Meskûn Bölge Hidrolojisinde Hazne Hacimlerinin Hesabı

Retention Basin Design in Urban Hydrology

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Yağmur suyu haznelerinin kapasitesine tesir eden faktörlerin mümkün olduğu kadar çoğunu göz önünde tutan, basit fakat güvenilir bir metoda ihtiyaç vardır. Teklif edilen metod, drenaj alanına ait S - eğrisine dayanmakta olup lüzumlu hazne hacmini, yağış alanının fiziksel ve hidrolojik karakteristikleri cinsinden bulmaya imkân vermektedir. Analitik ifadesi verilen iki S - eğrisi için metodun tatbik şekli gösterilmiştir.

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There remains a need to develop a simple but reliable method which considers as much factors effecting the capacity of the retentions basins used in urban drainage as possible. The proposed method is based on the S - curve of the drainage area and enables to find the variation of the required volume of the basin with respect to the physical and hydrological characteristic of the water shed. The application of the method has been shown for the two S - Curves whose analytical equations are given.

Introduction

Retention basins are very important elements of storm or combined sewerage systems. They store a part of storm water inflowing during intense rains. After the storm ceases they discharge the stored water slowly into downstream channels, a pumping station, or a treatment plant. So, retention basins constructed at the end of the drainage area reduce the peak rate of runoff in the main sewer. In this way smaller diameters are possible in the main sewer, resulting considerable economy. Retention basins equipped with the overflows reduce the waste water load flowing into the river; so they are very advantageous from sanitary point of view. Retention basins are made empty by means of pumps where there is no sufficient slope.

All kinds of storage problems related with water economy are solved with the help of inflow hydrographs or mass curves. Examples are water supply reservoirs, dams and retention basins. The capacity of water supply reservoirs, for example, can be determined according to the variation of water consumption of the community with respect to time. In this case inflow is constant, outflow fluctuates. However great difficulties are encountered for determination of required capacity of retention basins because in this case inflow hydrograph changes from one storm to the other, and depends upon the shape of the water shed considerably. It is necessary to compute many inflow hydrographs for various storm durations and find the storage corresponding to each storm. The maximum storage computed gives the required capacity of the retention basin. However this is a very long and tedious procedure. For this reason a number of methods (See References) appear in the literature which have been presented to solve the problem with many simplifications. They are far from satisfaction. A method considering the real shape of the S - Curve of the drainage area is required for all kinds of practical problems. From this point of view the following method has been developed and its application on two dimensionless S - curves has been shown.

A New Method Developed for Computation of Retention Basin Capacity

In this paper the inflow hydrograph has been considered for a storm with uniform intensity. Hence the rate of runoff at a moment t is directly obtained as the difference between the ordinates of the two S-curves shifted horizontally by a distance equal to storm duration t_d (see Figure 1)

Let us express in general the storm intensity as $r = r_r \varphi(t_d)$. If the S-curve is plotted once for a storm r_r, there is no need to plot the S-curves for other storms. In order to get the S-curves for other storms, it is enough to multiply the ordinates of the S-curve corresponding to the storm r_r by φ; or to use a discharge scale increased by $1/\varphi$. r_r will be called as reference storm.

Let the S-curve resulting from the storm r_r be plotted as shown in Fig. 1 for a location where the retention basin will be constructed. Let us express the outflow from the basin as $Q_c = \eta \cdot Q_r$ where

$$Q_r = \Psi \cdot r_r A \tag{1}$$

A being the surface area of the drainage basin. The storage for a storm with the duration t_d is obtained by substracting the value

$$FA = \frac{\eta}{\varphi} \cdot Q_r \tag{2}$$

from the ordinates of the inflow hydrograph. Since the inflow hydrograph is obtained as the difference between the ordinates of the two S-curves shifted horizontally by a distance t_d, this substraction can be done directly in Fig. 1. If the storm intensity r is given, *FA* can

