

RAZVOJ BESKONAČNIH ELEMENATA ZA SIMULACIJU NEOGRANIČENOG MEDIJA

DEVELOPMENT OF INFINITE ELEMENTS FOR SIMULATION OF UNBOUNDED MEDIA

Kemal EDIP
Vlatko SHESHOV
Julijana BOJADJIEVA
Aydin DEMIR
Hakan OZTURK

ORIGINALNI NAUČNI RAD
ORIGINAL SCIENTIFIC PAPER
UDK: 004.421
doi: 10.5937/GRMK1803003E

1 INTRODUCTION

In simulation of physical problems, the area of interest for analysis is usually very small when compared to the dimensions of the surrounding medium such as in underground openings, tunnelling, mining operations etc. It is important to differentiate these problems by considering the boundaries remote from the area of interest, thus, extending the medium to infinity. In finite element (FE) analysis of this kind of problems extending of mesh and applying fixed boundaries is disadvantageous since numerous finite elements should be included.

In order to overcome this problem, infinite elements were proposed by Bettess [1] in the literature. In the work of Beer [2] the infinite elements were used to analyze problems in an underground excavation in pre-stressed infinite medium. Curnier [3] in his work uses infinite element to model unbounded domain economically by using less number of elements and different descent shape functions. In the work of Bettess and Jacqueline [4] the static infinite element is derived extending to infinity in one direction.

On the other hand, the transient wave propagation in elastic media takes place in many fields such as soil dynamics analysis, earthquake engineering, geo-technical engineering etc. The improved simulation is the one that gives a seismic response of the soil model identical to that obtained in the case of the prototype, i.e., the semi-infinite soil layer 1D response under vertically propagating shear waves[5]. Numerical simulation experiences of wave propagation show that simulation of surrounding boundaries is the point which should be considered with special care. In particular, due to these boundaries problems in soil dynamics are more difficult than the problems in structural dynamics because of the radiating waves at the boundaries. The wave propagation problems are solved using the finite element method although the numerical simulation takes place only in the finite element domain. Efficient modelling of infinite media is important since unbounded spatial domain presents a challenge for numerical modelling for simulation of infinite domains.

In the work of Bettess and Zienkiewicz [6] the appropriate boundary conditions are presented for solving the wave problem at the boundaries. In the work of Medina and Penzien[7] an axisymmetric infinite element is developed to solve elastic wave propagation problems in unbounded domain in which the elements are capable of transmitting Rayleigh, shear and compression waves in frequency domain. This type of formulation uses Gauss-Laguerre quadrature scheme for computing the infinite elements numerically. In the work of Medina and Taylor [8] several improvements have been done to the infinite elements. Namely, the approximation accuracy of the infinite elements is obtained by a) selecting realistic shape functions containing approximately the form of the expected

Kemal Edip, Institute of Earthquake Engineering and Engineering Seismology (IZIIS), University "Ss. Cyril and Methodius", Skopje, Macedonia
Vlatko Sheshov, Institute of Earthquake Engineering and Engineering Seismology (IZIIS), University "Ss. Cyril and Methodius", Skopje, Macedonia
Julijana Bojadjeva, Institute of Earthquake Engineering and Engineering Seismology (IZIIS), University "Ss. Cyril and Methodius", Skopje, Macedonia
Aydin Demir, Department of civil engineering, Faculty of Engineering, Sakarya University, Sakarya, Turkey
Hakan Ozturk, Department of civil engineering, Faculty of Engineering, Sakarya University, Sakarya, Turkey

solution and b) increasing the number of element nodes in the infinite direction. In the work of Chuhan and Chongbin[9] with the consideration of the behaviour of wave amplitude attenuation and phase delay the formulation of a frequency-dependent compatible infinite elements have been derived.

On the other hand, the transmitting boundaries have been simulated by many other authors Lysmer and Kuhlmeyer[10], White et. al. [11], Kausel[12] and Cundall et al. [13] who have presented various promising techniques. Toward this end, absorbing infinite elements have been developed by combination of static infinite elements with absorbing layers. In the work of Haggblad [14] a six node infinite element is developed adding the absorbing layer. Similarly, in the work of Edip [15], static infinite elements with $1/n$ decay function has been combined with absorbing boundaries. The main advantage in using these type of infinite elements lays in the simplification of usage of conventional Gauss-Legendre abscissae and weights. Further usage of absorbing infinite elements can be found in the works of other authors [16-19].

In this work the infinite elements have been developed considering both static and dynamic conditions. Next, the development and verification of the infinite elements is given.

2 DEVELOPMENT OF STATIC AND ABSORBING INFINITE ELEMENTS

Mapping the infinite domain to finite element seems the most logical solution to develop infinite elements using nodes at special points even in infinity. The figure 1 illustrates the mapping functions logic.

The element field variables are approximated in such a way that standard interpolation functions are used to obtain a decay function with $1/r$ rate for the field variables, where r stands for the distance from the 'pole' of the element to a point in the infinite domain.

In defining the positioning of the pole(s) in infinite elements, special attention should be given to the geometry and the field variable expansion. Moreover, the uniqueness of the mapping and the continuity of the

solution between elements with common sides has to be considered by choosing the shape functions according to the number of nodes. The main advantage of the mapped infinite elements is the usage of the conventional Gauss-Legendre abscissa and weights. The main benefit of the proposed infinite elements is in the number of nodes which allow coupling with four or eight noded finite elements. Element matrices are constructed by using the procedures as described in Bathe[20]. By adding together the parts from each element constituent, the governing incremental equations for equilibrium in dynamic analysis are obtained.

