EURO J Transp Logist (2019) 8:249–268
https://doi.org/10.1007/s13676-018-0123-1

RESEARCH PAPER

Estimating the operational and service efficiency of bus transit routes using a non-radial DEA approach

Samet Güner[1] · Erman Coşkun[1]

Received: 14 February 2017 / Accepted: 10 April 2018 / Published online: 13 April 2018

Abstract In public transportation literature, researchers increasingly tend to evaluate the service efficiency of bus transit units in addition to their operational efficiency. Evaluation of both operational and service efficiencies provides a more comprehensive performance analysis for decision-makers. This paper proposes a non-radial data envelopment analysis (DEA) approach to measure the operational and service efficiencies simultaneously. Benefits of the proposed approach are demonstrated by applying the model to assess the performance of a bus transit company's routes. The analysis results show that the proposed approach ensures optimal operational and service efficiency scores and provides applicable input targets for each route.

Keywords Bus transit routes · DEA · Efficiency · Non-radial · Operational · Service

1 Introduction

Data envelopment analysis (DEA), which was first introduced by Charnes et al. (1978), is a non-parametric mathematical method for assessing the relative efficiency of decision-making units with multiple inputs and multiple outputs. These multiple inputs and outputs are used to construct a piecewise linear best-practice frontier by accepting the best-performing units as a reference point and evaluating

✉ Samet Güner
sguner@sakarya.edu.tr

Erman Coşkun
ermanc@sakarya.edu.tr

1 School of Business, Sakarya University, Esentepe Campus, 54187 Serdivan, Sakarya, Turkey

the other units' efficiencies comparatively according to their distances from the efficient frontier. Units located on the efficient frontier are classified as efficient and rated with an efficiency score of one. Units located below the efficient frontier are considered inefficient and rated with scores between zero and one. Therefore, DEA allows managers to distinguish between efficient and inefficient units and to measure the degree of inefficiency by assigning the best-practice units as benchmarks.

The main advantages of DEA that make it a suitable tool for estimating the efficiency of public transportation systems are: (1) it can handle multiple inputs and outputs simultaneously; (2) it is unit invariant and not affected by the measurement units of inputs and outputs; (3) it does not require information on prices; and (4) it does not impose any functional form on the underlying technology. Because of these advantages, past studies have shown that DEA has been widely and effectively used for efficiency analysis of bus transit systems. In these studies, DEA is mainly used to measure the operational efficiency, which is related to financial and technical performance to convert bus transit units' resources to outputs. In addition to operational efficiency, the service efficiency of bus transit systems, which pertains to the optimization of the service levels provided, has started attracting the attention of researchers. Therefore, multiple objectivity has emerged as an important issue in public transportation studies (Güner and Coşkun 2016).

To analyze the comprehensive efficiency of bus transit systems, Chu et al. (1992) developed three distinct DEA models. Two were designed to assess the different aspects of operational efficiency and provide two means of managerial control, one by functional areas and one by budgetary objects. The third DEA model was designed to measure the service efficiency (or effectiveness) of transit agencies. While the operational efficiency models include financial inputs (operational expenses, maintenance expenses, etc.) the service efficiency model includes input variables related to the provided service level (operating hours, population density, number of households without an automobile, and total financial support per passenger). The total number of unlinked passenger trips was used to measure the output consumed in this model. Kerstens (1999) developed two distinct DEA models for operational and service efficiency. Each model includes the same input parameters (average number of vehicles used, average number of employees, and total fuel consumption) but different output measures; the output parameter of the operational efficiency model is vehicle kilometers, and the output of the service efficiency model is the number of passengers carried. Similarly, Karlaftis (2004) and Karlaftis and Tsamboulas (2012) evaluated transit operators' performances in terms of their operational and service efficiency using two distinct DEA models. In these studies, the authors used the same inputs for both models (labor, fuel, and capital) but used different outputs. Total annual vehicle mileage was the output of the operational efficiency model, and total annual ridership was the output of the service efficiency model. In their study, Barnum et al. (2008) constructed a DEA model to estimate the service quality of bus transit routes. This model includes physical resources (seat kilometers and seat hours) as inputs and service quality indicators (ridership, span of service, average frequency, maximum frequency, and on-time performance) as outputs. Lao and Liu (2009) created two distinct DEA models to measure the operational efficiency and the spatial effectiveness of transit companies. The number of passengers transported was determined to be the unique output for both models. The operational efficiency model was characterized by three input variables: operation time, round-trip distance, and number of bus stops. Spatial effectiveness was characterized by three input variables: commuters who use buses, the 65 years and older population, and persons with disabilities within 0.25 miles of a bus stop. Sanchez (2009), on the other hand, added quality-related outputs such as frequency, accessibility, comfort, and safety directly to his technical efficiency model. More recently, Güner and Coşkun (2016) developed two distinct DEA models to measure the operational and service efficiencies of bus transit routes. The average number of unlinked passenger trips per day was determined as the unique output for both models. The operational efficiency model included three input variables (number of buses, route distance, and fuel consumption) that reflected the technical performance. On the other hand, the service efficiency model included five input variables (frequency, service minutes, stops per km, deviation from shortest distance, and travel time) that reflected the effectiveness of the provided service level. The authors adopted the two-model approach, which was introduced by Shimshak and Lenard (2007), to determine the best-performing benchmarks for inefficient routes.

It is important to note that there are two methods of simultaneous investigation of the operational and service efficiencies of a public transportation unit. The first method aggregates the operational and service efficiency variables in a single model (see Barnum et al. 2008 and Sanchez 2009). This approach provides ease of implementation. However, it does not allow decision-makers to manage the tradeoffs between operational and service efficiencies (Sherman and Zhu 2006). Furthermore, aggregated models (which include indicators of more than one performance dimension) weaken the ability of a DEA model to distinguish the individual effects of these performance aspects (Klimberg and Puddecombe 1999).

