The New Logics For Linear Programming Problems

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Abstract

This monograph proposes two new logics for LP problems, namely i) choosing optimal active constraints and non - zero variables, ii) production of the adding constraint (CONAD). Direct Approach Method, Simplex Method, and Revised Simplex Method are modified based on these new logics.

The present modifications provide many advantages, such as the reductions of the computing time and size of dimension, no degeneracy troubles, and less possibility to meet unbounded and no - feasible solutions.

A short historical brief survey of the solution methods, main features of advanced computer codes of LP, and a variety of special topics in solution ways are also presented.

The proposed Direct Approach Method and Simplex Method have been demonstrated by several numerical examples. The similar demonstration can be made for the proposed Revised Simplex Method also.

Introduction

The well - known linear programming has great power and applicability in mathematical programming. The Author thinks that all power of LP was started after discovering simplex method in 1947 mathematician George B. Dantzig. Then, many investigators were interested in linear

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programming. Also at that time the high - speed digital computer was quickly developed. This opportunity gave many changes to the preparation of computer program or code for LP.

The Simplex Method of LP is a dynamic trial and error solution method in which has most of time, the number of the trial steps is just same as the number of the extreme points in the problem. It can be shown that the Simplex Method generally converges in between m and $2m$ trials, where m is the number of inequalities.

The Revised Simplex Method 14), 17), 18), 21), 44), 45) was developed by Dantzig, Orchard - Hays and others at the RAND cooperation as an efficient computational procedure for solving linear programming - problems on digital computers. The Revised Simplex Method solves a linear programming - problem in the same way as the Simplex Method.

The decomposition procedure 16), 17), 22) in LP, it is the simplifying the solution of linear programming - problems in certain cases. After this simplifying, method for solution is the Revised Simplex or the Simplex Method.

There are two other ways to solve linear programming problems in special cases. The Interval Linear Programming and Integer Linear Programming (or Discrete Programming). Interval Linear Programming is clearly equivalent to an enlarged linear programme. In the Integer Linear Programming some or all variables must take integer values in certain cases.

The history 12) of the Integer Programming is very briefly, as follows ; in 1958 Gomory 30) devised a method, known as the method of integer Forms, for solving pure Integer Programming problems. An outline of this was published at the time. In 1960, he devised another method, the All - Integer Method (Gomory, 1963 b) 31).

Recently linear programming has been widely used in the Refining and petrochemical fields. Always chemical plants have non - linear relations between chemical operation variables. After that linear programming was extended into non - Linear area then Mixed Integer Programming was developed by Gomory 31).

The field of mixed Integer Programming is less far advanced. Today, many investigators and research centers have been hardly working on Mixed Integer Programming which is the most recent (and most succesfull) in a series of techniques for handling non - linear data within linear programming format, also this is a fast - moving field in the mathematical programming.

More Special Topics on Linear Programming

In this section are considered a veriety of special topics. Some of these deal with the possibility of simplifying the solution of linear programming problems in certin cases. Others deal with ways of making parameter variation analyses and sensitivity studies. All of these deals are developed for preparing some programmes with digital computer techniques.

Some solution procedures of LP are developed, these can give permision to solve problems with several objective functions but with the same constraints or the determination of optimal solutions for a series of RHS parameter groups (Multiple Objective Functions and Right-Hand Sides Procedures[34]). For this purpose another selection criterion is generally used and it is based upon the so called Dual Simplex method.

There are other computation procedures that can save much time, money, and energy. After a computation has terminated and the solution is inspected, it may occur that either a constraint or a variable has been left out (GETOFF and RESTART procedures).

It is possible to vary the objective function coefficients in a continuous fashion starting from the original objective function and its optimal solution. This called 'Parametric programming or parametrization' 25), 35), 37), 38) on the objective function. To do this it is necessary to add, or subtract, multiples of specified changes to each coefficient in the objective function.

Some special algorithm is developed based on the specialities of problem in this case it can be shown ; transportation algortihm, Upper and Lower Bounds 15), that can be used to solve well for 'Capacitated Transportation problem' the Algorithm that is most efficient for solving such problems is known as the 'Out - of - Kilter algorithm. It was designed to solve a more general class of problems known as Capacitated Network problems' which capacitated transportation problem is a special case.

Advanced Computatioanl Features of LP Computer Codes

The first 40) successful solution of a linear programming problem on a high - speed electronic digital computer occured in January, 1952, on the National Bureau of Standards computer, te SEAC. The computatinal method used was the original simplex procedure, and the application was an Air Force Programming, problem dealing with the deployment and support of an aircraft to meet stipulated requirements. Since that time, the simplex algorithm, or variations of this procedure, has been coded for most of the intermediate and large generalpurpose electronic computer.

After this fact the all linear programming investigators tried to prepare a good computer program, or code for LP problems. On the other hand the applicability of LP was increased into planning a cooporate level, and regional and national planning. On each passed day, dimensions of LP problem has been increassing thus linear programming become into computer programming art.

The most efficient digital computer program using Simplex Technique has been developed by Dantzig, Orden, and others at the RAND corporation 45). That program is called as two - phase method using full tableau. Phase I is to get feasible area, phase II is to getting optimal feasible solution.

After developing Revised Simplex Method that permits so many options which are not available in the full tableau method, at the time Orchard - Hays Revised Simplex Program is well - known (on an I.B.M. 704 computer, and the maximum number of restrictions allowed by this program was 255).

After 1960 several computer codes were developed using variants of integer linear programming method have been written; and have successfully solved many real problems. The most spectacular work in this area has been that of Glenn Martin. His code for the I.B.M. 7 94 computer uses a variant of the Method Integer Forms called the Accolerated Euclidean Algarithm 19) (Martin, 1963). It has solved a number of problems with about 100 equations and 2000 variables. Up to the beginning of 1964 the largest single problem had about 215 equations and about 2600 variables.

A mixed integer programming procedure 42), published by Healy (1964) under the title of Multiple Choise Programming has been programmed for the I.B.M. 7090 computer and found to solve practical problems, although its arctical status is obscure. Its emphasis is that problems in which a some of the non - negative integer - valued variables must add up to 1 is appropriate, since many practical problems have this structure.

Driebeck 28) (1964) and Dakin 32) (1964) have developed programmes using a 'branch and bound' method for mixed integer programming. As the 1970, many digital computer manufacture company are developed some new programmes of LP for saving time, and money. Most of these programmes are developed based on two phase, full tableau simplex method or Revised Simplex Method. There are a few programmes based on Multiple Objective Functions and Right - Hand sides Procedures. These programmes does not save so much time. The programmes based on GETOFF and RESTART procedures, can save more time. Also the programmes based on parametrization procedure and same algorithm which are based on the properties of problem (Upper and lower bound., Out - of Kilter algorithims) can save time in own cases.