The formulation of infinite elements follows the same procedure as for the finite elements. The mapping functions are added to present the mapping of the domain. The infinite element is obtained as a six and eight node element as shown in Figure 2.

The element displacement in u and v direction is interpolated with the usual shape functions $N1, N2, N4$ and $N5$:

$$\begin{aligned} u &= [N_1 \quad N_2 \quad 0 \quad N_4 \quad N_5 \quad 0] \mathbf{u} \\ v &= [N_1 \quad N_2 \quad 0 \quad N_4 \quad N_5 \quad 0] \mathbf{v} \end{aligned} \quad (1)$$

The vectors \mathbf{u} and \mathbf{v} of expression (1) are nodal point displacements in global coordinates.

$$\begin{aligned} N_1 &= -(1-s)r(1-r)/4 \\ N_2 &= (1/2)(1-r^2)(1-s) \\ N_4 &= -(1+s)r(1-r)/4 \\ N_5 &= (1/2)(1-r^2)(1+s) \end{aligned} \quad (2)$$

For coordinate interpolation in r - s coordinate system a one-dimensional mapping is applied.

$$\begin{aligned} r &= [M_1 \quad M_2 \quad 0 \quad M_4 \quad M_5 \quad 0] \mathbf{r} \\ s &= [M_1 \quad M_2 \quad 0 \quad M_4 \quad M_5 \quad 0] \mathbf{s} \end{aligned} \quad (3)$$

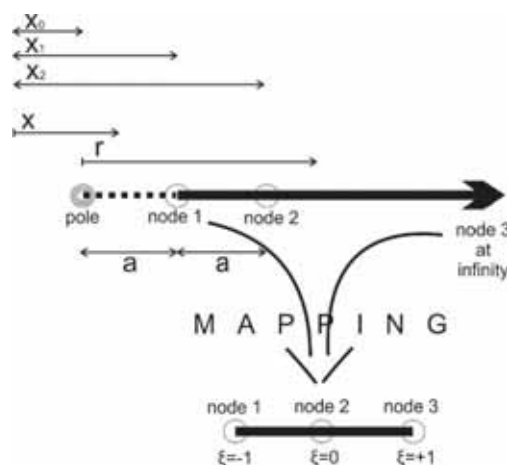


Fig. 1 Representation of the domain in infinite element

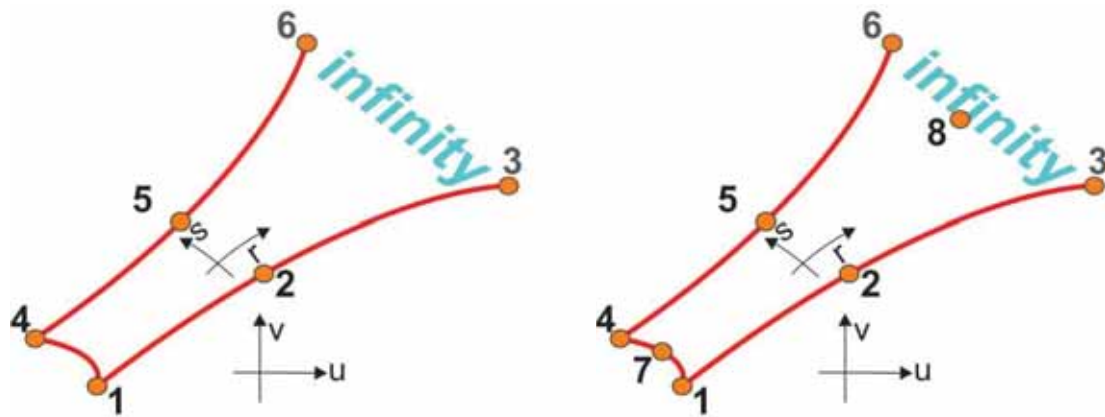


Fig. 2. Infinite element with 6 nodes and 8 nodes

where

$$\begin{aligned}
 M_1 &= -\frac{(1-s)r}{1-r} \\
 M_2 &= -\frac{1}{2} \frac{(1-s)(1+r)}{1-r} \\
 M_4 &= -\frac{(1+s)r}{1-r} \\
 M_5 &= -\frac{1}{2} \frac{(1+s)(1+r)}{1-r}
 \end{aligned} \quad (4)$$

In eight node infinite element, the element displacement in u and v direction is interpolated with the usual shape functions as given below for 8 node infinite element:

$$\begin{aligned}
 u &= [N_u^1 \ N_u^2 \ 0 \ N_u^4 \ N_u^5 \ 0 \ N_u^7 \ 0] \bar{u} \\
 v &= [N_u^1 \ N_u^2 \ 0 \ N_u^4 \ N_u^5 \ 0 \ N_u^7 \ 0] \bar{v}
 \end{aligned} \quad (5)$$

The shape functions are given as follows:

$$\begin{aligned}
 N_u^1 &= -(r-1)(-1+s)(s+1+r)/4 \\
 N_u^2 &= (r-1)(1+r)(-1+s)/2 \\
 N_u^4 &= -(r-1)(1+s)(s-1-r)/4 \\
 N_u^5 &= -(r-1)(1+r)(1+s)/2 \\
 N_u^7 &= (-1+s)(1+s)(r-1)/2 \\
 N_p^1 &= (s-1)(r-1)/4 \\
 N_p^4 &= -(s+1)(r-1)/4
 \end{aligned} \quad (6)$$

