Yozlaşma Durumu için Lineer Programlamanın Simpleks Metodunda bir Direkt Yaklaşma

A Direct Approach in the Simplex Method of Linear Programming of Degeneracy Case

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Lineer programlama problemi, tahdit şartları ve gaye fonksiyonu karakteristikleri esas alınarak bir kaç tipe ayrılmış, ve her tipe uygulanabilen simpleks metotda bir direkt yaklaşım gösterilmiş bu yaklaşım Amundson'dan alınan bir örnekle kontrol edilmiştir.

The classifying the linear programming into the several thypes based on the characteristics of the objective function and the constraints, and a direct approach in the simplex method which can be applicable for the all types has been presented, and examined with the illustration which is carried out by Amundson.

Introduction

The linear programming has been known as one way of the solution methods for the optimization problems, and has great power and appticability for the many special problems in chemical industry.

Generally the simplex method of linear programming has degeneracy trouble with complicated calculation procedures, however, the presented method by mean of the logic way does not involve such as troubles.

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Before commencing with the main argument, based on the constraints and the objective function type, the LP problems used in this paper are classified by the following items.

A) The standard from

Choose the quantities

$$x_j \geqslant 0 \qquad (j=i, \ldots\ldots n)$$

to maximize

$$\sum_{j=1}^{n} c_j x_j$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (i=1, \ldots. m)$$

B) The nonstandard from

a) Type I

Chose the quantities

$$x_j \geqslant 0 \qquad (j=1, \ldots\ldots n)$$

to maximize

$$\sum_{j=1}^{n} c_j x_j$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad (i=1, \ldots. m)$$

b) Type II

Choose the quantities

$$x_j \geqslant 0 \qquad (j=1, \ldots\ldots n)$$

to minimize

$$\sum_{j=1}^{n} c_j x_j$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad (i=1, \ldots. m)$$

c) Type III

Choose the quantities

$$x_j \geqslant 0 \qquad (j=1, \ldots\ldots n)$$

to minimize

$$\sum_{j=1}^{n} c_j x_j$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_j$$

d) Type IV

Choose the quantities

$$x_j \geqslant 0 \qquad (j=1, \ldots\ldots n)$$

to maximize

$$\sum_{j=1}^{n} c_j x_j$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} x_i \lesseqqgtr b_i \quad (i=1, \ldots. m)$$

e) Type V

Choose the quantities

$$x_j \geqslant 0$$

to minimize

$$\sum_{j=1}^{n} c_j x_j$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} x_j \lesseqgtr b_i \quad (i=1,....m)$$

The policy of the logic way for in each classification mentioned above are shown in Table 1.

Table 1 Entering Policy of the LP problem

	Nonstandard form of LP (B)				
Standard form of LP (A)	a) Type	b) Tyae II	c) Type III	d) Type IV	e) Type V
Lowest value of the key capacity factor	higest value of the key capacity factor	higest value the key capacity factor	lowest value of the key capacity factor	key capacity factor must be on valid area for the present capacity	
biggest Δ_j	biggest Δ_j	smallest Δ_j	smallest Δ_j	biggest Δ_j	smallest Δ_j

Then, the authors called all $b_i (i=1,......m)$ a capacity or power to use for the optimum of the x's combinations. Each feasible solution is a fact or way which includes a part of the capasities termed by $\sum_{j=1}^{n} a_{ij}$ $(j=1,......n)$. All the remaining capacities are formed a slack x's combination by slack variables.

Now, let us use all capacities for the most profitable decision, and also let us consider it at the begining of each step. Then the steps for the solution way are shown in blow:

1 — Convert the inequalities to equalities with nonnegative slack variables which form feasible solution.

2 — Determine the first feasible solution. The variables which form this feasible solution are calles the basic set.

3 — Choose the entering for the problem by accordance of Table 1. The entering decision must be in accord to properities of the problem. This is the first speciality of logic way in this method.

4 — The feasible solution is now tested for optimality. This is the second speciality, and the details will be shown in later. Each variable which is not part of the feasible solution is evaluated by computing.

$$\Delta_j = c_j - \sum_{j=1}^{n} a_{ij} c_i$$

If one or more of the Δ_j are positive, the feasible solution is non-optimal for standard form, type I, and type IV. If $\Delta_j \leqslant 0$ for all j's solution has been found, an the simplex method terminates. If one or more of Δ_j are negative, the feasible solution is nonoptimal for type II, III and V. If $\Delta_j \geqslant 0$ for all j's, the solution has been found, and the simplex method terminates.