Proposed New Direct Approach Method

Definitions and Preliminaries

The general linear programming problem can be stated as: given a set of m linear equations and / or inequalities involving n variables find the nonnegative values of these variables which satisfy the equations and inequalities and also maximize or minimize a linear objective function.

The 'standard form' of linear programming problem may be stated mathematically as follows

chose the quantities

$$x_j \geqslant 0 \qquad (j = 1, \dots n)$$

to maximize

$$p = \sum_{j=1}^{n} c_j x_j \tag{1}$$

subject to the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (i=1, \ldots .. m)$$

This problem may be written in matrix - vector form

$$\begin{array}{c} \max p^T x \\ Ax \leqslant b \\ x \geqslant 0 \end{array} \tag{2}$$

Extreme points : The solution of a linear optimization problem is to be found only at one of several distinguished locations, called extreme points are defined by the constraints on the problem and are relatively few in number.

Optimal Extreme Points : This one of Extreme points which is including only optimum conditions values of variables.

Inactive Constraints : If one of constraints can not be determined by any extreme point, this is called Inactive Constraints. It means inactive constraints is in, out of feasible area which is determined by active constraints.

Active constraints : These constraints pass on one extreme point.

Optimal active constraints : These constraints pass on optimal extreme point.

Feasible region : This is the collection of all feasible solutions to the problem. Any point that is not in the feasible region cannot be a feasible solution to the *LP* problem.

Proof 1 The optimal conditions of linear programming can be determined one point which is optimal extreme point on an n - dimensional Euclidian space. The optimal conditions never can be determined by plane or hyperplane on an n - dimensional Euclidian space.

Proof 2 An n - dimensional Euclidian space, symbolized by E^n, is defined as the collection of all vector (points) $a = [a_1, \ldots a_n]$ where n is the dimension. If $n=2$, the optimal extreme point can be determined by two vectors, if $n=3$ the optimal extreme point can be determine by three vectors. It means we need to determine the optimal point only constraints that is constraints number is equal to the dimension number (n).

Prof 3 The Optimal Active Constraints can determine optimal feasible solution without nonactive and active constraints. Because the optimal active constraints determine optimal extreme point. It means only optimal active constraints have intersection with each others an optimal extreme point.

The new Direct Approach Method choose optimal active constraints. it eliminates other unnecessary constraints (non active and active). Therefore many unnecessary calculations in solution way were left out. After choosing of optimal active constraints, this new method produces one adding contraint that has same slope as the slope of objective functions. It will be shown later on how to choose optimal active constraints, and how to produce adding constraint in calculation procedure.

After above rearrangement original linear programming problem becomes into a subproblem. But subproblem has same result for optimal solution as original problem.

This subproblem can apply easily on any conventional solution method of linear programming problem as an ordinary case. But this manustcript also has been shown a special solution way of *LP*.

Before using this new method one can understand which variables will take zero values without using the solution procedure.

The Procedure for the determination of Subproblem of Original Linear Programming Problem

The new Direct Approach Method uses matrix calculation to get the subproblem. The following pieces of information thus have to be determine before rearrangement to get subproblem.

1) The values of the LHS coefficients (a_{ij})
2) The values of the RHS parameter (b_i)
3) The values of the objective function coefficients (c_j)
4) The type of constraint relationship, i, e ... , ($\leqslant$), ($=$), or ($\geqslant$)
5) Whether to maximize or minimize the objective function
6) The number of variables on the number of constraints

NOTICE :

1) «Inacvite» and «Nonactive» are used in the same meaning.

2) «Group» and «Class» are used in the same meaning.

The following nomenclature is given for this section.

$A0(I,J)$	:	Original matrix
$A1(I,J)$	:	Transposed matrix of $A0(I,J)$
$P(J)$	:	Original objective function as a vector.
$PM(J)$	:	Multiple objective function as a vector
$PT(J)$	:	Deviated objective function as a vector
$PTK(J)$	:	Differenced and deviated objective function as a vector
$PS(J)$	:	Differenced original objective function as a vector
CONAD(J)	:	Adding constraint as a vector

The Steps of calculations procedure to choose optimal active constraints and non zero variables

This procedure is given for standard LP problem.

1) The transposation of $A0(I,J)$ into $A1(I,J)$
2) Getting multiplied objective function $|PM(I)|$ by process (3)

$$P(J) \times A1(I,J) = PM(I) \tag{3}$$

3) Getting deviated objective function $|PT(J)|$ by process (4)

$$PM(I) - P(J) = PT(I) \tag{4}$$

4) The checking $PT(I) \geqslant P(J)$, for ($\leqslant$) constraints or $PT(I) \leqslant P(J)$

Some constraints is optimal active constraints which is ensure $PT(J) \geqslant P(J)$ relation for ($\leqslant$) constraints. All the other constraints were left out in this step.

5) Getting differenced and deviated objective function $|PTK(I)|$ by process (5)

$$PT(I) - PT(I+1) = PTK(I) \tag{5}$$

6) Getting differenced original objective function $|PS(J)|$ by process (6)

$$P(J) - P(J+1) = PS(J) \tag{6}$$

7) Checking process (7)

$$PTK(I) \leqslant PS(J) \quad \text{or not} \qquad (7)$$

For the maximal objective function; if $PTK(I)$ is less than $PS(J)$, this variables is equal zero value; for the minimal objective function if $PTK(I)$ is great than $PS(J)$, this variable is equal to a non - zero value.

Notice : If Maximization is desired, for minimum objective function LP problem, the objective function has to be multiplied through by (— 1). Because, the optimal values for the decision variables obtained by minimizing the objective function are exactly the same as those obtained by maximizing the negative of the same objective function, and $|\min z = -\max(-z)|$.

All of the mentioned above steps will be expressed after giving a numerical examples, the reason of this is well expression.

Example 1 (for how to choose optimal active constraints and nonzero variables.) This problem is carried out 7).