The coordinate interpolation in the infinite element from global to local is completed using the expressions (5) and (6). The infinite element is one sided meaning that only the positive r direction extends to infinity. Following Fig. 1 the mapping functions for coordinate interpolation considering displacement degrees of freedom are defined as follows:

$$\begin{aligned}
 r &= [M_u^1 \ M_u^2 \ 0 \ M_u^4 \ M_u^5 \ 0 \ M_u^7 \ 0] \bar{r} \\
 s &= [M_u^1 \ M_u^2 \ 0 \ M_u^4 \ M_u^5 \ 0 \ M_u^7 \ 0] \bar{s}
 \end{aligned} \quad (7)$$

Where the mapping functions are given as:

$$\begin{aligned}
 M_u^1 &= -\frac{(1-s)rs}{1-r} \\
 M_u^2 &= -\frac{(1-s)(1+r)}{2(1-r)} \\
 M_u^4 &= -\frac{(1+s)rs}{1-r} \\
 M_u^5 &= -\frac{(1+s)(1+r)}{2(1-r)} \\
 M_u^7 &= -\frac{2r(1+s)(1-s)}{(1-r)}
 \end{aligned} \quad (8)$$

In expressions (7) and (8), u and r stand for vectors of nodal point displacement. On the infinity side where values of r approach unity ($r=1$), no mappings are defined. The number and location of the common nodes connecting finite and infinite elements should coincide to guarantee the continuity condition between the elements. The proposed infinite elements have possibility of assigning different number of nodes for displacement allowing coupling with different finite elements. Last but not the least, the newly developed infinite elements are advantageous in correct representation of the boundary conditions by user defined number of integration points in the infinite elements. The only disadvantage of the developed infinite elements is in the fact that the more integration points are used the slower the calculation is performed.

3 VERIFICATION OF STATIC INFINITE ELEMENTS - PLATE WITH CIRCULAR HOLE

In geotechnical problems extending to infinity one of the most difficult tasks is to model the infinite region. In the next problem, an infinite plate having a circular hole of

radius $R=1.0$ m length is subjected to a pressure $p=1\text{kPa}$ which is uniformly distributed. The domain is presented in Fig. 3.

The domain being a plane strain space has the following material properties: Young's modulus $E=1\text{kPa}$ and Poisson's ratio $\nu=0.25$. The discretization of the domain is done using 80 finite elements in total. The infinite elements of 6 and 8 nodes are used to simulate the boundaries. The results comparing the analytical solution as given in the work of T.T. Abdel-Fattah [21] are given in Fig. 4.

As it is seen from the Fig. 4 the correctness of the analytical results with infinite elements is adequately good although; the 8-node infinite elements have better correctness in the results. Thus, in absorbing infinite elements, the 8-node infinite elements should be used further.

4 ABSORBING INFINITE ELEMENTS IN ABAQUS

In the study, efficiency of the developed infinite element is also compared with the results of the numerical simulations conducted by the FE-code ABAQUS [21]. ABAQUS is a general-purpose analysis software that can solve a wide range of linear and nonlinear problems comprising the static and dynamic response of the components [22].

Built-in infinite elements are provided in ABAQUS to solve boundary value problems defined in unbounded domains or problems in which the region of interest is small compared to the dimensions of the surrounding medium. While standard finite elements are used to model the region of interest, the infinite elements are utilized to model the far-field region. Infinite elements can be used together with first- and second-order planar, axisymmetric, and three-dimensional finite elements [22].

The infinite elements provided in ABAQUS are based on the study of Zienkiewicz et al. [23] for static response and of Lysmer and Kuhlemeyer [9] for dynamic response. They provide stiffness in static solid continuum analyses and quiet boundaries to the FE model in dynamic analyses. The static behaviour of the infinite elements is based on modelling the basic solution variable u according to spatial distance r measured from a "pole" of the solution, so that $u \rightarrow 0$ as $r \rightarrow \infty$, and $u \rightarrow \infty$ as $r \rightarrow 0$. The interpolation provides terms of order $1/r$, $1/r^2$ and, when the solution variable is a stress-like variable $1/r^3$ as $r \rightarrow 0$. Additionally, dynamic response of the infinite elements is considered in case of plane body waves travelling out of the domain. All assumptions near to the boundary are of small amplitude to develop responds only in a linear elastic domain [22].

Infinite elements available in ABAQUS are given and defined in Table 1 and their naming convention is depicted in Fig. 5.