The second method involves developing two distinct DEA models for operational and service efficiencies and evaluating them independently (see Chu et al. 1992; Kerstens 1999; Karlaftis 2004; Lao and Liu 2009; Karlaftis and Tsamboulas 2012; and Güner and Coşkun 2016). This method allows managers to assess operational and service performances separately and provides a more comprehensive efficiency analysis. Moreover, it makes the tradeoffs more evident. Despite these advantages, it has some issues. First, undesirable benchmarking could occur, which improves the operational efficiency but worsens the service efficiency, and vice versa (Shimshak and Lenard 2007; Güner and Coşkun 2016). Furthermore, as discussed in detail in Sect. 2, this approach results in contradictory input targets for inefficient routes.

Therefore, this study proposes a non-radial DEA approach to simultaneously assess the operational and service efficiency models. The proposed approach considers the mathematical interrelationships between these models and can determine optimal efficiency scores that provide applicable input/output targets for each bus transit route. To do this, several additional constraints are added to the basic non-radial DEA model. It is expected that the proposed methodology will be used effectively to conduct a more comprehensive efficiency analysis for public transportation systems. The research is based on a case study of Sakarya City Metropolitan Municipality Transportation Bureau (SMMTB), a public transit bus authority in Sakarya, Turkey.

The remainder of the paper is organized as follows: Sect. 2 discusses the problem statement, and Sect. 3 introduces the non-radial DEA approach. Section 4 describes the demonstration and application of this approach, and Sect. 5 concludes the paper.

2 Problem statement

DEA is designed to evaluate single efficiency models. It is unsuitable for evaluating decision-making units in multiple-efficiency environments; consequently, it behaves like a single objective method (Klimberg and Puddecombe 1999; Cook et al. 2000). However, in practice, managers evaluate multiple performance aspects of their organizations. As Charnes and Cooper (1985) have noted, although managerial performance can also be evaluated regarding propriety (i.e., the objectives pursued and methods used) and effectiveness (i.e., stating and attaining objectives), DEA focuses on a single efficiency aspect and overlooks other performance dimensions. This structure of DEA limits decision-makers to a single management performance assessment and deprives them of a comprehensive performance evaluation.

Along with the increasing importance of service efficiency in public transportation literature, researchers are developing two distinct DEA models to evaluate operational and service efficiencies independently. This approach overlooks the interrelationships among the input variables of operational and service efficiency models and results in contradictory input targets for inefficient routes, making it impossible to implement DEA results in practice (Güner 2014).

Let us explain this situation by offering an example from the public transportation industry. Assume that two distinct models are constructed to estimate the operational and service efficiencies of a set of bus transit routes. The operational efficiency model includes two inputs: capacity and number of stops. The service efficiency model is characterized by three input variables: frequency, stops per km, and service hours. The unique output measure for both models is unlinked passenger trips. See Sect. 4.1 for detailed explanations of these input/output variables and their uses.

To analyze the operational and service efficiencies, two distinct DEA models under variable returns to scale should be constructed as presented below.

$$\theta^* = \min \theta \quad \text{(a)}$$
subject to
$$\sum_{j=1}^{J} \lambda_j c_j \le \theta c_r j = 1, \ldots, r, \ldots, J \text{(b)}$$
$$\sum_{j=1}^{J} \lambda_j s_j \le \theta s_r j = 1, \ldots, r, \ldots, J \text{(c)}$$
$$\sum_{j=1}^{J} \lambda_j p_j \ge p_r j = 1, \ldots, r, \ldots, J \text{(d)}$$
$$\sum_{j=1}^{J} \lambda_j = 1 \text{(e)}$$
$$0 \le \theta \le 1$$

$$\vartheta^* = \min \vartheta$$
subject to
$$\sum_{k=1}^{K} \mu_k f_k \le \vartheta f_r k = 1, \ldots, r, \ldots, K$$
$$\sum_{k=1}^{K} \mu_k s'_k \le \vartheta s'_r k = 1, \ldots, r, \ldots, K$$
$$\sum_{k=1}^{K} \mu_k h_k \le \vartheta h_r k = 1, \ldots, r, \ldots, K$$
$$\sum_{k=1}^{K} \mu_k p_k \ge p_r k = 1, \ldots, r, \ldots, K$$
$$\sum_{k=1}^{K} \mu_k = 1$$
$$0 \le \vartheta \le 1$$

The initial model estimates the operational efficiency, where θ refers to the operational efficiency rating of the route being evaluated; J is the number of routes being compared; r is the route being evaluated; c_j is the amount of capacity used by route j; c_r is the amount of capacity used by the route being evaluated; s_j is the number of bus stops used by route j; s_r is the number of bus stops used by the route being evaluated; p_j is the number of unlinked passenger trips provided in route j; p_r is the number of unlinked passenger trips provided in the route being evaluated; and λ_j refers to the linear combinations of routes to compare to the route being evaluated. Therefore, the other routes with non-zero λ form the benchmark set for the route being evaluated.

The objective function (a) seeks efficiency by minimizing the operational efficiency of the route being evaluated. The objective function is subjected to four constraints. The first constraint (b) emphasizes that the weighted sum of the capacity amount of the other routes is less than or equal to the route being evaluated. Similarly, the second constraint (c) emphasizes that the weighted sum of the bus stops of the other routes is less than or equal to the route being evaluated. The third constraint (d) asserts that the weighted sum of the unlinked passenger trips of the other routes is greater than or equal to the route being evaluated. Finally, the fourth constraint is the convexity constraint (e), which ensures the variable returns to scale and implies that the route being evaluated is only compared to routes of roughly similar size.

The latter model estimates the service efficiency, where ϑ refers to the service efficiency rating of the route being evaluated; K is the number of routes being compared; r is the route being evaluated; f_k is the frequency of route k; f_r is the frequency of the route being evaluated; s'_k is the number of bus stops per km in route k; s'_r is the number of bus stops per km in the route being evaluated; h_k is the service hours in route k; h_r is the service hours in the route being evaluated; p_k is the number of unlinked passenger trips provided in route k; p_r is the number of unlinked passenger trips provided in the route being evaluated. μ_k refers to the linear combinations of routes to compare to the route being evaluated. Therefore, the other routes with non-zero μ form the benchmark set for the route being evaluated.