Problem $\max z = 20x_1 + 10x_2 + 5x_3$

$$5x_1 + 3x_2 + x_3 \leqslant 1050 \qquad (1)$$
$$4x_1 + 3x_1 + 2x_3 \leqslant 1000 \qquad (2)$$
$$x_1 + 2x_2 + 2x_3 \leqslant 400 \qquad (3)$$

1) The transposation of $A0(I,J)$ into $A1(I,J)$

$$A0 \begin{bmatrix} 5 & 3 & 1 \\ 4 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix} \longrightarrow A1 \begin{bmatrix} 5 & 4 & 1 \\ 3 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

2) Getting muptiplied objective function $|PM(I)|$ by process (3)

$$P\,[20 \ 10 \ 5] \times A1 \begin{bmatrix} 5 & 4 & 1 \\ 3 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix} = PM[135 \ 120 \ 50]$$

3) Getting deviated objective function $[PT(I)|$ by process (4)

$$135 - 20 = 115$$
$$120 - 10 = 110$$
$$50 - 5 = 45$$

4) Checking $PT(I) > P(J)$, or not

$$115 > 20$$
$$110 > 10$$
$$45 > 5,$$

All constraints are optimal active constraints

5) Getting differenced and deviated objective function $[PTK(I)]$ by process (5)

$$115 - 110 = 5$$
$$115 - 45 = 70$$
$$110 - 45 = 65$$

6) Getting differeced original objective function $[PS(J)]$ by process (6)

$$20 - 10 = 10$$
$$20 - 5 = 15$$
$$10 - 5 = 5$$

7) Checking process (7)

$$5 < 10 \quad (*)$$
$$70 > 15$$
$$65 > 5$$

for thise problem if $PTK(I)$ is less than $PS(J)$, thise variables (*) is equal zero value. It means ($x_2 = 0$). We can eliminate constraint number (2) in accordance with the proof 2 (see proof 2).
Subproblem is mentioned below.

$$\max (20x_1 + 5x_3)$$
$$5x_1 + x_3 = 1050$$
$$x_1 + 2x_3 = 400$$

Expression of choosing criterion of optimal active constraints : This expression is given for ($\leqslant$) constraints. If, some constraints are to ensure $PT(J) \leqslant P(J)$ relation. It means, these constraints can not do any action for the determination of iptimal extreme point or we can express other constraints that can determine the optimal extreme point without using these constraints which is to ensure $PT(J) \leqslant P(J)$ relation. This criterion gives a possibility to choose the optimal active constraints. On the other handt it can do left out nonoptimal active and nonactive constraints the logic of this criterion can be understood well after example 1 that is mentioned above.

Expression of determining criterion of the choosing nonzero variables

This expression is given determining criterion for LP problem that has maximize objective function. If *PTK(I)* is less than *PS(J)*, this variable is equal to a zero value, which means, Differenced and Derivated abjective function $|PTK(I)|$ determines the power of the getting optimal conditions for each decision variables. On the other hand *PS(J)* determines a comperative degree each others for variables the attending of power the description of optimal conditions. It can be expressed more clear as follows ; $PTK(I) \leqslant PS(I)$ means this variable will lose its given real value (that is given by original objective function) after attending to determine of optimal conditions. The attending power of the variables for the determination of the optimal conditions must be in the $PTK(I) \leqslant PS(J)$ condition.

The procedure of the production of adding constraint (CONAD)

Nomenclature;

A3(I, I)	:	Multiplied Matrix
B(I)	:	*RHS* parameter
A1(I, I)	:	Unit matrix of *A0(I, J)* that determines the type of constraints relationship.

It is given the following steps for production of adding constraint (CONAD)

1) Transposation *A0(I, J)* into *A1(I, J)*

2) Multiplication of *A0(I, J)* with *A1(I, J)* for the getting *A3(I, I)* by process (8)

$$A0(I, J) \times A1(I, J) = A3(I, I) \tag{8}$$

3) The getting simultaneous equations by process (9)

$$A3(I,I) + AI(I,I) = A4(I,I) \tag{9}$$

4) The solving of the following simultaneous equations

$$AI(I,I) = B(I)$$

After solving, result is $X0(I)$ that transposes into $X0(J)$

5) The getting $X(J)$, $X(I)$ with following multiplication by process (10)

$$X0(I) \times A0(I,J) = X(J)$$
$$X(J) \text{ transposes } X(I) \tag{10}$$

6) The getting of the RHS of adding constraint by process (11)

$$P(J) \times X(I) = RHS \text{ of CONAD} \tag{11}$$

7) The adding constraint is determined as follows by process (12)
Assumed objective function

$$P(J) \geqslant RHS \text{ of CONAD} \tag{12}$$

Example 2 This example is given for the production adding constraint.

Problem

$$\text{Min } (3x_1 - x_2)$$
$$-x_1 + 2x_2 \leqslant 8$$
$$x_1 + 3x_2 \leqslant 18$$
$$x_1 + x_2 \leqslant 12$$

This problem can be rewrites in matrix and vector form as follows

$$A0\begin{bmatrix} -1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}, \quad AI\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P[3 \;\; -1], \quad B\begin{bmatrix} 8 \\ 18 \\ 12 \end{bmatrix}$$

Solution

1) Transposationaly *A0(I , J)* into *A1(I , J)*

$$A0 \begin{bmatrix} -1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} \textit{ Transposes } A1 \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

2) Getting *A3(I, I)* by process (8)

$$A0 \begin{bmatrix} -1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} \times A1 \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} = A3 \begin{bmatrix} 5 & 5 & 1 \\ 5 & 10 & 4 \\ 1 & 4 & 2 \end{bmatrix}$$

3) The getting simultaneaus equations by process (9)

$$A3 \begin{bmatrix} 5 & 5 & 1 \\ 5 & 10 & 4 \\ 1 & 4 & 2 \end{bmatrix} + AI \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A4 \begin{bmatrix} 6 & 5 & 1 \\ 5 & 11 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

4) Solution of following simultaneous equations

$$\begin{bmatrix} 6 & 5 & 1 \\ 5 & 11 & 4 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \\ 12 \end{bmatrix}$$

Result of above mentioned simultaneous equations are follows

$$X0 \begin{bmatrix} 0.8214 \\ -0.1785 \\ 3.9641 \end{bmatrix} \xrightarrow{\text{Trans.}} X0\,[0.8214 - 0.1785 \;\; 3.9641]$$

5) Getting of *X(J)* by process (10)

$$X0[0.8214 - 0.1785 \;\; 3.9541] \times A \begin{bmatrix} -1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} = X[2.96 \;\; 5.0714]$$

6) Getting of the RHS of CONAD by process (11)

$$P[3 - 1] \times X \begin{bmatrix} 2.96 \\ 5.0714 \end{bmatrix} = 3.81$$

7) Produce of CONAD by process (12)

$3x_1 - x_2 \leqslant -3.81$ (Because, objective function is minimize)
This problem can be converted into a subproblem as follows.

Subproblem

$$\begin{aligned} &\text{Min } (3x_1 - x_2) \\ &3x_1 - x_2 \leqslant -3.81 \\ -&x_1 + 2x_2 \leqslant 8 \\ &x_1 + 3x_2 \leqslant 18 \\ &x_1 + x_2 \leqslant 12 \end{aligned}$$

The expressing of good point of above mentioned subproblem : The subproblem has same optimum solution as original problem. But, the subproblem and original problem has different *feasible region.* These two feasible regions have the same point in the feasible region that are near the optimal feasible point. The CONAD gives a possibility to reduce the original feasible region to a small new feasible region around the optimal feasible point. It means, many unnecessary feasible point can be left out of the solution way by CONAD. After adding CONAD into original problem, it can be solve in the smaler iteration number than the iterations number of the solution of original problem by any ordinary solution method of LP. (see figure 1) The feasible region of original problem are shown by points A, B, C, D the feasible points of subproblem are E, D, F. The feasible points A, B, C were left out of the solution way by CONAD.