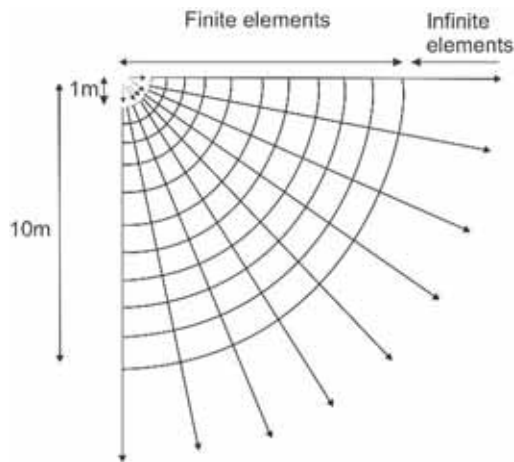


Fig. 3. Mesh of finite and infinite elements

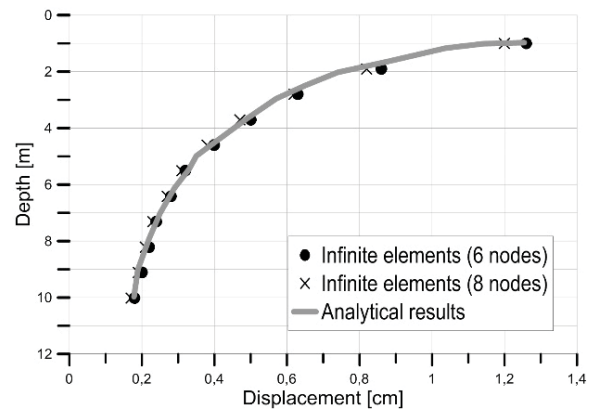


Fig. 4. Comparison of analytical and numerical results

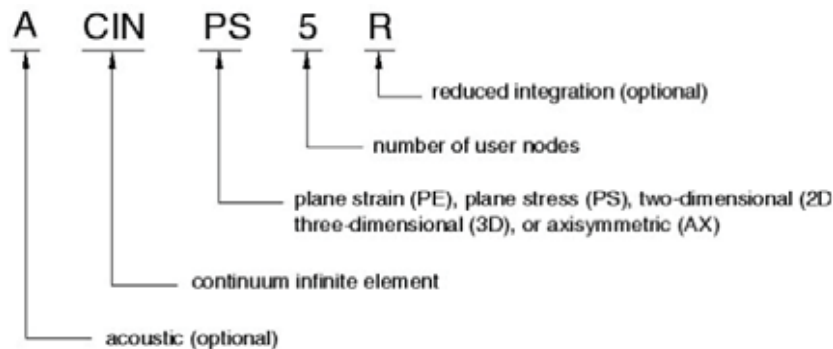


Fig. 5. Naming Convention of Infinite Elements in ABAQUS [22]

Table 1. Infinite Elements provided in ABAQUS

Type of Infinite Element	Naming Convention	Definition
Plane strain solid continuum	CINPE4	4-node linear, one-way infinite
	CINPE5R(S)	5-node quadratic, one-way infinite
Plane stress solid continuum	CINPS4	4-node linear, one-way infinite
	CINPS5R(S)	5-node quadratic, one-way infinite
3D solid continuum	CIN3D8	8-node linear, one-way infinite
	CIN3D12R(S)	12-node quadratic, one-way infinite
	CIN3D18R(S)	18-node quadratic, one-way infinite
Axisymmetric solid continuum	CINAX4	4-node linear, one-way infinite
	CINAX5R(S)	5-node quadratic, one-way infinite

A solid section definition is used to define the element's section properties which should be associated with a region of the model. A thickness should be defined for two-dimensional, plane strain, and plane stress elements as an element property definition. By default, a unit thickness is assumed. On the contrary, it is needless to specify a thickness for three-dimensional and axisymmetric solid elements. Material definition of infinite elements is assumed to be linear, therefore only linear behaviour can be associated with infinite elements. Moreover, the material response of the infinite elements in dynamic analysis is assumed to be isotropic and it should also match the material properties of the adjacent finite elements in the linear domain [22].

The node numbering of infinite elements should be assigned in a way that the first face is the face that is connected to the finite element part of the mesh. Node ordering and face numbering on elements, and numbering of integration points for output is depicted in Fig. 6 for PS and PE solid continuum elements. The formulation of the solid medium elements is based on the fact that the far-field solution along each element edge that stretches to infinity is centred about an origin, called the "pole". The position of the nodes in the infinite direction should be selected appropriately with respect to the pole. It is important to position the second node along each edge pointing in the infinite direction in order that it is twice as far from the pole as the node on the same edge at the boundary between the finite and the

infinite elements. In addition to this length consideration, the second node in the infinite direction should be specified in the way that the element edges in the infinite direction do not cross over, which would give non-unique mappings (Fig. 7). ABAQUS will give an error message if such a problem occurs. These second nodes in the infinite direction can be projected conveniently from a pole node. The positions of the pole and the nodes on the boundary between the finite and the infinite elements are used. However, in explicit dynamic analysis the infinite element nodes that are not part of the first face are treated differently. Those nodes are located away from the finite element mesh in the infinite direction. Loads and boundary conditions should not be specified using these nodes since the location of these nodes is not meaningful for explicit dynamic analysis [22].

In the study of 4-node linear, one-way plane strain solid continuum infinite elements (CINPE4) are used to simulate the propagation of waves in a soil domain in which the region of interest is small in size compared to the surrounding medium. Linear dynamic behaviour of the soil is analyzed by ABAQUS/Explicit analysis procedure. A solid section definition is used to define the element's section properties and it is associated with the region of the model. Unit thickness is assumed for thickness of the two-dimensional plane strain elements. Moreover, similar mesh size and material properties given in the reference studies are assigned to the numerical models.

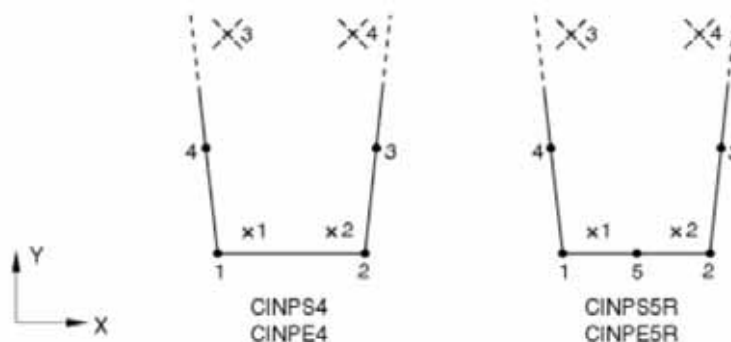


Fig. 6. Node ordering and face numbering on elements, and numbering of integration points for PS and PE solid continuum elements [22].

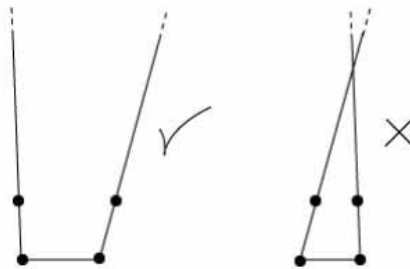


Fig. 7. Examples of an acceptable and an unacceptable two-dimensional infinite element [22].

