Limit osilasyonlu hidrolik servo pozisyonlama sistemleri üzerine bazı teorik düşünceler

Some theoretical remarks on limit-cycling hydraulic positional servo systems

Doç. Dr. Aybars ÇAKIR [1]

Tasvir fonksiyonu analiz metodunun kullanılması ile hidrolik, lineer olmayan, pozisyonlama servo mekanizmalarının, hareketsiz ve sonlu giriş hız miktarlarında, dinamik karakteristikleri üzerine kızak yollarında mevcut kuru (Coulomb) sürtünme ve sıfır boşluk nonlineeritelerinin tesirlerinin gösterilmesi.

Representation of the influences of the dry (Coulomb) friction (existing on the guideways) and the zero - lap nonlinearities on the dynamic characteristics (involving limit cycling about stand - still and finite input velocity rates) of the hydraulic non - linear positioning servomechanisms by using describing function analysing method.

1. Introduction

The servomechanism considered is an integral positional hydraulic servo system, that is, in practice, a hydrocopying device, comprising a zero - lap valve.

The limit cycling behaviour of the system has been analysed not only at the state of absolute rest, but also at the oscillatory state

(1) Doç. Dr. Müh. İ.T.Ü. Makina Fakültesi Makina Elemanları Kürsüsü (Technical University of Istanbul Faculty of Mechanical Engineering, Machine Design Division)

about rest and at finite medium input - output velocities, when non - linear Coulomb dry friction at the guideways is essential.

Guideway dry friction is one of the most important non - linearities of such a system with non - linear valve characteristics ; the friction has been considered as a pure Coulomb friction.

The quasi - linear analysis of the hydraulic servomechanism with non - linear characteristics, has been carried out using dual - input describing functions methods applied to the non - linear elements to replace the system non - linearities by their quasi - linear approximators.

Only the elasticity of the oil columns in the actuator and the inertia of the movable parts of the system have been allowed for in the analysis.

The results are here presented.

2. System theoretical analysis

The hydraulic servomechanism, which is here analysed, can be presented by the functional block diagram of Fig. 1. It should be noted that a positive displacement in x, the input, results in positive displacement of y, the output, ie. in the same direction.

The equations for the different links of the system are as follows.

The flow toward the hydraulic motor is a function of the valve aperture and the load differential pressure :

$$\frac{Q}{A} = \frac{V_k}{2}\,\varepsilon \left| \begin{array}{c|c} \left(1 - \dfrac{p_L}{2p_s}\right) & \varepsilon > 0 \\ \left(1 + \dfrac{p_L}{2p_s}\right) & \varepsilon < 0 \end{array} \right. \tag{1}$$

where $\varepsilon = x - y$; p_s supply pressure ; V_k velocity gain.

For the values of $\varepsilon > 0$ and $\varepsilon < 0$ equation (1) can be rewriten as :

$$\frac{Q}{A} = \frac{V_k}{2}\,\varepsilon - \frac{V_k\, p_L}{4\, p_s} \,|\, \varepsilon \,| \tag{2}$$

The output position of the hydraulic motor can be expressed in terms of the flow toward the hydraulic motor and the load differential pressure :

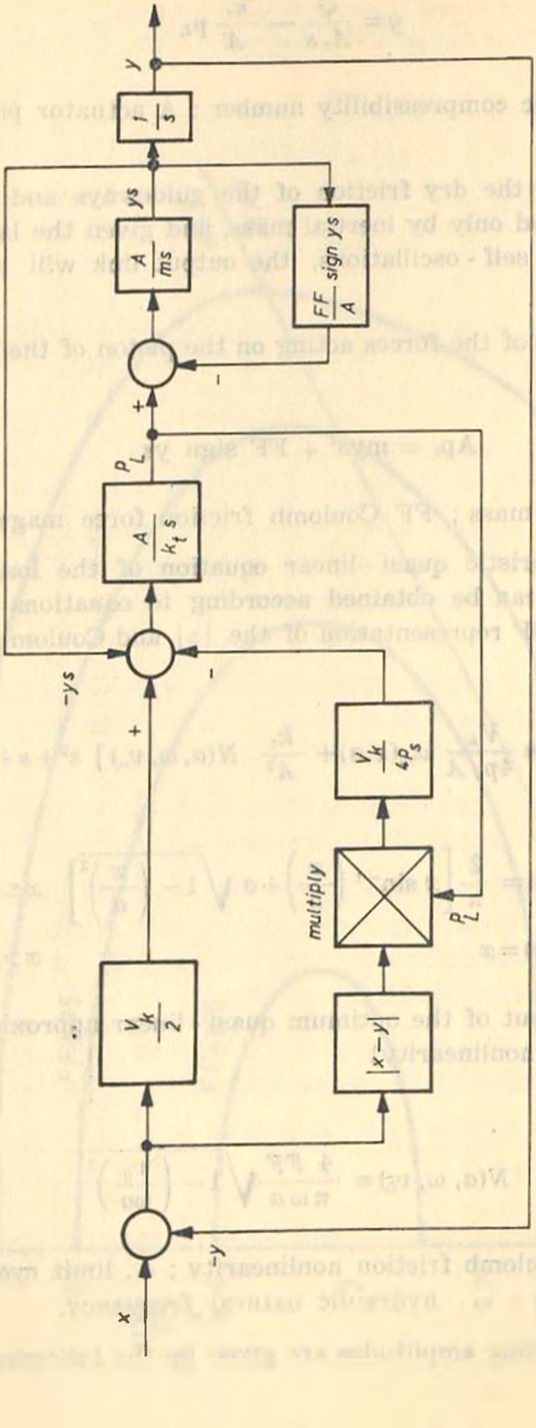

Fig. 1.