The proposed new process for Linear program problems : The above proposed two modifications (elimination of nonoptimal active constraints and zero value variables, production of CONAD, gives three new procedures for solution as follows

1) The approach to solution is without any iteration. (Dirert Approach Method)

2) The approach to solution in smallest iterations number as well as possible in certain cases.

3) The modified revised Simplex Method.

The Approach in solution without any iteration (Direct Approach Method)

This solution way includes following steps.

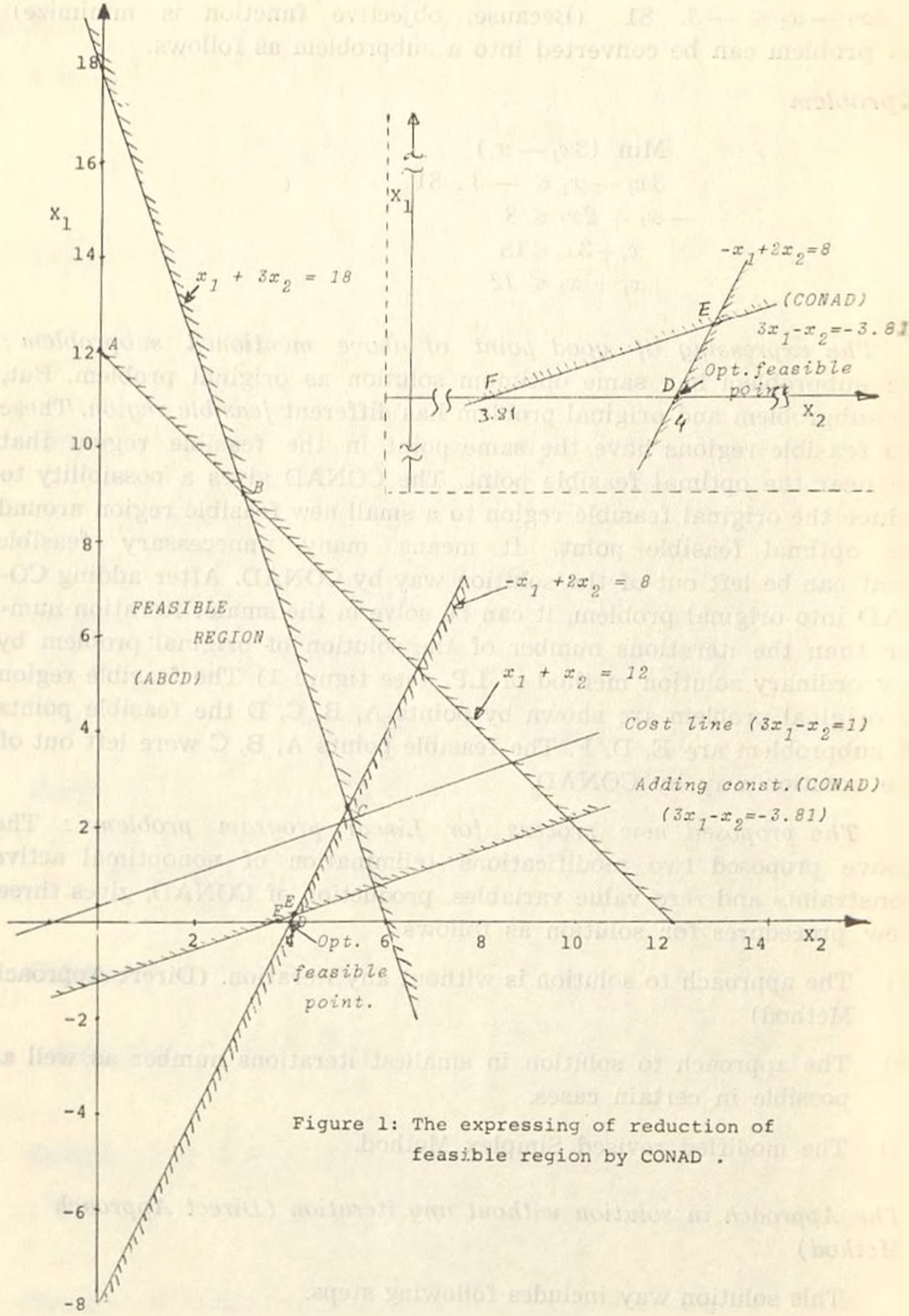

Figure 1: The expressing of reduction of feasible region by CONAD.

A) Determining subproblem; This step is equivalent to getting optimal active constraints and doing left out of zero value varibles in seven steps. It is not necessary to write before mentioned seven steps again here. Our subproblem of example 1 was as follows.

$$\text{Max } (20x_1 + 5x_3)$$
$$5x_1 + x_3 = 1050$$
$$x_1 + 2x_3 = 400$$

B) The solution of subproblem; This solution of subproblem can be reduce as the solution of a set of simultaneaus equations by knowing method (Gauss elimination, sweep out, etc). Without using objective function. Because it can be obtained with same number of equations and variables. In this case m is equal n, where m is the number of equations, n is the number of variables. The solution of example 1 is given by direct Approach as follows. Example 1 is carried out [7]. (p. 215 - 18)

$$5x_1 + x_3 = 1050$$
$$x_1 + 2x_3 = 400$$
$$x_1 = 188.88 \ , \quad x_3 = 105.55 \ , \quad x_2 = 0$$

(from step A)

This problem was solved in three iteration by Mc Millian [7].

Appendix A has some numerical example that is solved by Direct Approach Method. An important part of this mentioned case is the verification that the linear program is numerically sound. Extremely large and small numbers appearing in the same problem should, if possible, be avaided, since their simultaneous presence increases the possibilities for large error accumulation. Then it will be given down a normalization procedure of simultaneous equations. If this normalization procedure is not enough we can use a more excellent normalizations procedure.

The Normalization procedure for Direct Approach Method

1) After getting subproblem (for this case) *LHS* (a_{ij}) coefficient of subproblem transposes new. LHS

2) Multiplication of old *LHS* (a_{ij}) coefficient by transposed *LHS* (a_{ij}) coefficient then we will get new *LHS* (a_{ij}) coefficient.

3) Building new set of simultaneous equation by new LHS coefficient and B(I), after solving this new simultaneous equations we get subresult of solution.

4) Final result can be get, after multiplications of subresult with untransposed *LHS (aij)* coefficient of subproblem.

Example 3 This example can be solved for Direct Approach Method. This problem is carried out literature No 20 page 72, Figure 4 - 5.