5 VERIFICATION OF ABSORBING INFINITE ELEMENTS

The newly developed absorbing infinite elements are verified by several examples. Comparisons are made with the software ABAQUS.

5.1 Wave propagation in 1D

First, wave propagation in one dimensional medium is simulated to verify the newly programmed infinite elements. The soil parameters are taken from Plaxis [22] example and comparisons are made accordingly. Simulation is done using a soil column composed of finite elements with fixed and infinite element boundaries. Soil domain is presented in Figure 8. It has a length of 10m and is discretized with 40 elements. In the middle of the soil column a point A is chosen as a reference point for the comparison of the results. The soil domain of interest has a length of 10m and is discretized with 40 elements. The properties of soil are taken as: Young's modulus $E=18000\text{kPa}$, Poisson's ratio $\nu=0.2$ and Density $=2.04\text{ton/m}^3$. The p wave velocity is given in PLAXIS manual [22]. The p-wave velocity is given in the equation 9 below as follows:

$$V_p = \sqrt{\frac{(1-\nu)E}{\rho(1+\nu)(1-2\nu)}} \quad (9)$$

Using the values above, the p wave velocity can be computed as $V_p=99\text{m/s}$. In simulation of wave propagation the application of vertical displacement on the top of the domain is done in three different types. Namely,

- Heaviside step function
- Impulse function
- Sine function

In this case the applied displacement is considered to be of Heaviside step type with a magnitude of $u_y=0.001\text{m}$. Time history of the travelling P wave at point A is given in Figure 9.

In Figure 9, it is clearly shown that in the case of fixed boundaries the P wave is reflected while using the infinite elements the P wave is absorbed in the boundary hindering the back propagation of the wave. When estimated the time needed for the P-wave to reach the point A it is easily seen that the wave velocity equals the

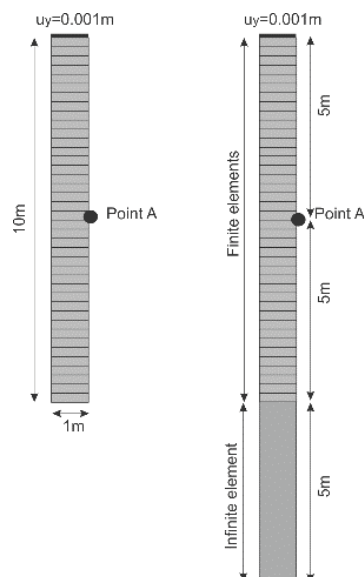


Fig. 8. Domain of soil column

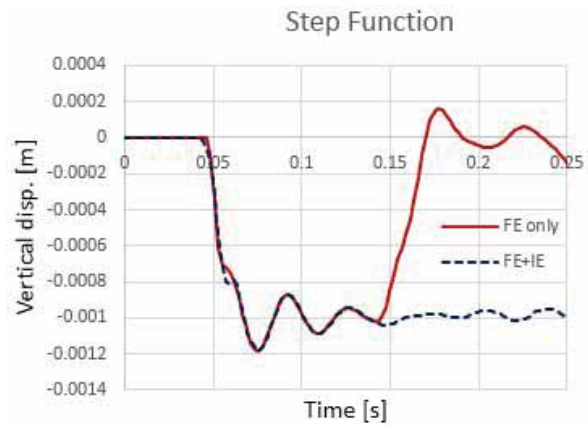
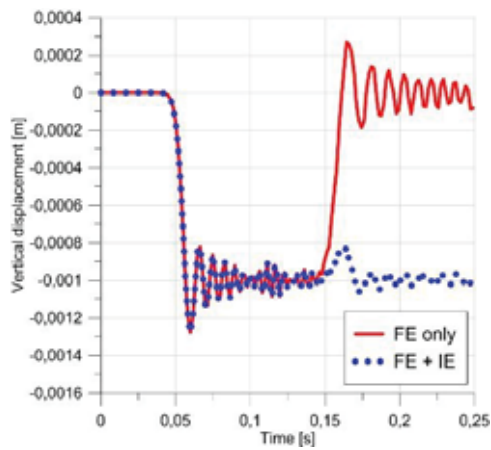


Fig. 9. Time displacement history-Heaviside step function ANSYS left, ABAQUS right

value of equation 9 ($V_p=99\text{m/s}$). On the other hand, the simulation with ABAQUS reveals similar results proving that the newly developed infinite elements in ANSYS provide correct results.

Displacement as an Impulse function

In applying the displacement as an impulse function interesting results have been found. The impulse function is applied only for 0.00166s and then the domain is left to vibrate freely up to 0.3seconds. In Figure 10, displacement comparison is given in the mid-point of the domain (Point A).

As can be seen from Fig. 10 that infinite elements absorb the wave in a correct manner not letting the reflection to take place back in the domain. The first impulse occurs at time=0.05 seconds which ensures that the velocity of P-wave is 99m/s and is in accordance with equation (9). The comparison between the infinite elements used in ANSYS and ABAQUS software shows that both infinite elements absorb the wave in a correct manner although the newly developed infinite elements in ANSYS software seem to decrease the oscillations back in the domain of finite elements. This is mainly due

to the reason of the increased number of integration points which are used in the newly developed infinite elements.

Displacement as a sine function

In order to evaluate the simulation of the applied displacement as a sine function a circular frequency of 0.5Hz has been selected. The sine function is applied at the top of the model for 10 seconds and the results are compared accordingly.