Please note that these models evaluate the operational and service efficiencies separately and may provide different efficiency scores for each bus transit route. For instance, a bus transit route could show high operational efficiency but service inefficiency, and vice versa. However, although operational and service efficiency models are constructed and evaluated independently, their input variables are closely interrelated (see Fig. 1). Accordingly, capacity and frequency are closely interrelated

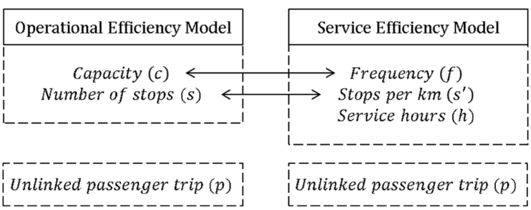

Fig. 1 Operational and service efficiency models

because the capacity is the product of the frequency and vehicle capacity that operates on this route. A similar interrelationship also exists between the number of stops and stops per km because the number of stops is a product of stops per km and route distance. These interrelationships demonstrate the linear interdependence among capacity–frequency and number of stops–stops per km. Please see the appendix for mathematical formulations of these interrelationships. Although these input variables appear in distinct DEA models, an increase or decrease in one of them will directly affect the other in a certain proportion.

Now, suppose that the operational efficiency of a route was estimated at 0.6 while its service efficiency was 1. This indicates that this route is fully service efficient but 40% operationally inefficient. Its operational inputs, including capacity and number of stops, should be decreased by 40% to be considered fully operationally efficient. This reduction in capacity and number of stops will automatically reduce the frequency and stops per km by 40%, respectively. However, since this route was considered fully service efficient, the service efficiency model suggests not decreasing the frequency and stops per km. In this case, the decision-makers face a dilemma, and the DEA results would not be applied to the practice.

To obtain applicable results, operational and service efficiency models should generate the same efficiency scores for each route because they would provide proportional changes for capacity–frequency and number of stops–stops per km. However, since service hours are not interrelated with any operational efficiency input, it should be considered independently and needs non-proportional changes. To address these issues, we propose a methodology based on non-radial DEA in the next section.

3 Non-radial DEA model

3.1 The need for non-radial methodology

To address the aforementioned problems, operational and service efficiency models should be evaluated in a single linear model that considers the mathematical relationships among the input variables of both models. In this single model, equiproportionality of interrelated inputs (capacity–frequency and number of stops–stops per km) can be satisfied by equating the operational and service efficiency scores of each route. Thus, each route receives the same operational and service efficiency scores, and it is possible to find the optimal efficiency scores for both models, which provide proportional changes for capacity–frequency and number of stops–stops per km.

However, the problem is not limited to the proportionality of interrelated input variables. Distinct DEA models may include some input variables that are not interrelated to another input of the latter model. For instance, in our case, service hours are not interrelated to any input variable of the operational efficiency model, which means that while capacity–frequency and number of stops–stops per km need to be considered together and require proportional changes (although they appear in distinct models), service hours should be considered independently and require a non-proportional change.

The original radial DEA model deals with the maximum proportional reduction in all inputs, and it is not able to provide individual efficiency scores for each input variable that appears in the same model. As a result, this approach entails radial (or proportional) changes for each input variable and does not allow measuring specific scores for independent variables. For instance, if the service efficiency score of a route is estimated as 0.8, the radial model suggests reducing all service inputs (frequency, stops per km, and service hours) by 20%.

In contrast, the non-radial model that was first introduced by Färe and Lovell (1978) puts aside the proportionality assumption of radial models and attaches individual efficiency scores to each input variable in the process (Athanassopoulos, 1996). Thus, non-radial models provide non-proportional changes for each input variable, providing an efficiency indicator for each of the variables in the process. This property of non-radial DEA makes it an appropriate method to provide proportional changes for interrelated inputs of operational and service efficiency models while providing non-proportional changes for independent input variables. Non-radial DEA offers other advantages as well. First, non-radial DEA has a higher discriminating power compared to radial DEA in evaluating the efficiencies of a set of decision-making units (Barros et al. 2012). Second, it is able to incorporate preference structures that reflect decision-makers' preferences by assigning different weights to different input/output variables (Zhu 1996). Third, non-radial DEA models do not allow the existence of non-zero input/output slacks and provide more accurate efficiency scores that reflect the performance of the units more accurately (Tone 2001).

3.2 Modeling the non-radial DEA

Two independent non-radial DEA models for operational and service efficiency are presented below. Please note that these models generate specific efficiency scores for each input variable. θ_c and θ_s are the efficiency scores of capacity and number of stops, respectively, and ϑ_f, $\vartheta_{s'}$ and ϑ_h refer to the efficiency scores of frequency, stops per km, and service hours, respectively. $\hat{\theta}$ refers to overall operational efficiency.$\hat{\theta}$ is the average of θ_c and θ_s and calculated as $[(\theta_c+\theta_s)/2]$. On the other side, $\hat{\vartheta}$ refers to overall service efficiency. $\hat{\vartheta}$ is the average of ϑ_f, $\vartheta_{s'}$ and ϑ_h and calculated as $\left[\left(\vartheta_f + \vartheta_{s'} + \vartheta_h\right)/3\right]$.

$$\hat{\theta}^{*} = \min\left(\frac{\theta_c+\theta_s}{2}\right)$$

subject to

$$\sum_{j=1}^{J}\lambda_j c_j = \theta_c c_r j = 1,\ldots,r,\ldots,J$$
$$\sum_{j=1}^{J}\lambda_j s_j = \theta_s s_r j = 1,\ldots,r,\ldots,J$$
$$\sum_{j=1}^{J}\lambda_j p_j \geq p_r j = 1,\ldots,r,\ldots,J$$
$$\sum_{j=1}^{J}\lambda_j = 1$$
$$0 \leq \hat{\theta} \leq 1$$
$$0 \leq \theta_c \leq 1$$
$$0 \leq \theta_s \leq 1$$

$$\hat{\vartheta}^{*} = \min\left(\frac{\vartheta_f+\vartheta_{s'}+\vartheta_h}{3}\right)$$

subject to

$$\sum_{k=1}^{K}\mu_k f_k = \vartheta_f f_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{k=1}^{K}\mu_k s'_k = \vartheta_{s'} s'_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{k=1}^{K}\mu_k h_k = \vartheta_h h_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{k=1}^{K}\mu_k p_k \geq p_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{k=1}^{K}\mu_k = 1$$
$$0 \leq \hat{\vartheta} \leq 1$$
$$0 \leq \vartheta_f \leq 1$$
$$0 \leq \vartheta_{s'} \leq 1$$
$$0 \leq \vartheta_h \leq 1$$