Problem

Constraint No	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8		$B(I)$
(1)	1	1	0	0	0	0	0	0	$\leqslant$	700
(2)	0	0	1	1	0	0	0	0	$\leqslant$	600
(3)	0	0	0	0	1	1	0	0	$\leqslant$	900
(4)	0	0	0	0	0	0	1	1	$\leqslant$	500
(5)	1	0	1	0	1	0	1	0	=	1300
(6)	0	1	0	1	0	1	0	1	=	800
(7)	—2	0	—0.5	0	—3	0	11	0	$\leqslant$	0
(8)	0	—2	0	—0.5	0	—3	0	11	$\leqslant$	0
(9)	8	0	—3	0	—10	0	10	0	$\geqslant$	0
(10)	0	8	0	—6	0	—13	0	8	$\geqslant$	0
Min	(7.2	7.2	4.35	4.35	3.8	3.8	4.3	4.3)		

After elimination of nonoptimal active constraints and zero value variables. This problem reduces into below subroblem that is equivalent of original problem without Row 1, 3, 4 and Column 4. The type of all constraints are (=).

After above mentioned elimination, this LP problem can be solve as a set of simultaneous equations that has seven variables and same number equations. This simultaneous equations can be shown in matrix form as follows.

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ -2 & 0 & -0.5 & -3 & 0 & 11 & 0 \\ 0 & -2 & 0 & 0 & -3 & 0 & 11 \\ 8 & 0 & -3 & -10 & 0 & 10 & 0 \\ 0 & 8 & 0 & 0 & -13 & 0 & 8 \end{bmatrix} \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ x(5) \\ x(6) \\ x(7) \\ x(8) \end{bmatrix} = \begin{bmatrix} 600 \\ 1300 \\ 800 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

II. The Approach to solution in smallest iterations number as well as possible in certain cases.

In these cases, after determining a subproblem of the original problem, it can be solve using the Simplex Technique. It can be given five different appliations.

a) The first, CONAD is produced by process 8, 9, 10, 11, 12 then subproblem will have more one constraint that is CONAD. This subproblem can be applied Simplex Technique.

Example (Example 2)

Original problem	subproblem (a)
min $(3x_1 - x_2)$	Min $(3x_1 - x_2)$
$-x_1 + 2x_2 \leqslant 8$	$3x_1 - x_2 \leqslant -3.81$
$x_1 + 3x_2 \leqslant 18$	$-x_1 + 2x_2 \leqslant 8$
$x_1 + x_2 \leqslant 12$	$x_1 + 3x_2 \leqslant 18$
	$x_1 + x_2 \leqslant 12$

b) For this application style, our subproblem is equivalent to original problem without zero value varibles and non - optimal active constraints.

Example (Example 1)

original problem	subproblem (b)
Max $z = 20x_1 + 10x_2 + 5x_3$	Max $(20x_1 + 5x_3)$
$5x_1 + 3x_2 + x_3 \leqslant 1050$	$5x_1 + x_3 = 1050$
$4x_1 + 3x_2 + 2x_3 \leqslant 1000$	$x_1 + 2x_3 = 400$
$x_1 + 2x_2 + 2x_3 \leqslant 400$	

c) The CONAD can be aded into subproblem (b)

original problem	subproblem
$\text{Max } z = 20x_1 + 10x_2 + 5x_3$	$\text{Max}(20x_1 + 5x_3)$
$5x_1 + 3x_2 + x_3 \leqslant 1050$	$20x_1 + 5x_3 \geqslant$ CONAD RHS
$4x_1 + 3x_2 + 2x_3 \leqslant 1000$	$5x_1 + x_3 = 1050$
$x_1 + 2x_2 + 2x_3 \leqslant 400$	$x_1 + 2x_3 = 400$

d) The number of constraints of optimal active constraints can be reduced by an operation that is the addition of some constraints on each other.

Example (Example 3)

The original problem is Example 3, subproblem is determined as follows

Added constraint No	x_1	x_2	x_3	x_5	x_6	x_7	x_8		$B(I)$
(2+7+8)	—2	—2	0.5	—3	—3	11	11	=	600
(4+5)	1	1	1	1	1	1	1	=	2100
(9+10)	8	8	—3	—10	—13	10	8	=	0
Min	(7.2	7.2	4.35	3.8	3.8	4.3	4.3)		

Subproblem (d) can be solve by direct approach with normalization or simplex method, also subproblem (d) can be run to simplex technique after adding its CONAD into constraints of subproblem (d) in the three iterations.

III. The proposed re - Revised Simplex Method

In the revised simplex method, the objective function is essentially treated as of it were another constraint. Then, it can be considered as another constraint equation for which z is to be made as large as possible in the notations of LP problem (13).

$$
\begin{aligned}
&z-c_1x_1-\cdots\cdots\cdots-c_nx_n=0\\
&a_{11}x_1+\cdots\cdots\cdots+a_{1n}x_n=b_1\\
&\quad\vdots\qquad\qquad\vdots\\
&a_{m1}x_1+\cdots\quad\cdots+a_{mn}x_n=b_m
\end{aligned}
\tag{13}
$$

then (13) can be considered to be a system of $m+1$ simaltaneous linear equations in $n+1$ variables $z, x_1, \ldots, x_n$. We wish to find a solution to (13) such that z is as large as possible (and unrestricted in sign), subject to the non-negativity restrictions $x_j \geqslant 0$ $(j=1, \ldots n)$. If we put $z=x_0$, $-c_j=a_j$ into (7—2).

$$
\begin{aligned}
&x_0+a_{01}x_1+\cdots\cdots\cdots+a_{0n}x_n=0\\
&a_{11}x_1+\cdots\cdots\cdots+a_{1n}x_n=b_1\\
&\quad\vdots\qquad\qquad\vdots\\
&a_{m1}x_1+\cdots\cdots\cdots+a_{mn}x_n=b_m
\end{aligned}
\tag{14}
$$

Re - revised Simplex Method is going a proof as follows.

[Original objective function - Multiplied objective function = 0]. The above mentioned equation can be considered as other constraint. Then, the LP models of proposed Revised Simplex Method is as follows

$$
\begin{aligned}
&c_1x_1+\cdots\cdots\cdots+c_nx_n-PM(J)=0\\
&a_{11}x_1+\cdots\quad\cdots+a_{1n}x_n=b_1\\
&\quad\vdots\qquad\qquad\vdots\\
&a_{m1}x_1+\cdots\cdots\cdots+a_{mn}x_n=b_m
\end{aligned}
\tag{15}
$$

If we use above LP model our proposed Re - revised Simplex Method is more efficient than the revised simplex method in solution way.

How to applied the all proposed new solution procedures of LP to Digital Computer

The presented stepts of modification of original linear programming problem (description of optimal active constraints and zero value variables) can be easily programmed, as they have been for nearly all computers. Before it is presented new solution procedure of LP in three groups. It is given some points on how to applied to digital computer machine. These new procedures in same three classes are as follows.

1) New Direct Approach Method

After getting correct subproblem, it can be applied any computer machine as a FORTRAN program of simultaneous equations.