As can be seen from Fig. 11 the time displacement histories of sine functions have different magnitudes. This proves that boundary conditions play an important role in simulation of wave propagation. The values that are compared occur at the middle of the domain in the way that the wave magnitude in the case of fixed boundaries is doubled. Once more, the usage of infinite elements proves to play important role in simulation of the wave propagation problems. The comparison between ANSYS and ABAQUS show similar magnitudes although in ABAQUS the frequency is considered as 0.1Hz.

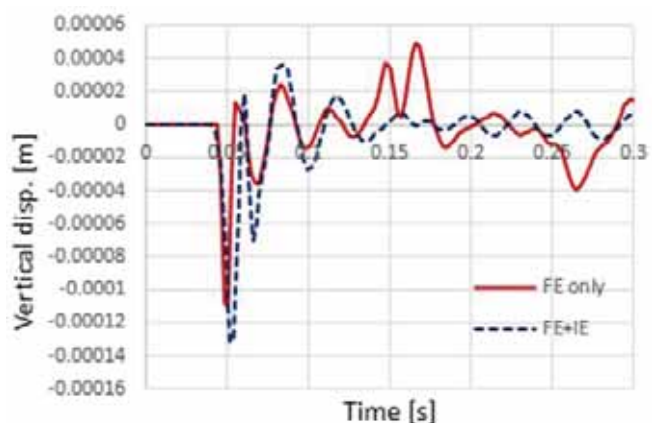
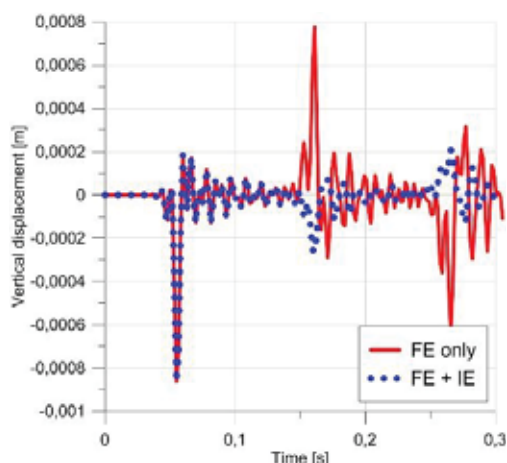


Fig. 10. Time displacement history-impulse function ANSYS left, ABAQUS right

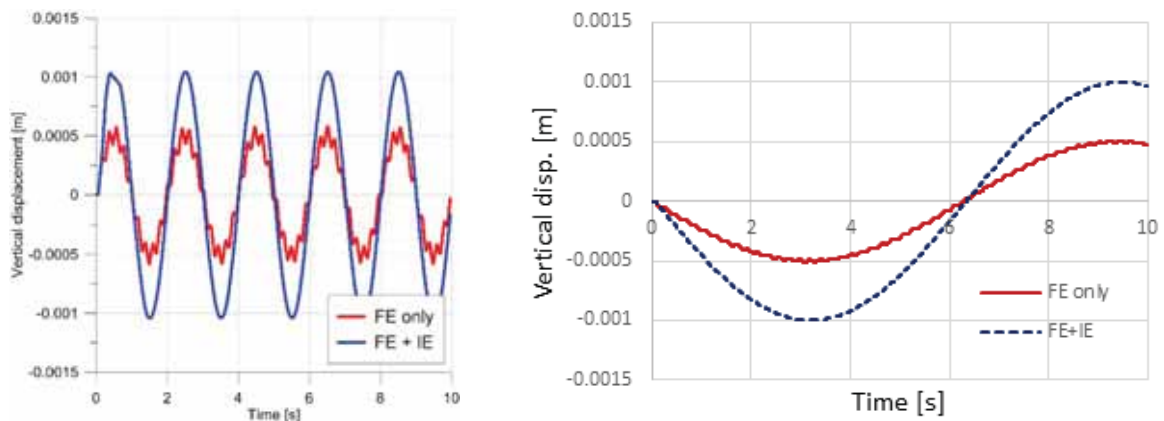


Fig. 11. Time displacement history-sine function ANSYS left, ABAQUS right

5.2 Two dimensional wave propagation

Two dimensional wave propagation through a quarter-space is considered for the sake of completeness. The material parameters are the same as in the case of 1D simulations above. The soil medium as given in Figure 12, and it is a combination of finite and infinite elements. There are 15 infinite elements in total. At the uppermost part a prescribed displacement of magnitude 0.001m is prescribed. Comparisons of the results are given in Fig.13.

In Fig. 13 simulation of wave propagation using both software of ANSYS and ABAQUS is shown. The wave

propagation is smooth up to the point where the wave arrives at the boundary, as can be seen from Fig. 13. In the case of finite elements only the wave reflects back to the medium, thus, colliding with the other waves propagating in the medium. In the case of boundaries represented by infinite elements, the wave does not return back to the medium. Namely, the wave is absorbed at the infinite elements. The comparison of newly programmed infinite elements provides acceptable results when compared with the infinite elements in ABAQUS software.