$$y = \frac{Q}{A.s} - \frac{k_t}{A} p_l \qquad (3)$$

where k_t hydraulic compressibility number ; A actuator piston effective area.

Allowing for the dry friction of the guideways and the hydraulic motor being loaded only by inertial mass, and given the large inertia of the load, during self - oscillations, the output link will not stop when $x=0$.

The equation of the forces acting on the piston of the hydraulic motor is :

$$Ap_l = mys^2 + FF \text{ sign } ys$$

where m inertia mass ; FF Coulomb friction force magnitude.

The characteristic quasi - linear equation of the loaded hydraulic servomechanism can be obtained according to equations (2), (3) and (4) by using DIDF representation of the $|\varepsilon|$ and Coulomb friction nonlinearities :

$$m \frac{k_t}{A^2} s^3 + \left[m \frac{V_k}{4p_s A} x_m(x,a) + \frac{k_t}{A^2} N(a, \omega, v_o) \right] s^2 + s + \frac{V_t}{2} = 0 \qquad (5)$$

where

$$x_m(x,a) = \frac{2}{\pi}\left[x \sin^{-1}\left(\frac{x}{a}\right) + a\sqrt{1 - \left(\frac{x}{a}\right)^2} \right] \qquad x \leq a$$

$$x_m(x,a) = x \qquad x > a$$

that is mean output of the optimum quasi - linear approximator. (DIDF signal for the $|\varepsilon|$ nonlinearity)

And

$$N(a, \omega, v_0) = \frac{4\, FF}{\pi \omega a}\sqrt{1 - \left(\frac{v_0}{\omega a}\right)^2}$$

DIDF for the Coulomb friction nonlinearity ; a, limit cycle amplitude ; v_o, input velocity ; ω, hydraulic natural frequency.

The limit cycling amplitudes are given by the following equations :