2) The approaches to solution in smallest iterations number as well as possible in certain cases.

In the (a), it can be prepared a program in three steps namely i) getting subproblem ii) getting feasible region iii) getting optimal solution for this case. If the preparing program can get feasible area without iterations this case can be more efficient than within iteration of getting feasible region. If user has a LP program it can be applied after getting subproblem easily. In this case the subproblem is equivalent to original problem by adding CONAD.

Also in the b), c), d), e) it can be prepared a program with includes both steps, getting subproblem in each certain case and LP program of subproblem.

A program can be prepared for re - revised simplex method by using above the mentioned LP model of re - revised simplex method (15).

If the LP problem is really large it may be that one can meet some error accumulations in solution way, like row errors and column errors. Every LP code or computer program enable the specification of an error frequency at which errors are being checked. For example, after each 50 iterations the errors may be checked and if they exceed a specified error tolerance a basis inversion may be automatically triggered, the user must be careful about error checking style.

APPENDIX A : Examples on Direct Approach Method.

Example 4 : This problem is carried out literature No. 20, chap. 4, page 46 - 48, Example 1

Problem :

$$\text{Min } z = 2x_1 + 3x_2$$

$$5x_1 + 10x_2 \geqslant 90 \qquad (1)$$

$$4x_1 + 3x_2 \geqslant 48 \qquad (2)$$

$$0.5x_1 + \geqslant 1.5 \qquad (3)$$

$$2x_1 + x_2 \geqslant 20 \qquad (4)$$

$$x_1 \, , x_2 \geqslant 0$$

Solution :

1) The transposation of A0 into A1

$$A0 \begin{bmatrix} 5 & 10 \\ 4 & 3 \\ 0.5 & 0 \\ 2 & 1 \end{bmatrix} \longrightarrow A1 \begin{bmatrix} 5 & 4 & 0.5 & 2 \\ 10 & 3 & 0 & 1 \end{bmatrix}$$

2) Getting multiplied objective function (PM) by process 3. After chanching objective function into maximize. (Max $z = -2x_1 - 3x_2$).

$$P[-2 \; -3] \times A1 \begin{bmatrix} 5 & 4 & 0.5 & 2 \\ 10 & 3 & 0 & 1 \end{bmatrix} = PM[-40 \; -1 \; -7]$$

3) Getting deviated objective function *(PT)* by process (4)

$$-40-(-2) = -38$$
$$-17-(-3) = -14$$
$$-1-0 = -1$$
$$-7-0 = -7$$

4) Checking $PT \leqslant P$ or not. Because, constraint type ise $(\geqslant)$

$$-38 < -2$$
$$-14 < -3$$
$$-1 < 0$$
$$-7 < 0$$

These four constraints attend the determining of optimal conditions in power comperison as follows ; 1) constraint (1) 2) constraint (2) 3) constraint (3) 4) constraint (4). But, constraint (1) and (2) are enough for the solution. Because, there are two variables in the problem.

5) Getting differenced and deviated objective function *(PTK)* by process (5). After this step we will do all the calculation for constraint (1) and (2). Because constraint (3) and (4) was eliminated.

$$-38-(-14) = -24$$

6) Getting differenced original objective function *(PS)* by process (6)

$$-2-(-3) = 1$$

7) Checking process (7)

$$-24 < 1 \qquad \textit{(Because, all constraints are } (\geqslant)\textit{)}$$

The subproblem of this equivalent without constraint (3) and (4) of original problem as follows;

There is no elimination for variables.

$$5x_1 + 10x_2 = 90$$
$$4x_1 + 3x_2 = 48$$
$$\textit{Result:} \quad x_1 = 8.4 \,,\ x_2 = 4.8 \ .$$

Example 5 : This problem is carried out literature No. 20, chap. 4, page 49 - 51, example 2

Problem :

$$Max\ z = 15x_1 + 15x_2$$
$$2/5x_1 + 3/5x_2 \leqslant 8 \qquad (1)$$
$$x_1 + 3/2x_2 \leqslant 15 \qquad (2)$$
$$1/3x_1 + x_2 \leqslant 8 \qquad (3)$$
$$8/3x_1 + 2x_2 \leqslant 32 \qquad (4)$$
$$x_1 , x_2 \geqslant 0$$

Solution : This problem is solved in the following steps.

1) $A0 \begin{vmatrix} 2/5 & 3/5 \\ 1 & 3/2 \\ 1/3 & 1 \\ 8/3 & 2 \end{vmatrix}$ *Transposes* $A1 \begin{vmatrix} 2/5 & 1 & 1/3 & 8/3 \\ 3/5 & 3/2 & 1 & 2 \end{vmatrix}$

2) $P[15\ \ 15] \times A1 \begin{vmatrix} 2/5 & 1 & 1/3 & 8/3 \\ 3/5 & 3/2 & 1 & 2 \end{vmatrix} = PM[15\ \ 75/2\ \ 20\ \ 70]$

3)

$$15 - 15 = 0$$
$$75/2 - 15 = 45/2$$
$$20 - 0 = 20$$
$$70 - 0 = 70$$

4)

$$0 < 15 \quad \textit{can not attend}$$
$$45/2 > 15$$
$$20 > 0$$
$$70 > 0$$

The four constraints attend the determining of optimal conditions in power compresion as follows; 1) constraint 2, 2) constraint 4, 3) constraint 3.

But, constraint 2 and 4 are enough for the solution. We choosed constraint 4 according constraint 3. Because the attendance power of constraint 4 is bigger than the constraint 3.

5) $45/2 - 70 = -95/2$

6) $15 - 15 = 0$

7) $-95/2 < 0$ There is no elimination for variables

The subproblem of this problem is given as follows;

$$x_1 + 3/2x_2 = 15$$

$$8/3x_1 + 2x_2 = 32$$

$$\textit{The result :} \quad x_1 = 9 \;,\; x_2 = 4 \;.$$

Example 6 : This problem is carried out lineraturе No. 35 Chap. 4, page 105.

Problem : $Max\ (x_1 - x_2)$

$$2x_1 - x_2 \leqslant 4 \qquad (1)$$

$$x_1 - 2x_2 \leqslant 2 \qquad (2)$$

$$x_1 - x_2 \leqslant 5 \qquad (3)$$

$$x_1 \;,\; x_2 \geqslant 0$$

Solution :

1) $A0 \begin{bmatrix} 2 & -1 \\ 1 & -2 \\ 1 & 1 \end{bmatrix}$ $\quad Transposes\ A1 \begin{vmatrix} 2 & 1 & 1 \\ -1 & -2 & 1 \end{vmatrix}$

2) $P[1 \ \ -1] \times A1 \begin{vmatrix} 2 & 1 & 1 \\ -1 & -2 & 1 \end{vmatrix} = PM[3 \ \ 3 \ \ 0]$

3) $3 - 1 = 2$

$3 - (-1) = 4$

$0 - 0 = 0$

4) Checking $PT > P$ or not, Because, constraint type is $(\leqslant)$.

$$2 > 1$$
$$4 > -1$$
$$0 = 0 \quad \textit{can not attend for the optimality.}$$

The constraint 3 is eliminated by above process.