5 — Determine new amount of the b_i and a_{ij} according to the properties of the problem's in the step IV.

6 — Return to step III, and continue untile the solution tests for an optimum solution.

A Numerical Example

The following numerical example is carried out from the problem by Amundson[1]. This problem is choosen as a degeneracy problem and also given its solution with special algoritm. Now it will be solve by logic way as a ordinary problem

$$\text{Max } (x_1 - x_2)$$

$$2x_1 - x_2 \leqslant 4$$

$$x_1 - 2x_2 \leqslant 2$$

$$x_1 + x_2 \leqslant 5$$

1 — Converting inequalities to equaltities with z_1, z_2 and z_3 slack variables.

$$2x_1 - x_2 + z_1 = 4$$

$$x_1 - 2x_2 + z_2 = 2$$

$$x_1 - x_2 + z_3 = 5$$

2 — Thus, Tableau I gives first feasible solution.

Tableau I

c_i	z_i	x_1	x_2	z_1	z_2	z_3	capacity
0	z_1	2	—1	1	0	0	4
0	z_2	1	—2	0	1	0	2
0	z_3	1	1	0	0	1	5
	c_j	1	—1	0	0	0	
	$\sum_{j=1}^{n} a_{ij} c_i$	0	0	0	0	0	
	Δ_j	1	—1	0	0	0	
		↑					

3 — Choose the entering policy of problem as shown mark ↑ in Tableau I.

4 — In this problem all Δ_j must be negative or zero. But there is one positive Δ_j in the tableau I. Because of this feasible solution is not optimum solution.

5 — Based on the steps 4) and 5), Tableau II must be formed choosing the key capacity fastor as

$$4/2=2, \quad 2/1=2, \quad 5/1=5$$

Consequently, as mentioned in table 1, 2 is the key capacity factor, choosing the new capacity (b_i) again

$$2, \quad 2-2\times1=0, \quad 5-2\times1=3$$

Now a_{ij} are as follows

$$x_1=2z_1+z_2+z_3 \rightarrow z_1=(1/2)x_1-(1/2)z_2-(1/2)z_3$$

$$x_2=-z_1-2z_2+z_3$$

$$=-(1/2)x_1+(1/2)z_2+(1/2)z_3-2z_2+z_3$$

$$=-(1/2)x_1-(3/2)z_2+(3/2)z_3$$

Tableau II given optimum solution. Because there is no positive Δ_j in it.

Tableau II

c_i	z_i	x_1	x_2	z_1	z_2	z_3	Capacity
1	x_1	1	−1/2	1/2	0	0	2
0	z_2	0	−3/2	−1/2	1	0	0
0	z_3	0	3/2	−1/2	0	1	3
	c_j	1	−1	0	0	0	
	$\sum_{j=1}^{n} a_{ij}c_i$	1	−1/2	1/2	0	0	
	Δ_j	0	−1/2	−1/2	0	0	

Results are : $x_1=2$ $x_2=0$

Of course, the obtained results agree with those of by Amundson[1] as mentioned above, this method does not need any resolution.

Fig. 1 is shows the solution of this problem by Graphic Method.

The presented steps can be easily programmed, as they have been for nearly all computers. The computing time of the logic way in the simplex technique is always less than the computing time of the ordinary simplex technique.

Discussion

There are three solution weakness of LP problem termed by i) unbounded solutions, ii) no feasible solutions and iii) degeneracy.

The objective function increase for maximization or decrease for minimization beyond bound, without leaving the feasible region. But some times objective function vector (or line) never hits an extreme point. Then it call that this solution is unbounded that arises from the mistaking of problem formulation or incomplete formulation.