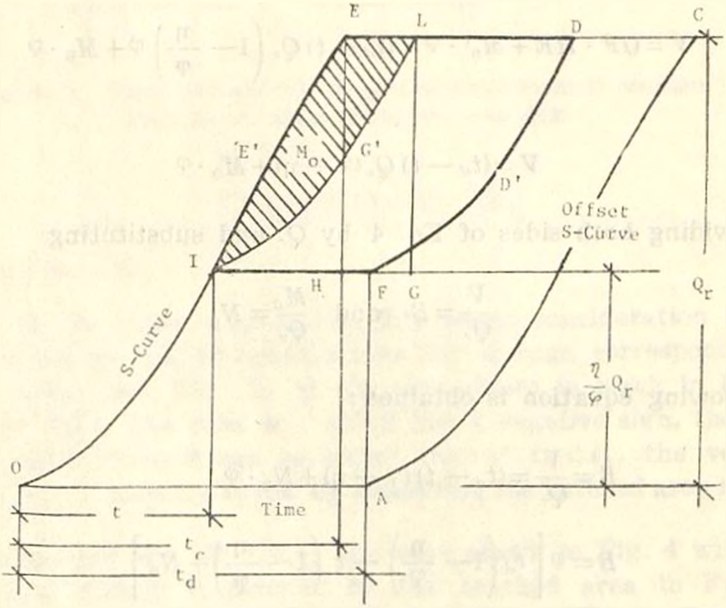

Figure 1. The general case in determination of the water stored in the basin during a storm of duration t_d.

be easily computed. There is no storage till the point I because in this case the outflow is greater than the inflow. If we go on to subtract the value FA from the ordinates of the inflow hydrograph, we obtain the horizontal line IF. The area above this line denotes the volume to be stored in the retention basin. Thereafter we plot the curve $FD'D$ which is obtained by shifting the S-curve vertically by a distance FA and horizontally by a distance t_d, so that the area $IFD'DEE'I$ multiplied by φ gives the volume of storage for a storm with the duration t_d.

Let us draw a curve $IG'L$ from the point I parallel to the curve $FD'D$. Of course this is an offset S-curve displaced horizontally and vertically starting from the point I. If we denote the hatched area with M_0, so the area $IFD'DEE'I$ becomes equal to the sum of the parallelogram $IFD'DLG'I$ and M_0. From Figure 1 we can write:

$$IF = t_d - t\,; \qquad HE = \left(1 - \frac{\eta}{\varphi}\right) \cdot Q_r$$

Hence the storage becomes:

$$V = (IF \cdot HE + M_0) \cdot \varphi = (t_d - t)\, Q_r \left(1 - \frac{\eta}{\varphi}\right) \varphi + M_0 \cdot \varphi \tag{3}$$

or

$$V = (t_d - t)\, Q_r (\varphi - \eta) + M_0 \cdot \varphi \tag{4}$$

Dividing both sides of Eq. 4 by Q_r and substituting

$$\frac{V}{Q_r} = B \quad \text{and} \quad \frac{M_0}{Q_r} = N_0$$

the following equation is obtained:

$$B = \frac{V}{Q_r} = (t_d - t)(\varphi - \eta) + N_0 \cdot \varphi \tag{5}$$

$$B = \varphi \left[t_d \left(1 - \frac{\eta}{\varphi}\right) - t \left(1 - \frac{\eta}{\varphi}\right) + N_0 \right] \tag{6}$$

The following two special cases will be discussed:

Special case I: Let the point D of the curve $FD'D$ coincides with the end point of the S-curve OIE ($t_d = t_k$ in Fig. 2). This is a limiting case and even here Eq. 3 can be applied, that is

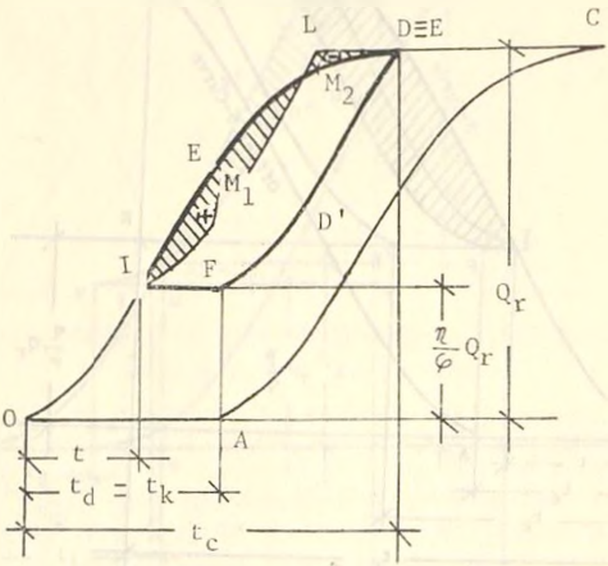

Figure 2. Limit position for the general case: Point D concides with the end point of the S-curve OIE.