5) $2 - 4 = -2$

6) $1 - (-1) = 2$

7) $-2 < 2$, it means $x_2 = 0$

The subproblem is given as follows;

$$2x_1 = 4$$

Result : $x_1 = 2$, $x_2 = 0$.

Example 7 : This problem is carried out literature No. 20, Chap. 8, page 163 - 179

Problem :

$$\begin{aligned} Max\ z = {} & 351{,}360 - 0.301R1 - 2.2C1R2 + 3.5C1M - 0.25C2R1PA \\ & - 0.55C2R1PB - 1.85R2 + 2.45C2M - 1.4C1R1Y1 \\ & - 1.7C1R1Y2 - 1.6C2R1Y1 - 2.1C2R1Y2 + 1.8RR1 \\ & + 2.1RR2 + 2.5HR1M0 + 2.7HR2M0 - 0.3GR1M3 \\ & - 0.4GR1M4 - 0.8GR1M6 - 8.4GM6 - 0.3LR1M3 \\ & - 0.4LR1M4 - 0.3HR1M3 - 0.4HR1M4 - 0.25GR2M3 \\ & - 0.3GR2M4 - 0.25LR2M3 - 0.3LR2M4 \\ & - 0.25HR2M3 - 0.3HR2M4 \end{aligned}$$

(C1AV) $C1R1 + C1R2 + C1M = 30{,}000$ (1)
(C2AV) $C2R1PA + C2R1PB + C2R2 + C2M = 50{,}000$ (2)
(PACAP) $C2R1PA \leqslant 9{,}000$ (3)
(PBCAP) $C2R1PB + C2R2 + C2M \leqslant 44{,}000$ (4)
(R1CAP) $C1R1 + C2R1PA + C2R1PB \leqslant 40{,}000$ (5)
(R2CAP) $C1R2 + C2R2 \leqslant 20{,}000$ (6)
(R2GPRO) $0.54C1R2 + 0.39C2R2 - GR2 = 0$ (7)
(R2LPRO) $0.14C1R2 + 0.16S2R2 - LR2 = 0$ (8)
(R2HPRO) $0.25C1R2 + 0.35C2R2 - HR2 = 0$ (9)
(R2RPRO) $0.06C1R2 + 0.08C2R2 - RR2 = 0$ (10)
(C1YIEL) $C1R1 - C1R1Y1 - C1R1Y2 = 0$ (11)

(C2YIEL)	$C2R1PA + C2R1PB - C2R1Y1 - C2R1Y2 = 0$	(12)
(Y2CAP)	$C1R1Y2 + 1.4C2R1Y2 \leqslant 25{,}000$	(13)
(R1GPRO)	$0.49C1R1Y1 + 0.67C1R1Y2 + 0.35C2R1Y1 + 0.59C2R1Y2 - GR1 = 0$	(14)
(R1LPRO)	$0.15C1R1Y1 + 0.19C1R1Y2 + 0.14C2R1Y1 + 0.18C2R1Y2 - LR1 = 0$	(15)
(R1HPRO)	$0.29C1R1Y1 + 0.10C1R1Y2 + 0.40C2R1Y1 + 0.15C2R1Y2 - HR1 = 0$	(16)
(R1RPRO)	$0.07C1R1Y1 + 0.03C1R1Y2 + 0.10C2R1Y1 + 0.06C2R1Y2 - RR1 = 0$	(17)
(R1GAV)	$GR1 - GR1M3 - GR1M4 - GR1M6 = 13{,}100$	(18)
(R2GAV)	$GR2 - GR2M3 - GR2M4 = 6{,}800$	(19)
(M3GDEM)	$GR1M3 + GR2M3 = 6{,}100$	(20)
(M4GDEM)	$GR1M4 + GR2M4 = 4{,}200$	(21)
(M6GDEM)	$GR1M6 + GM6 = 1{,}800$	(22)
(R1LAV)	$LR1 - LR1M3 - LR1M4 = 4{,}300$	(23)
(R2LAV)	$LR2 - LR2M3 - LR2M4 = 2{,}600$	(24)
(M3LDEM)	$LR1M3 + LR2M3 = 2{,}200$	(25)
(M4LDEM)	$LR1M4 + LR2M4 = 900$	(26)
(R1HAV)	$HR1 - HR1M3 - HR1M4 - HR1M0 = 4{,}200$	(27)
(R2HAV)	$HR2 - HR2M3 - HR2M4 - HR2M0 = 3{,}800$	(28)
(M3HDEM)	$HR1M3 + HR2M3 = 3{,}200$	(29)
(M4HDEM)	$HR1M4 + HR2M4 = 800$	(30)
(MAXHR1)	$HR1M0 \leqslant 6{,}000$	(31)
(MAXHR2)	$HR2M0 \leqslant 2{,}000$	(32)

There is a total of 35 decision variables and 32 constraints (exclusive of the nonnegativity constraints and slack variables).

The constraints PBCAP, R1CAP, MAXHR1 are eliminated by the procedure of choosing optimal active constraints.

The variables *C1R2, C1M, C2R1Y2, GR2M3, LR2M3, HR1M4* are eliminated by the procedure of choosing nonzero variables. The subproblem of Example 7 is equivalent the original problem without constraints PBCAP, R1CAP, MAXHR1 and variables *C1R2, C1M, C2R1Y2, GR2M3, LR2M3, HR1M4*. Then subproblem includes 29 variables and 29 constraints. It can be solved as a set of simultaneous equations. After solution, result is given as follows;

C1R1 = 30,000, C2R2 = 20,000, C2M = 20,714.3, C2R1PA = 9,000., C2R1PB = 285.7, GR2 = 7,800., LR2 = 3,200., HR2 = 7,000., RR2 = 1,600., C1R1Y1 = 5,000., C1R1Y2 = 25,000., C2R1Y1 = 9,286., GR1 = 22,450., LR1 = 6,800., HR1 = 7,664.3, RR1 = 2,028.6, GR1M3 = 6,100., GR1M4 = 3,200., GR1M6 = 50., GR2M4 = 1000., GM6 = 1,750., LR1M3 = 2,200., LR1M4 = 300., LR2M4 = 600., HR1M3 = 2,800., HR2M3 = 400., HR2M4 = 800., HRM0 = 664.3, HR2M0 = 2,000. .

The variables *C1R2, C1M, C2R1Y2, GR2M3, LR2M3, HR1M4 are zero.*

This problem was solved using LPGOGO in 39 iterations. The new Direct Approach Method solves it without iterations as (29 variables and 29 equations) simultaneous equations.