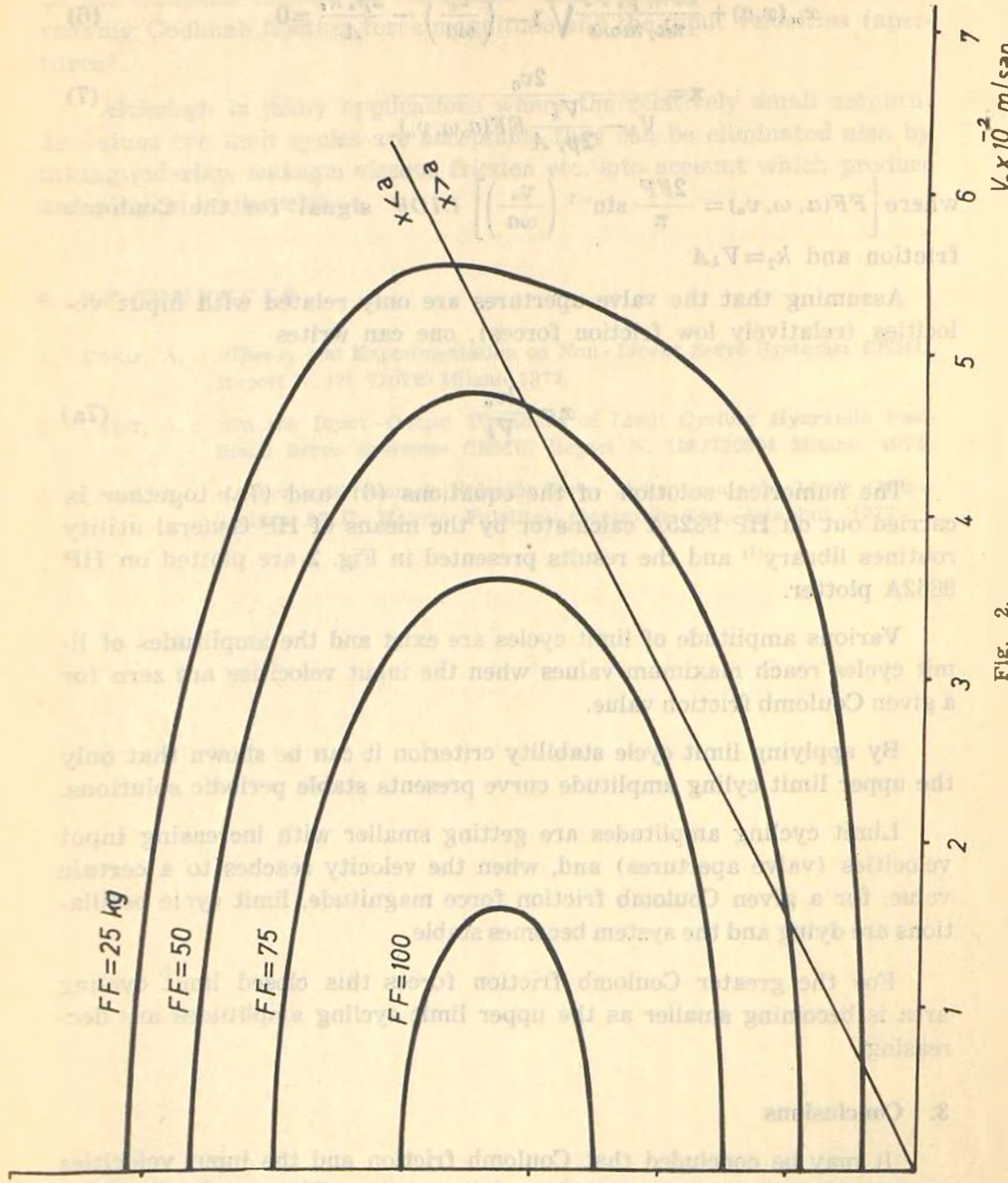

Fig. 2.

$$x_m(x,a) + \frac{16\,k_t\,p_s\,FF}{\pi k_2\,m\omega a}\sqrt{1-\left(\frac{v_0}{\omega a}\right)^2} - \frac{2p_s\,k_t}{A} = 0 \tag{6}$$

$$x = \frac{2v_0}{V_k - \dfrac{V_t}{2p_s\,A}\,FF(a,\omega,v_o)} \tag{7}$$

where $\left[FF(a,\omega,v_o) = \frac{2FF}{\pi}\sin^{-1}\left(\frac{v_o}{\omega a}\right)\right]$ *DIDF* signal for the Coulomb friction and $k_2 = V_k A$

Assuming that the valve apertures are only related with input velocities (relatively low friction forces), one can writes

$$x = \frac{2v_o}{V_k} \tag{7a}$$

The numerical solution of the equations (6) and (7a) together is carried out on HP 9825A calculator by the means of HP General utility routines library[1] and the results presented in Fig. 2 are plotted on HP 9862A plotter.

Various amplitude of limit cycles are exist and the amplitudes of limit cycles reach maximum values when the input velocities are zero for a given Coulomb friction value.

By applying limit cycle stability criterion it can be shown that only the upper limit cyling amplitude curve presents stable periodic solutions.

Limit cycling amplitudes are getting smaller with increasing input velocities (valve apertures) and, when the velocity reaches to a certain value, for a given Coulomb friction force magnitude, limit cycle oscillations are dying and the system becomes stable.

For the greater Coulomb friction forces this closed limit cycling area is becoming smaller as the upper limit cycling amplitudes are decreasing.

3. Conclusions

It may be concluded that Coulomb friction and the input velocities (apertures) increase the stability of the system. The stabilising effect

(1) Iterative root finder for user - defined function, file 28, 29. HP General utility routines part No. 09825 - 10001.

of the Coulomb friction on the system becomes more effective with increasing Coulomb friction force magnitude and the input velocities (apertures).

Although in many applications where the relatively small amplitude values for limit cycles are acceptable they can be eliminated also by takingunderlap, leakage, viscous friction etc. into account which produce damping on limit cycles.

4. REFERENCES

1. Çakır, A. : «Theory and Experimentation on Non - Linear Servo Systems» CEMU, Report N. 123/720725 Milano, 1972.
2. Çakır, A. : «On the Input - Output Dynamics of Limit Cycling Hydraulic Positional Servo Systems» CEMU, Report N. 138/730504 Milano, 1973.
3. Çakır, A. : «Coulomb Sürtünmeli Hidrolik Servo Mekanizmalarda Limit Osilasyonlar» İ.T.Ü. Makina Fakültesi Doçentlik Tezi, İstanbul, 1973.

EDITORIAL POLICY and SCOPE

The aim of the Bulletin is to allow rapid dissemination of interesting results in the field of Engineering and Science of the Staff of The State Academy of Engineering and Architecture of Sakarya.

The Executive Editor has authorized to publish the papers of the authors who do not belong to the Academy.

GUIDE FOR AUTHORS

Bulletin of The School of Engineering and Architecture of Sakarya is published with issues appearing in July, October, January and April. The Executive Editor has authorized to publish extra issues.

Papers for publication should be submitted with two copies to Editorial Secretary of Bulletin of The State Academy of Engineering and Architecture of Sakarya, Adapazarı/TURKEY.

Papers should be written in English, French and German and contain an abstract of about 150 words.

Further details are included in the booklet «Information for Authors and Manuscript Preparation Requirements» avaible from Editorial Secratary of Bulletin

MATBAA TEKNİSYENLERİ KOLL. ŞTİ. — İSTANBUL 1979