$$V = IFD'DLI + M_1 - M_2 \tag{7}$$

where $M_0 = M_1 - M_2$

But if the case seen in Fig. 3 is under consideration ($t_d < t_k$) Eq. 3 can not be applied because here the storage corresponds to the hatched area (see Fig. 3). If the area shown in black in Fig. 3 is subtracted from the area M_2, which has a negative sign, then Eq. 3 can be appled. Thus, it can be stated that if $t_d < t_k$, the volume of storage will be directly found by measuring the hatched area in Fig. 3.

Special case II: If $t_d < t$, the case shown in Fig. 4 will be involved. The storage is denoted by the hatched area in Fig. 4. A limiting position of this special case is shown in Fig. 5 where $V = 0$.

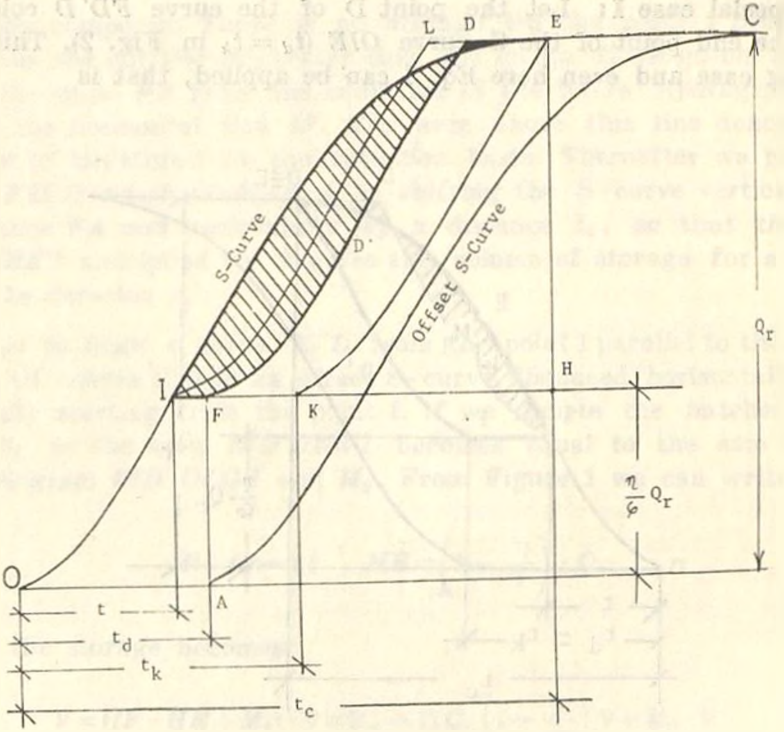

Figure 3. Special case I in calculation of retention basin capacity: The point D is located on the left side of the end point of the S - curve ($t_d < t_k$). The volume of water stored is equal to the hatched area in this case.

From this study it is seen that the storage is found in general by applying Eq. 3 and Eq. 6 and, in special cases by measuring the hatched areas shown in Fig. 3 and Fig. 4.

The S-curve should be shifted without turning in order to plot these hatched areas. A transparent paper as shown in Fig. 6 can be used for this purpose. For example let us say it is desired to shift the S-curve by a distance t and $\frac{\eta}{\varphi} \cdot Q_r$ horizontally and vertically respectively (Fig. 6). The S-curve starts from the origin O. The shifted S-curve will begin from the point I. So the points O and I are marked on the transparent paper, and a small triangular opening is made at the point I in order to make a pencil sign. If the transparent paper is moved without turning, in such a way that the point O moves al-