APPENDIX B : A special procedure of the getting optimal active constraints for special problem.

If, the original problem has so different numbers of the constraints and variables, for instance a problem has 50 variables, 600 constraints, at that time «the procedure of the choosing of optimal active constraints and non - zero variables» can be done in a little changes.

If we follow a decomposation procedure of those procedure that divide problem into several problems. The constraints of each dividing group (that has same number of constraints as a number of variables) can be compared by mentioned necessary procedures. Then, it is picked up useful constraints and variables from each dividing group. There is not so much such problem in the real life LP problems.

Now, let us see how to do above mentioned decomposation with resolution of Example 5.

$$A0 \begin{bmatrix} 2/5 & 3/5 \\ 1 & 3/2 \\ 1/3 & 1 \\ 8/3 & 2 \end{bmatrix} \text{ decomposes } \begin{cases} A01 \begin{vmatrix} 2/5 & 3/5 \\ 1 & 3/2 \end{vmatrix} \\ A02 \begin{vmatrix} 1/3 & 1 \\ 8/3 & 2 \end{vmatrix} \end{cases}$$

The process 1), 2), 3), 4), 5), 6), 7) are applied on *A01* and *A02* as different problem but, objective functions of two decomposed problems are same.

The calculations for *A01*

1) $A01 \begin{bmatrix} 2/5 & 3/5 \\ 1 & 3/2 \end{bmatrix} \rightarrow A11 \begin{bmatrix} 2/5 & 1 \\ 3/5 & 3/2 \end{bmatrix}$

2) $P[15 \;\; 15] \times A11 \begin{bmatrix} 2/5 & 1 \\ 3/5 & 3/2 \end{bmatrix}$

$= PM1[15 \;\; 75/2]$

3) $15 - 15 = 0$

$75/2 - 15 = 45/2$

4) $0 < 15$ *can not attend*

$45/2 > 15$

The calculations for *A02*

1) $A02 \begin{bmatrix} 1/3 & 1 \\ 8/3 & 2 \end{bmatrix} \rightarrow A12 \begin{bmatrix} 1/3 & 8/3 \\ 1 & 2 \end{bmatrix}$

2) $P[15 \;\; 15] \times A12 \begin{bmatrix} 1/3 & 8/3 \\ 1 & 2 \end{bmatrix}$

$= PM2[20 \;\; 70]$

3) $20 - 15 = 5$

$70 - 15 = 55$

4) $5 < 15$ *can not attend*

$55 > 15$

After above calculations, we can pick up constraint 2 and 4 for the subproblem. Then we can follow the application of process 5), 6), 7) as before.

Discussion and Conclusion

The proposed two modification procedures of LP problem (namely i) choosing optimal active constraints and non - zero variables. ii) production of adding constraint (CONAD) give a powerful possibility for the reduction of LP problem dimension. This opportunity (hat is reduction of LP problem dimension) gives the production of many new methods in three class. The first, the proposed New Direct Approach Method is really powerful in middle size problem area of LP, over that size New Direct Approach Method is already powerful, but computing time will include more matrix nomalization time than the real computation time.

The second group includes five different applications that work with a co - working of ordinary solution way of LP Problem Application a) is most convenient in the application of the original problem with CONAD for two phase full tableau simplex method digital computer program, for instance MPS/360.

b), c) of second group is more convenient than the a), if we don't have a digital computer program which can not get feasible region without iterations.

d) and e) of the second group are most powerful applications in their cases. But, applicater must be careful the reduction of the number of optimal active constraints. In this case the number of contraints of subproblem is approximately equal 1/2 of the number of non - zero variables of subproblem in the small size problem, 1/3 of the number of non - zero variables of subproblem in the large size problem. In the other case, applicater may meet accumulation of row error and column error in the solution.

In the b), c), d), and e) there is shorter time for the reading of data. Because many constraints and variables can be left out, and also it does not need any slack variables in any case of LP problem. For instance in the original problem of example (3) has 8 variables and 10 constraints. After necessary elimination for constraints and variables, the getting new subproblem has 7 variables and 7 constraints. In the second group d), e) this problem has a subproblem that has 7 variables and 3 constraints. In the example (7), original problem has 35 variables and 32 constraints, on the other hand the subproblem of example (7) has 29 variables and 29 constraints as the equations, then we don't need any slack variables. It is clear this reduction of dimension of LP problem and without slack variables give a solution in the shortest time in the calculation way.

The calculation of the getting subproblem does not take much time, that can be assumed as one iteration in the solution, for the making a comparison with other conventional solution technique of LP problem.

The proposed re - revised simplex technique can reduce much computing time. Because it can be left out many iterations in the solution way of revised simplex technique.

The many linear programming literatures are devoted to the development of special algorithms. The transportation algorithm (Out - of - Kilter algorithm etc.) is a case in point. Computer programmes have been written to solve this special class of linear programmes alone. Also, the proposed necessary modifications can modify that special algorithm.

There are three solution weaknesses of LP problem termed by i) unbounded solutions, ii) no feasible solutions, and iii) degeneracy.

The objective function increases for maximization or decreases for minimization beyond bound, without leaving the feassible region. But some times objective function vector (or line) never hits an extreme point. Then it calles that this solution is unbounded that arises from the mistaking of problem formulation or incomplete formulation.

No feasible solution means that it is not possible to find non - negative values for all decision variables. In this case something went wrong in the problem formulation. No feasible solution problem is not so much in the real life LP problems.

When degeneracy is present, the objective function may not change when one moves from a basic feasible solution to another. If one wants to solve such as degeneracy problem by simplex method, one can not catch an optimal feasible solution. But there is only one optimal feasible solution when one follows to try simplex nechnique, each trying calles on the condition of no optimal feasible solution. It will never give optimal solution. One must do the resolution of the degeneracy problem of LP by hand and digital computer.

Two different approaches to the resolution of the degeneracy problem have been developed. One is the perturbation method of Charnes 1), The other, developed by Dantzig 17), Orden 4), and Wolfe 29).

However, the present paper docs not need to discuss for the resolution way of the degeneracy problem.

Finally the present paper wants to say when one uses above mentioned new solution methods of LP problem, one meets never degeneracy problem and does not need any resolution procedure. Example (6) of this present paper bc solved in the literature 35) as a degeneracy problem by special algorithm.

If, the original problem has so different numbers of the constraints and variables, for instance a problem has 50 variables, 600 constraints, at that time «the procedure of the choosing of optimal active constraints and non - zero variables» can be done in a little changes. If we follow a decomposation procedure of those procedures that divide problem into several problems. The constraints of each dividing group (that has the same number of constraints as a number of variables) can be compared by mentioned necessary procedures. Then, it is picked up useful constraints and variables from each dividing group. There is not so much such problems in the real life of LP problems.

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