No feasible solution means that it is not possible to find nonnegative values for all decision variables. In this case something went

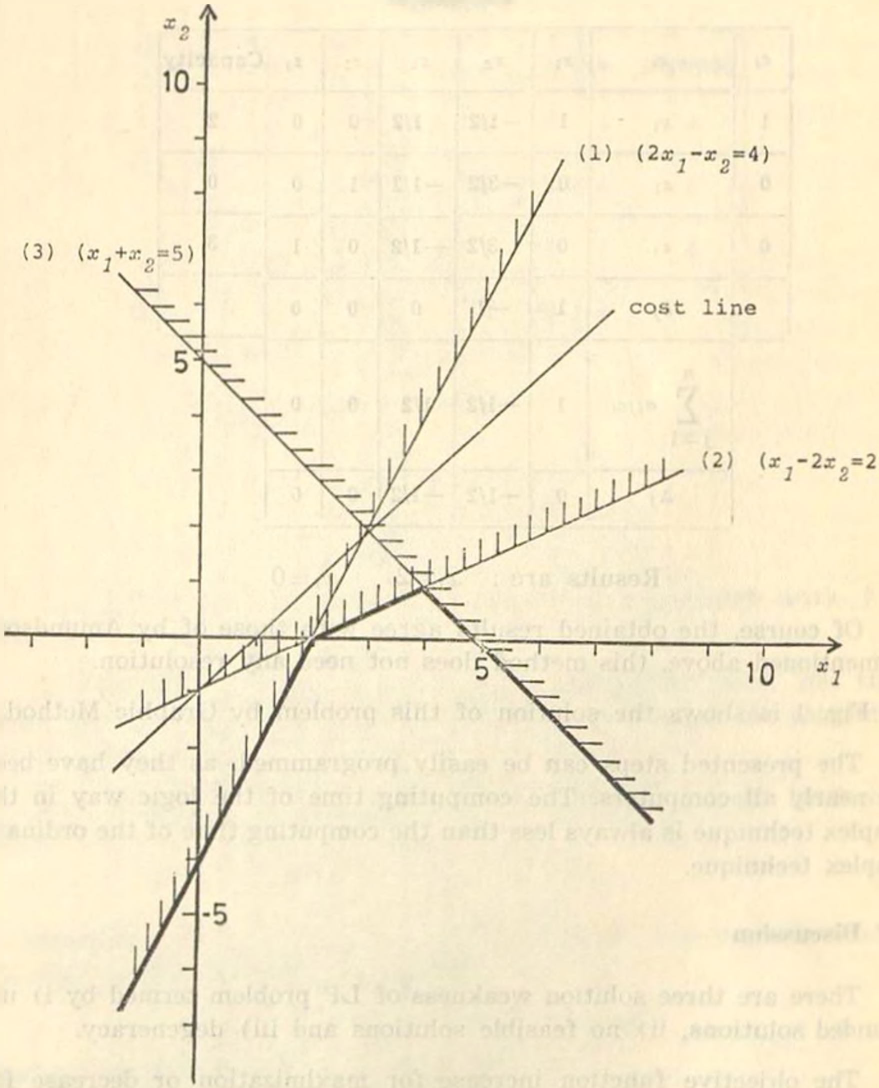

Fig. 1 Graphical presentation of the illustration

wrong in the problem formulation. No feasible solution problem is not so much in the real life LP problems.

When degeneracy is present, the objective function may not change when one move from a basic feasible solution to another. If one want to solve such as degeneracy problem by Simplex Method (by hand or digital computer). One can not catch optimal feasible solution. But there is only one optimal feasible solution. When one follow to try simplex technique, each trying call one the condition of no optimal feasible solution. It will give never optimal solution. One must do the resolution of the degeneracy problem both case by hand and digital computer.

Two different approaches to the resolution of the degeneracy problem have been developed. One is the perturbation method of Charnes[2,3,4]. The other, developed by Dantzig[5], Orden[5], and Wolfe[6]. However, the present paper does not need to discuss for the resolution way of the degeneracy problem. Finally the present paper wants to say when one use Logic Way in simplex technique, one meet never degeneracy problem and doesn't need any resolution procedure.

Nomenclature

a_{ij} : coefficient of x_{ij} [—]

b_i : parameter of constraint i which means capacity [—]

c_i : coefficient of objective function of z_i

c_j : coefficient of objective function of x_j [—]

m : number of constraints [—]

n : number of real variables [—]

p : profit by objective function [—]

p^* : profit by realized objective function [—]

x_j : real variable [—]

z_i : slck variable [—]

Δ_j : difference between c_j and $\sum^{i} a_{ij}c_i$ [—]

Subscripts

i : constraint

j : real variable

Upperscripts

$*$: determined realized value

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