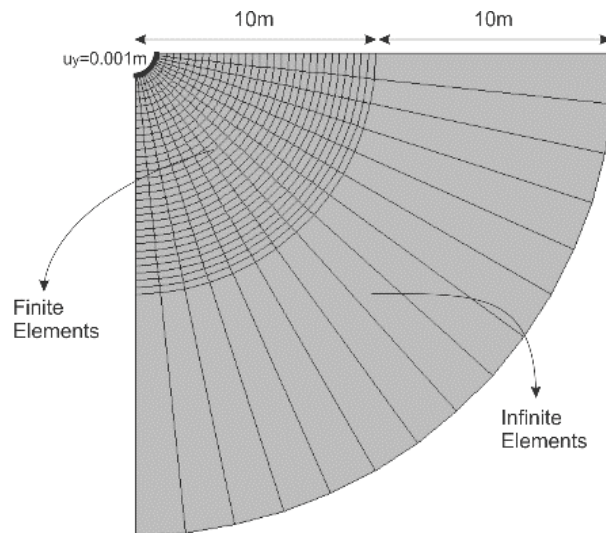


Fig. 12. 2D Domain of finite and infinite elements

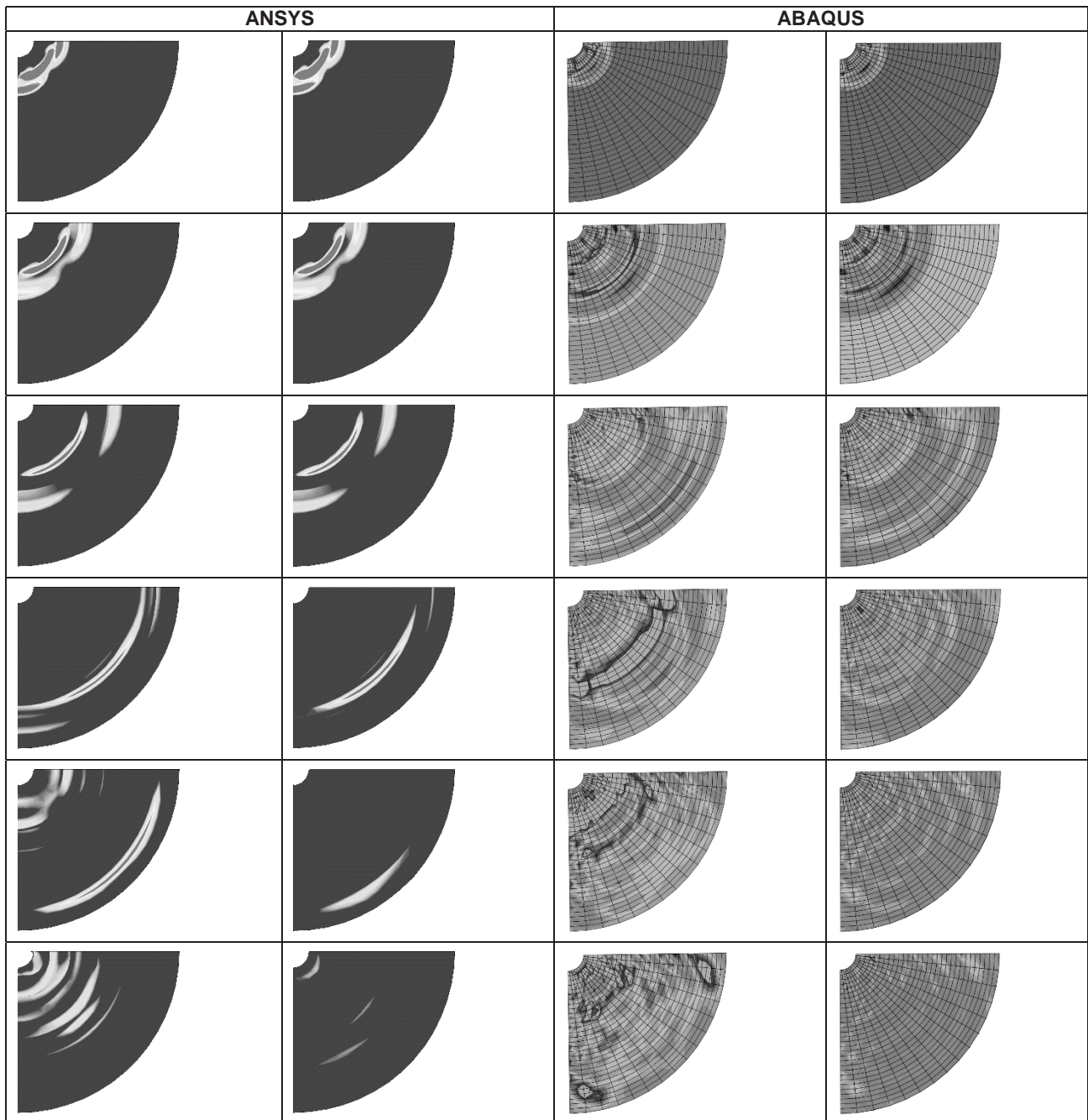


Fig. 13. Wave propagation at time: $t=0.04s, t=0.06s, t=0.15s, t=0.18s, t=0.21s, t=0.24s$ (left only finite elements; right finite and infinite elements)

6 CONCLUSION

In this work, new type of infinite elements with both six and eight nodes having a $1/n$ type decay function have been developed. In simulation of static problems the infinite elements have been shown to give satisfactory results. On the other hand, in dynamic conditions the same infinite elements have been added absorbing layer in order to prevent reflection of the displacement waves back to the region of interest i.e. finite elements. In the case of 1D and 2D wave

propagation, the obtained results show that the usage of absorbent infinite elements improves the results greatly. The obtained numerical results from ANSYS and ABAQUS software are reliable and further application of coupled finite and infinite elements could be considered in the field of soil structure interaction. Moreover, the newly developed absorbent infinite elements are in time domain which allow non-linearity of materials to be considered in the finite elements region.