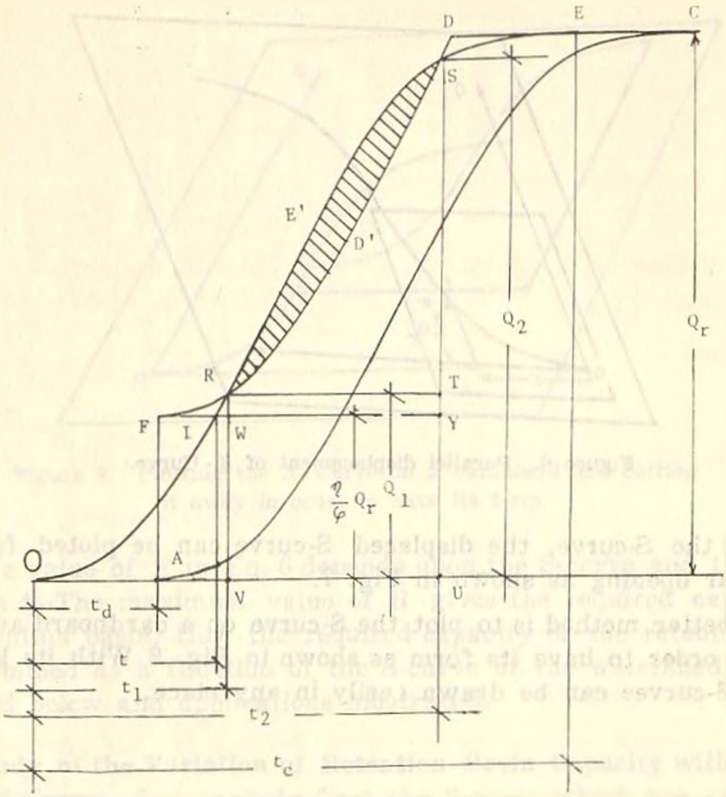

Figure 4. Special case 2 in calculation of retention basin capacity: The point *F* is located on the left side of the *S* - Curve. The volume of water stored is obtained by measuring the hatched area.

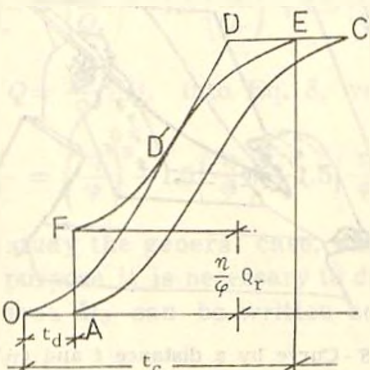

Figure 5. Limiting position for the special case II: The greatest ordinate of the runoff hydrograph is equal to the retention basin outflow, hence the required capacity of the retention basin is zero.

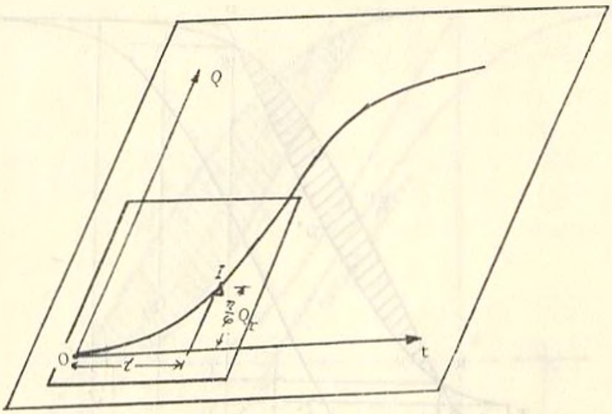

Figure 6. Parallel displacement of S - Curve.

ways on the S-curve, the displaced S-curve can be ploted from the triangular opening as shown in Fig. 7.

A better method is to plot the S-curve on a cardboard and cut it away in order to have its form as shown in Fig. 8 With its help, the shifted S-curves can be drawn easily in any place.