Nevertheless, these two models are constructed separately and are still far from considering the relationships among capacity–frequency and number of stops–stops per km. To assess these two distinct DEA models simultaneously, the basic non-radial models need to be aggregated into a single linear program, which can be presented as follows:

$$\emptyset = min\left(w_1\hat{\theta} + w_2\hat{\vartheta}\right) \quad (f_1)$$

subject to

$$\sum_{j=1}^{J}\lambda_j c_j = \theta_c c_r j = 1,\ldots,r,\ldots,J$$
$$\sum_{j=1}^{J}\lambda_j s_j = \theta_s s_r j = 1,\ldots,r,\ldots,J$$
$$\sum_{k=1}^{K}\mu_k f_k = \vartheta_f f_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{k=1}^{K}\mu_k s'_k = \vartheta_{s'} s'_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{k=1}^{K}\mu_k h_k = \vartheta_h h_r k = 1,\ldots,r,\ldots,K$$
$$\sum_{j=1}^{J}\lambda_j p_j \geq p_r j = 1,\ldots,r,\ldots,J$$
$$\theta_c = \vartheta_f \quad (f_2)$$
$$\theta_s = \vartheta_{s'} \quad (f_3)$$
$$0 \leq \emptyset \leq 1$$
$$0 \leq \theta_c \leq 1$$
$$0 \leq \theta_s \leq 1$$
$$0 \leq \vartheta_f \leq 1$$
$$0 \leq \vartheta_{s'} \leq 1$$
$$0 \leq \vartheta_h \leq 1$$
$$\lambda_j, \mu_k \geq 0$$

Simple additive weighting (SAW) is used to combine $\hat{\theta}$ and $\hat{\vartheta}$. SAW is one of the most popular, simplest, and widest used multi-objective decision-making method, and it is based on the weighted average of decision outcomes. Using the SAW method, f_1 indicates the new objective function, $\emptyset = \min\left(w_1\hat{\theta} + w_2\hat{\vartheta}\right)$, where w_1 is the priority assigned to $\hat{\theta}$ and w_2 is the priority assigned to $\hat{\vartheta}$. Finally, $\emptyset$ is the composite score and refers to the weighted average of $\hat{\theta}$ and $\hat{\vartheta}$. Therefore, $\emptyset$ ensures the simultaneous investigation of operational and service efficiency models.

Each objective is given a weight, and the sum of all weights must be equal to 1. This ensures the composite score is between 0 and 1 $\left(0 \leq \emptyset \leq 1\right)$. In this study, $\hat{\theta}$ and $\hat{\vartheta}$ are considered to have equal importance and both received the same weight $\left(w_1, w_2 = 0.5\right)$. This can easily be changed depending on the preference of the decision-makers. Similarly, each input variable $\left(\theta_c, \theta_s, \vartheta_f, \vartheta_{s'}, \vartheta_h\right)$ is considered to have equal importance and received identical weights of unity in this study. In practice, some input/output variables may be preferable than others for decision-makers. To incorporate value judgments, structures that reflect the decision-makers' preferences should be incorporated into the analysis. We refer to the paper by Zhu (1996) for a detailed discussion about specifying the preference weights in the non-radial DEA method.

In the second step, efficiency scores of the interrelated input variables should be equated to ensure the optimal efficiency scores, which may provide proportional changes. To ensure the proportional changes among $\theta_c - \vartheta_f$ and $\theta_s - \vartheta_{s'}$, they are equated to each other and added as new constraints to the model. f_2 equates to the efficiency scores of capacity and frequency $(\theta_c = \vartheta_f)$, and f_3 equates to the efficiency scores of the number of stops and stops per km $(\theta_s = \vartheta_{s'})$. Therefore, f_2 and f_3 indicate that the optimal efficiency scores should respect the proportionality between the interrelated input variables. This structure also ensures the independent evaluation of service hours (which is not interrelated to any operational efficiency input) and determines a measure-specific score.

4 Case study

We will demonstrate the non-radial DEA approach using data from the Sakarya City Metropolitan Municipality Transportation Bureau (SMMTB). SMMTB is a public transportation authority operating 30 routes in the Sakarya metropolitan area. For this study, each route is treated as a decision-making unit. To measure the performance of each route, we defined two major objectives for SMMTB: operational efficiency and service efficiency. Operational efficiency is defined as the optimization of physical resources (capacity and number of stops) for a given number of unlinked passenger trips. Service efficiency is defined as the optimization of the provided service level (frequency, stops per km, and service hours) for a given number of unlinked passenger trips. Data are from the year 2012, and approximately 6.6 million passenger trips were provided on these routes.

In this case, two different analyses were applied to the same dataset. First, we used the original radial DEA model that was presented in Sect. 2. This initial model evaluated the operational and service efficiency models independently and ignored the relationships between their input variables. This provided contradictory and inapplicable input targets for inefficient routes. In this step, we presented the results of the initial analysis and demonstrated the contradictory input targets.

In the second step, we applied the modified non-radial DEA model to the same dataset that was proposed in Sect. 3. This approach simultaneously investigates the operational and service efficiency models and considers mathematical relationships. Analysis results show that this approach defines the optimal efficiency scores for each route and overcomes contradictory input targets.

In these analyses, we utilized input orientation for both models because unlinked passenger trips do not display much variation, and SMMTB cannot control the number of passengers. Instead, as a public entity, resource utilization is more important for SMMTB. In addition, both models were analyzed under variable returns to scale to avoid the negative impacts of scale differences on the analysis results (Banker et al. 1984).

