:)*inv(As)*Bs(:,1)*Vin;
Iin=-Cs(2,:)*inv(As)*Bs(:,1)*Vin;

%Small signal model of the Boost Converter
a=[0 (-(1-D)/L);(1-D)/C -1/(R*C)];
b=[Vo/L;-Iin/C];
c=[0 1];
d=[0];

%Transfer function Vo/d
syss=ss(a,b,c,d);
[num,den]=ss2tf(a,b,c,d);
sys=tf(num,den);

%Discrete time system
T=10e-6;   %Sampling Time
sysd=c2d(syss,T);
[Ad,Bd,Cd,Dd] = ssdata(sysd);

Ax=[Ad,zeros(length(Ad),1);-Cd*Ad,1];
Bx=[Bd;-Cd*Bd];
Cx=[Cd 0];

%LQR Controller Design
Q(1,1)=(1e-3)/T;
Q(2,2)=(1e-2)/T;
Q(3,3)=1.7;
R =1;

Ke = dlqr(Ax,Bx,Q,R);
```

```
ki = -Ke(3)
k1=Ke(1)
k2=Ke(2)


%Check the system with the LQR Controller
K=[k1 k2];

sysx=ss([Ad-Bd*K,Bd*ki;-Cd*Ad+Cd*Bd*K,1-
Cd*Bd*ki],[0;0;1],[0,1,0],[0],T);
step(sysx)

%How to solve P from the Algebraic Ricaati Equation (ARE)
P=dare(Ax,Bx,Q,R)
k=inv(transpose(Bx)*P*Bx+R)*(transpose(Bx)*P*Ax)
```

RESUME

Mohammed Alkrunz was born on June 10, 1987 in Gaza, Palestine. He has completed his High school education in Gaza, 2005. He has earned his BSc in Electrical Engineering from Islamic University of Gaza in fall of 2010. Since his graduation, he worked as a coordinator at Projects and Research Center at Islamic University of Gaza, and he was a teaching assistance at the Electrical Engineering Department in the Islamic University of Gaza as he was one of the excellent students in his class. At the same moment, he was the Head of Control Systems Department at Palestine for Communication and IT. In September 2013, he started his Master Education, Department of Electrical and Electronics Engineering, Institute of Natural Sciences, Sakarya University, Sakarya, Turkey. He is confident that when he makes his next step into PhD's program, he will be building on a strong experience and gain valuable knowledge. His plans for PhD graduate study will be a prolonged one where he shall be consolidating his knowledge in a more specialized way and acquire the skills used for the research.