7 REFERENCES

- [1] Bettess, P., *Infinite elements*. INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 1977. **11**(1): p. 53-64.
- [2] Beer, G., *'Infinite domain' elements in finite element analysis of underground excavations*. International journal for numerical and analytical methods in geomechanics, 1983. **7**(1): p. 1-7.
- [3] Curnier, A., *A static infinite element*. INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 1983. **19**(10): p. 1479-1488.
- [4] Bettess, P. and J.A. Bettess, *Infinite elements for static problems*. Engineering Computations, 1984. **1**(1): p. 4-16.
- [5] Bojadjeva, J., *Dynamic behaviour of saturated cohesionless soils based on element and 1-g experiments*, in *Ss.Cyril and Methodius: Institute of Earthquake Engineering and Engineering Seismology*, 2015.
- [6] Bettess, P. and O.C. Zienkiewicz, *Diffraction and refraction of surface waves using finite and infinite elements*. INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 1977. **11**(8): p. 1271-1290.
- [7] Medina, F. and J. Penzien, *Infinite elements for elastodynamics*. Earthquake Engineering & Structural Dynamics, 1982. **10**(5): p. 699-709.
- [8] Medina, F. and R.L. Taylor, *Finite element techniques for problems of unbounded domains*. INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 1983. **19**(8): p. 1209-1226.
- [9] Chuhan, Z. and Z. Chongbin, *Coupling method of finite and infinite elements for strip foundation wave problems*. Earthquake Engineering & Structural Dynamics, 1987. **15**(7): p. 839-851.
- [10] Lysmer, J. and R.L. Kuhlmeyer, *Finite Dynamic Model for Infinite Media*. J.Engng Mech. Div., ASCE, 1969. **95**,: p. 859-877.
- [11] White, W., S. Valliappan, and I.K. Lee, *Unified boundary for finite dynamic models*. J. Engng Mech. Div., ASCE, 1977. **103**: p. 949-964.
- [12] Kausel Bolt, E.A.M., *Forced vibrations of circular foundations on layered media*, 1974, Massachusetts Institute of Technology.
- [13] Cundall, P., et al. *Solution of infinite dynamic problems by finite modelling in the time domain*. in *Proceedings of the Second International Conference on Applied Numerical Modelling*. 1978.
- [14] Häggblad, B. and G. Nordgren, *Modelling nonlinear soil-structure interaction using interface elements, elastic-plastic soil elements and absorbing infinite elements*. Computers & Structures, 1987. **26**(1-2): p. 307-324.
- [15] Edip, K., et al., *Numerical simulation of wave propagation in soil media*. 2011, *Proceedings of the 21st European Young Geotechnical Engineers' Conference Rotterdam, Holland*. ISBN 978-1-60750-807-6 (print) | 978-1-60750-808-3 (online)
- [16] Edip, K., *Development of three phase model with finite and infinite elements for dynamic analysis of soil media*, 2013, Ss. Cyril and Methodius: Institute of Earthquake Engineering and Engineering Seismology.
- [17] Edip, K., et al., *Boundary effects on seismic analysis of multi-storey frames considering soil structure interaction phenomenon*, in *Seismic Design of Industrial Facilities 2014*, Springer. p. 569-575.
- [18] M.Garevski, et al. *Effects Of Soil Medium On Response Of Base Isolated Multistorey Frame Structures*. in *15th World Conference on Earthquake Engineering*. 24 – 28 September, 2012. Lisbon, Portugal.
- [19] Sesov, V., et al., *Development of New Infinite Element for Numerical Simulation of Wave Propagation in Soil Media*, in *Experimental Research in Earthquake Engineering 2015*, Springer. p. 423-436.
- [20] K.J.Bathe, *Finite Element Procedures in Engineering Analysis* 1982, Englewood Cliffs. NJ, : Prentice-Hall.
- [21] Abdel-Fattah, T.T., H.A. Hodhod, and A.Y. Akl, *A novel formulation of infinite elements for static analysis*. Computers & Structures, 2000. **77**(4): p. 371-379.
- [22] Plaxis, B., *PLAXIS 2D Version 2015-Finite element code for soil and rock analysis*. AA Balkema, Delft, 2015.

SUMMARY

DEVELOPMENT OF INFINITE ELEMENTS FOR SIMULATION OF UNBOUNDED MEDIA

*Kemal EDIP
Vlatko SHESHOV
Julijana BOJADJIEVA
Aydin DEMIR
Hakan OZTURK*

Based on the elastic theory assumptions, an infinite element boundary which is frequency independent is derived. The infinite element development is based on mapping functions and viscous layer for damping propagating waves. In numerical modelling the general finite element software ANSYS using its User Programmable Features (UPF) is used. Related comparisons are done with PLAXIS and ABAQUS software. In simulation of propagating waves, the numerical approach is done considering several one-dimensional and two-dimensional wave propagation.

Key words: Infinite elements, numerical analysis, wave propagation

APSTRAKT

RAZVOJ BESKONAČNIH ELEMENATA ZA SIMULACIJU NEOGRANIČENOG MEDIJA

*Kemal EDIP
Vlatko SHESHOV
Julijana BOJADJIEVA
Aydin DEMIR
Hakan OZTURK*

Na osnovu pretpostavki elastične teorije izveden je granica beskonačnog elementa koja je nezavisna od frekvencije. Razvoj beskonačnog elementa zasnovan je na kartografskim funkcijama i viskoznom sloju za prigušivanje propagirajućih talasa. U numeričkom modeliranju koristi se opšti softver konačnih elemenata ANSYS koji koristi svoje korisničke programabilne funkcije (UPF). Za odgovarajuće uporedne analize korišćeni su rezultati dobijeni softverskim paketima PLAXIS i ABAQUS. U simulaciji propagirajućih talasa, a numerički pristup je obavljen uzimajući u obzir nekoliko jedno-dimenzionalnih i dvodimenzionalnih propagacija talasa.

Ključne reči: beskonačni elementi, numeričke analize, propagacija talasa