4.1 Model input and output measures

To measure the operational efficiency (θ) and service efficiency (ϑ) of SMMTB, the average unlinked passenger trips per day are used as the unique output measure. An unlinked passenger trip (p) refers to a trip on one transit vehicle regardless of the type of fare paid or transfer presented (APTA). It is a good measure of service efficiency because it represents the total number of passengers served (Chu et al. 1992; Karlaftis 2004; Lao and Liu 2009). At the same time, it is a good surrogate measure for total revenues and an ideal output for operational efficiency (Karlaftis 2003; Barnum et al. 2008; Lao and Liu 2009).

We selected two inputs to measure the operational efficiencies of transit routes: capacity and number of stops. Capacity (c) refers to the total passenger capacity of each route during service hours in a day, and it is calculated by multiplying vehicle capacities with frequency. For instance, if there is only one bus operating with a 66-passenger capacity for each trip with a total of 10 trips in a work day, the capacity of this route is calculated as 660.

SMMTB has three types of buses that have 66, 104, and 150 passenger capacities, which is different from other studies in the literature (Chang and Kao 1992;

Table 1 Statistical summary of operational inputs

	Capacity (c)	Number of stops (s)
Mean	2124.20	63.17
Median	1723	59
Standard deviation	1579	22
Minimum	528	35
Maximum	8070	133

Karlaftis 2003, 2004; Sanchez 2009). We decided to use "capacity" rather than "number of buses" as an input variable for operational efficiency. Since the bus capacities differ in this case, "number of buses" is an insufficient variable to reflect the real capacities of routes. While some routes operate with 150-passenger capacity buses, others operate with only 66-passenger capacity. Thus, considering the number of buses means that both low- and high-capacity buses are evaluated equally and the capacity differences are ignored. On the other hand, there are some routes that operate with only one bus. If these routes are considered inefficient, DEA results will suggest decreasing the number of buses to fewer than one. Since the number of buses is already at the minimum level, this suggestion is inapplicable.

The same problem also exists for "number of seats" and "bus capacity." Since public transport vehicles are also designed for standing passengers, the total capacity of the bus will be ignored if only the number of seats is considered. Similarly, the total capacity of the bus is not applicable because if a route is operating with a passenger capacity of 66, it is not possible to decrease this number if it is considered inefficient. Thus, in our study, using "capacity" instead of "number of buses," "number of seats" and "bus capacity" will provide more applicable results for decision-makers, who can adjust the capacity easily by decreasing either bus capacities or frequency.

Another input variable of operational efficiency is the number of stops (s), which refers to the total number of bus stops on a round-trip route. This input is used to balance the number of bus stops with the number of passengers transported in a day. All bus stops should be determined and signalized by SMMTB. The statistical summary of operational inputs is listed in Table 1.

Three inputs were defined for the service efficiency model: frequency, stops per km, and service hours. Frequency (f) is related to scheduling while service hours (h) are related to the length of service operation in a day. Stops per km (s') is obtained by dividing the length of the round-trip route by the number of bus stops and refers to the accessibility of the route. The statistical summary of service inputs is shown in Table 2.

4.2 Initial analysis

In this step, two distinct DEA models were constructed for operational and service efficiency, and they were evaluated independently using radial DEA models as presented in Sect. 2. Table 3 presents the operational and service efficiency scores of

Table 2 Statistical summary of service inputs

	Frequency (f)	Stops per km (s')	Service hours (h)
Mean	25.77	2.39	941.33
Median	24	2	993
Standard deviation	13.13	0.74	131.69
Minimum	8	1	585
Maximum	63	5	1230

Table 3 Efficiency scores and targets with radial DEA

No.	Route no.	θ	Targets		ϑ	Targets		
			c	s		f	s'	h
1.	#2	0.844	1837.755	40.501	0.805	16.930	1.748	748.57
2.	#3	0.764	2066.485	47.382	0.733	19.057	2.020	769.59
3.	#4	0.753	1889.233	69.302	0.769	29.212	2.029	784.12
4.	#5	0.888	1054.350	39.050	0.808	11.905	1.560	812.21
5.	#6	1.000	990.000	41.000	0.842	12.635	1.748	694.91
6.	#7	1.000	1518.000	57.000	0.849	19.521	2.086	738.39
7.	#9	0.799	949.109	42.342	0.601	10.814	1.939	594.76
8.	#10	0.809	801.368	57.472	0.713	10.695	2.033	613.15
9.	#12	1.000	594.000	54.000	1.000	9.000	1.333	935.00
10.	#14	1.000	1188.000	35.000	0.800	13.247	1.725	691.58
11.	#15	1.000	5826.000	57.000	1.000	50.000	2.533	1.015.00
12.	#17	0.664	1051.770	46.480	0.730	16.065	1.880	623.81
13.	#18	0.749	1186.534	38.952	0.746	13.030	1.651	760.96
14.	#19	0.763	1562.091	37.411	0.709	17.764	1.829	659.49
15.	#19/K	0.807	2266.467	51.657	0.784	21.174	1.984	815.60
16.	#20	0.923	974.259	41.517	0.699	11.189	1.868	618.87
17.	#20/A	0.538	959.538	42.000	0.701	8.000	2.000	585.00
18.	#21/A	0.513	1333.186	65.121	0.750	18.752	1.732	795.10
19.	#21/B	0.499	1244.897	66.335	0.732	17.573	1.771	765.15
20.	#21/H	0.908	838.798	54.467	1.000	14.000	1.840	600.00
21.	#22/A	0.708	1447.613	39.622	0.874	12.606	1.515	886.93
22.	#22/B	0.718	1469.443	42.374	0.914	14.029	1.669	781.12
23.	#23	0.826	763.469	52.055	0.771	10.793	1.592	790.19
24.	#24	0.749	2725.764	53.916	0.899	25.945	1.685	934.55
25.	#24/H	1.000	528.000	69.000	1.000	8.000	2.000	585.00
26.	#24/K	1.000	1664.000	75.000	1.000	16.000	1.531	1.005.00
27.	#26	1.000	8070.000	41.000	1.000	63.000	1.981	1.015.00
28.	#27	0.891	882.000	44.545	0.807	12.103	1.739	677.77
29.	#28	0.604	1890.517	35.612	0.614	13.987	1.692	755.07
30.	#29	0.660	1476.147	38.915	0.811	13.364	1.619	807.33
Mean	0.813				0.815			

each route. Accordingly, eight routes are considered operationally efficient (#6, #7, #12, #14, #15, #24/H, #24/K, and #26), while only six routes are considered service efficient (#12, #15, #21/H, #24/H, #24/K, and #26). The average service efficiency score (0.815) is slightly higher than the average operational efficiency score (0.813). Table 3 also presents the input targets for inefficient routes.

Since operational and service efficiency models were analyzed independently, each route received different operational and service efficiency scores. This structure is useful for evaluating the various performance aspects of bus transit routes

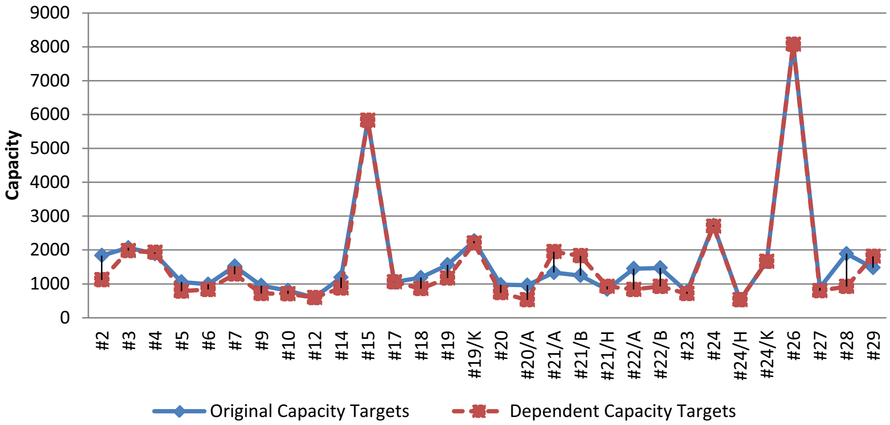

Fig. 2 Contradictions in capacity

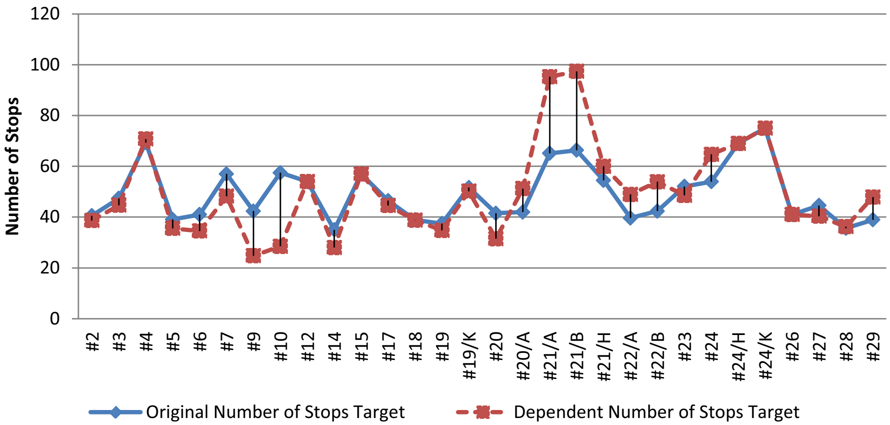

Fig. 3 Contradictions in number of stops

and defining areas that need improvement. For instance, analysis results show that #6, #7, and #14 are operationally efficient but service inefficient. This demonstrates a need to adjust the provided service levels on these routes. In addition, #21/H is service efficient but operationally inefficient, so the physical resources of this route should be adjusted.

However, different operational and service efficiency scores suggest different proportions of input reductions for inefficient routes in terms of capacity–frequency and number of stops–stops per km. Since these input variables are closely interrelated, analysis results may provide inapplicable input targets.

Take #6 as an example: #6 is considered fully operationally efficient. Based on this, it does not need to decrease its capacity or number of stops. However, #6 is service inefficient (0.842) and does need to decrease its service inputs (frequency, stops

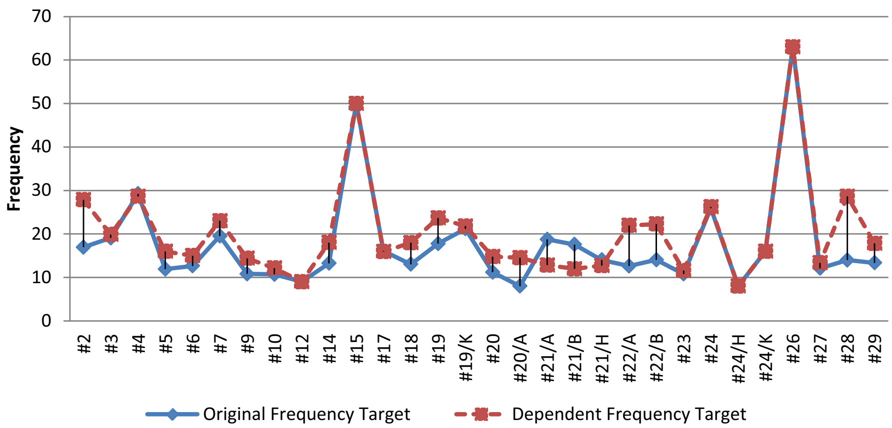

Fig. 4 Contradictions in frequency

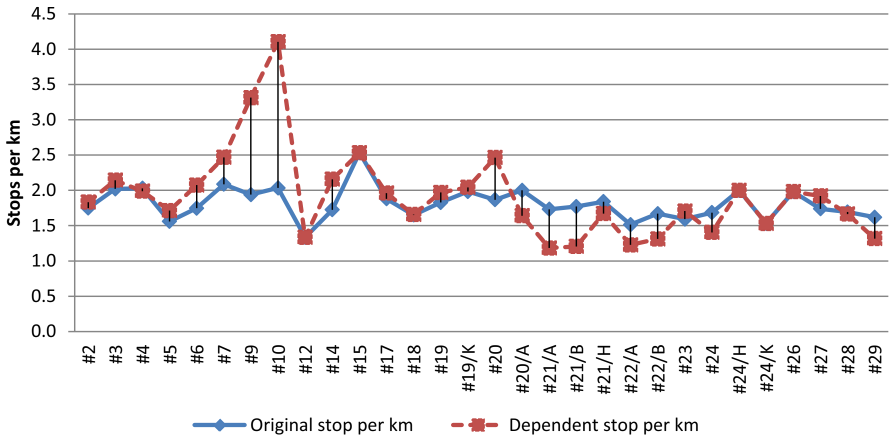

Fig. 5 Contradictions in stops per km

per km, and service hours) by 15.8% to become service efficient. This reduction in frequency and stops per km entails a 15.8% reduction in operational inputs (capacity and number of stops) because of the bilateral relationships. In such a situation, decision-makers will face two contradictory results. While the operational efficiency model proposes remaining at the same input level, the service efficiency model suggests a decrease of 15.8%.

This problem is not limited to #6 and can be observed in other routes that have different operational and service efficiency scores. Figures 2, 3, 4, and 5 demonstrate the contradictions in operational and service targets. Continuous lines represent the original input targets, which were obtained with DEA. Dotted lines, on the other hand, represent the dependent input targets, which were calculated indirectly from

Table 4 Efficiency scores and targets with non-radial DEA

No.	Route no.	∅	$\hat{\theta}$	θ_c	θ_s	Targets		$\hat{\vartheta}$	ϑ_f	$\vartheta_{s'}$	ϑ_h	Targets		
						c	***s***					***f***	***s′***	***h***
1.	#2	0.826	0.840	0.681	1.000	1482.805	48.000	0.811	0.681	1.000	0.751	22.467	2.172	698.515
2.	#3	0.755	0.765	0.780	0.750	2109.342	46.477	0.745	0.780	0.750	0.707	20.282	2.103	742.087
3.	#4	0.748	0.740	0.666	0.815	1670.185	74.973	0.755	0.666	0.815	0.783	25.306	2.151	798.638
4.	#5	0.834	0.880	0.790	0.970	938.404	42.694	0.787	0.790	0.970	0.601	14.218	1.873	604.297
5.	#6	0.960	1.000	1.000	1.000	990.000	41.000	0.920	1.000	1.000	0.759	15.000	2.075	626.511
6.	#7	0.975	1.000	1.000	1.000	1518.000	57.000	0.950	1.000	1.000	0.850	23.000	2.468	739.853
7.	#9	0.779	0.787	0.708	0.865	841.661	45.870	0.771	0.708	0.865	0.740	12.752	3.590	732.539
8.	#10	0.779	0.773	0.941	0.604	931.954	42.906	0.786	0.941	0.604	0.811	14.121	3.065	697.403
9.	#12	1.000	1.000	1.000	1.000	594.000	54.000	1.000	1.000	1.000	1.000	9.000	1.333	935.000
10.	#14	0.960	1.000	1.000	1.000	1188.000	35.000	0.920	1.000	1.000	0.760	18.000	2.158	657.689
11.	#15	1.000	1.000	1.000	1.000	5826.000	57.000	1.000	1.000	1.000	1.000	50.000	2.533	1015.000
12.	#17	0.669	0.657	0.679	0.634	1075.632	44.405	0.681	0.679	0.634	0.730	16.297	1.874	624.414
13.	#18	0.720	0.742	0.653	0.830	1034.798	43.183	0.698	0.653	0.830	0.611	15.679	1.838	623.698
14.	#19	0.752	0.756	0.638	0.874	1305.478	42.832	0.748	0.638	0.874	0.731	19.780	2.254	679.907
15.	#19/K	0.801	0.807	0.804	0.810	2258.207	51.832	0.794	0.804	0.810	0.769	21.714	2.049	799.684
16.	#20	0.884	0.911	0.822	1.000	868.154	45.000	0.857	0.822	1.000	0.748	13.154	2.671	661.727
17.	#20/A	0.550	0.517	0.367	0.667	653.336	52.052	0.583	0.367	0.667	0.714	9.899	2.033	596.369
18.	#21/A	0.649	0.594	0.552	0.636	1435.652	80.726	0.705	0.552	0.636	0.927	13.804	1.468	982.522
19.	#21/B	0.638	0.579	0.556	0.602	1387.081	80.056	0.698	0.556	0.602	0.936	13.337	1.456	978.373
20.	#21/H	1.000	1.000	1.000	1.000	924.000	60.000	1.000	1.000	1.000	1.000	14.000	1.840	600.000
21.	#22/A	0.739	0.704	0.551	0.856	1128.000	47.961	0.774	0.551	0.856	0.913	17.091	1.485	926.734
22.	#22/B	0.767	0.721	0.570	0.871	1165.908	51.405	0.814	0.570	0.871	1.000	17.665	1.592	855.000
23.	#23	0.799	0.843	0.751	0.936	693.559	58.951	0.755	0.751	0.936	0.578	10.508	1.933	592.044
24.	#24	0.788	0.759	0.651	0.867	2370.760	62.412	0.817	0.651	0.867	0.933	22.796	1.625	970.051

Table 4 (continued)

No.	Route no.	∅	$\hat{\theta}$	θ_c	θ_s	Targets		$\hat{\vartheta}$	ϑ_f	$\vartheta_{s'}$	ϑ_h	Targets		
						c	***s***					***f***	***s′***	***h***
25.	#24/H	1.000	1.000	1.000	1.000	528.000	69.000	1.000	1.000	1.000	1.000	8.000	2.000	585.000
26.	#24/K	1.000	1.000	1.000	1.000	1664.000	75.000	1.000	1.000	1.000	1.000	16.000	1.531	1005.000
27.	#26	1.000	1.000	1.000	1.000	8070.000	41.000	1.000	1.000	1.000	1.000	63.000	1.981	1015.000
28.	#27	0.847	0.872	0.784	0.961	775.876	48.029	0.823	0.784	0.961	0.724	11.756	2.070	607.833
29.	#28	0.533	0.531	0.385	0.676	1473.837	39.897	0.535	0.385	0.676	0.545	22.331	1.864	670.557
30.	#29	0.683	0.657	0.503	0.812	1125.714	47.908	0.709	0.503	0.812	0.812	13.581	1.621	807.940
mean		0.814	0.814	0.761	0.868			0.814	0.761	0.868	0.814			

the interrelated input variables. Note that the lines overlap each other if the route being evaluated received the same operational and service efficiency scores. For instance, #12, #15, #24/H, #24/K, and #26 are both operational and service efficient, and there is no contradiction in their input targets. However, the contradictions are as large as the difference of each route's operational and service efficiency scores.

4.3 Demonstration of the non-radial DEA

To provide applicable input targets for inefficient routes, the non-radial DEA methodology that was presented in Sect. 3 is used for the same dataset. The results are listed in Table 4 and illustrate that the non-radial DEA methodology provided the same optimal efficiency scores for capacity–frequency (θ_c and ϑ_f) and number of stops–stops per km (θ_s and $\vartheta_{s'}$), which suggests an input reduction at a certain proportion. Additionally, the model evaluated the service hours (ϑ_n) independently and provided non-proportional changes. Thus, the proposed methodology could provide applicable input targets for each route.

Let us examine the applicability of input targets for route #2. In the current situation, the frequency and vehicle capacity of #2 is 33 and 66, respectively. Since the capacity is the product of frequency and vehicle capacity operating on this route, the total capacity of #2 is calculated as 2178 ($33 \times 66 = 2178$). The non-radial DEA results show that the capacity efficiency of #2 is 0.681. If the decision-makers were to implement the operational efficiency results for #2, they would decrease the capacity by 31.9%. Thus, the capacity will decrease from 2178 to 1482.822. In this condition, since the vehicle capacity operating in #2 will not change, the frequency should be reduced by 31.9% and decreased from 33 to 22.467 ($1482.822/66 = 22.467$).

On the other hand, if the decision-makers were to implement the service efficiency results for #2, they would obtain the same results. As can be seen in Table 4, the frequency target for #2 is already 22.467. If the frequency is reduced to 22.467, the capacity will automatically be 1482.822 ($22.467 \times 66 = 1482.822$). Since both operational and service efficiency models generated the same capacity and frequency targets, the contradiction was eliminated.

The contradiction between number of stops and stops per km was also eliminated. Route #2 is considered fully efficient in terms of the number of stops, so decision-makers should keep the number of stops at 48. Accordingly, the stops per km must remain constant. On the other hand, #2 is considered fully efficient in terms of stops per km, so decision-makers should not reduce the stops per km. As a result, both models generated the same number of stops and stops per km. Please note that #2 is just an example among 30 routes, and other examples can be observed from other routes.

Based on analysis results, several managerial implications can be suggested for SMMTB. Nine routes (#6, #7, #12, #14, #15, #21/H, #24/H, #24/K, and #26) are considered operationally efficient, but only six routes (#12, #15, #21/H, #24/H, #24/K, and #26) are considered service efficient. Three operationally efficient routes (#6, #7, and #14) are considered service inefficient because of their surplus in service hours. Therefore, only six routes received a score of $\varnothing = 1$, which indicates full operational and service efficiency. In total, the analysis results suggest a reduction in the capacity and frequency by 23.9%, the number of stops and stops per km by 13.2%, and service hours by 18.6%. In addition, results demonstrate that bus transit routes received higher average efficiency scores in terms of θ_s and $\vartheta_{s'}$ than θ_c and ϑ_f (0.868 and 0.761, respectively). Thus, decision-makers should give more consideration to improve the capacity and frequency performance rather than bus stops because analysis results show that surpluses in capacity and frequency are the most important sources of operational and service inefficiency.

5 Conclusion

Multidimensional performance analysis, which allows units (such as transit operators and transit routes) to be evaluated from different perspectives, is an important issue in public transportation literature. Indeed, many researchers consider the operational and service efficiencies of public transportation systems together. To do this, researchers tend to develop two distinct DEA models for each performance dimension and evaluate them independently. Despite its advantages, this methodology ignores the mathematical interrelationships among the input variables of operational and service efficiency models, which have the potential to affect each other and result in contradictory input targets.

In this study, we proposed a non-radial DEA methodology to simultaneously investigate the operational and service efficiencies of bus transit routes, which takes the mathematical interrelationships between operational and service efficiency models into consideration. To do this, the basic non-radial DEA model is modified by adding several additional constraints. The proposed approach is then applied to the transit routes of a local bus authority. It was demonstrated that the proposed approach provides optimal efficiency scores and proportional changes for interrelated input variables of the distinct DEA models. It also evaluates independent input variables separately and provides non-proportional changes for them.

Consequently, it is proven that this methodology can be used effectively to investigate multiple DEA models simultaneously when any interdependency exists among the input/output variables. The main benefit of the proposed approach to the bus transit systems is to provide a more comprehensive performance analysis that allows monitoring both operational and service efficiencies. A secondary benefit is that the proposed methodology optimizes the physical and service resources for each bus transit route simultaneously and generates non-contradictory and applicable input targets for inefficient routes. It is also important to note that although the proposed methodology was demonstrated in the public transportation industry in this research, its application may not be limited to public transportation and can be applied to other industries where there is a need for simultaneous investigation of multiple DEA models.

Acknowledgements The authors would like to thank the two anonymous referees for their useful comments and suggestions.

Appendix

Capacity is the product of frequency and vehicle capacity that operates on this route. The relationship between capacity and frequency can be formulated as follows:

$$c_r = f_r \times c'_r \tag{1}$$

where c_r amount of capacity used by the route being evaluated; f_r frequency of the route being evaluated; c'_r vehicle capacity of the route being evaluated.

Frequency can also be calculated from the following formulation:

$$f_r = \frac{c_r}{c'_r} \tag{2}$$

The number of stops is a product of stops per km and route distance, and it can be formulated as shown below:

$$s_r = s'_r \times d_r \tag{3}$$

where s_r number of bus stops used by the route being evaluated; s'_r number of bus stops per km in the route being evaluated; d_r total round trip route distance of the route being evaluated.

To calculate the stops per km of each route, the formulation should be modified as below:

$$s'_r = \frac{s_r}{d_r} \tag{4}$$

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