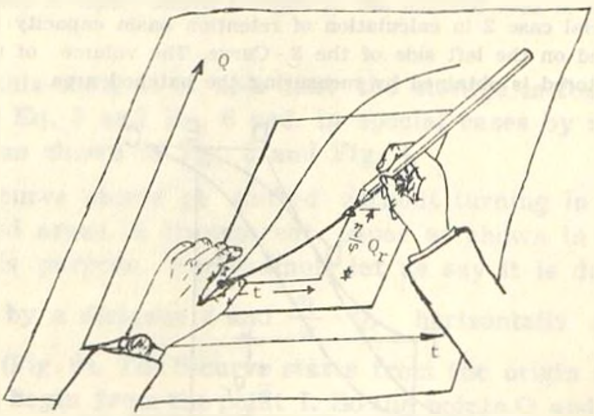

Figure 7. Shifting S - Curve by a distance t and (η/φ) Q_r horizontally and vertically respectively.

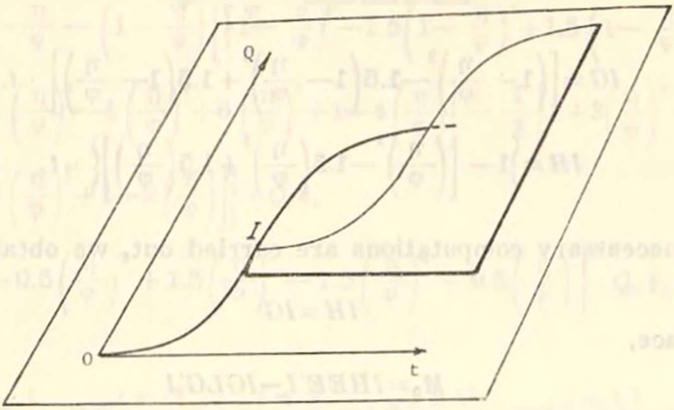

Figure 8. Plotting the S - Curve on a cardboard and cutting it away in order to have its form.

The value of B in Eq. 6 depends upon the S-curve and the storm duration t_d. The maximum value of B gives the required capacity of the retention basin. How the required capacity of the retention basin is determined as a function of the S-curve of the watershed, will be discussed below and applications illustrated.

Study of the Variation of Retention Basin Capacity with Respect to the S-Curve. Let us study first the S-curve which has an analytical equation as given below:

$$\frac{t}{t_c} = \left(\frac{Q}{Q_r}\right)^3 - 1.5\left(\frac{Q}{Q_r}\right)^2 + 1.5\left(\frac{Q}{Q_r}\right) \tag{8}$$

By substituting $Q = \frac{\eta}{\varphi} \cdot Q_r$ into Eq. 8, we obtain:

$$\frac{t}{t_c} = \left(\frac{\eta}{\varphi}\right)^3 - 1.5\left(\frac{\eta}{\varphi}\right)^2 + 1.5\left(\frac{\eta}{\varphi}\right) \tag{9}$$

At first we shall study the general case, and consider the special cases later. For this purpose it is necessary to determine M_0 in Equation 3. The hatched area M_0 can be written according to Fig. 1 as follows

$$M_0 = IHEE'I - IGLG'I + (IG - IH) \cdot HE \tag{10}$$

By Eq. 8

$$EH = LG = \left(1 - \frac{\eta}{\varphi}\right) \cdot Q_r \tag{11}$$

$$IG = \left[\left(1 - \frac{\eta}{\varphi}\right)^3 - 1.5\left(1 - \frac{\eta}{\varphi}\right)^2 + 1.5\left(1 - \frac{\eta}{\varphi}\right)\right] \cdot t_c \tag{12}$$

$$IH = \left\{1 - \left[\left(\frac{\eta}{\varphi}\right)^3 - 1.5\left(\frac{\eta}{\varphi}\right)^2 + 1.5\left(\frac{\eta}{\varphi}\right)\right]\right\} \cdot t_c \tag{13}$$

If neceassary computations are carried out, we obtain :

$$IH = IG$$

and, hence,

$$M_0 = IHEE'I - IGLG'I$$

can be written. Now we shall determine these areas:

$$IGLG'I = \left[\left(1 - \frac{\eta}{\varphi}\right) \cdot \frac{t}{t_c} - \int_{Q/Q_r=0}^{Q/Q_r=1-\eta/\varphi} \frac{t}{t_c}\, d\left(\frac{Q}{Q_r}\right)\right] \cdot Q_r\, t_c \tag{14}$$

where

$$\frac{t}{t_c} = \left(1 - \frac{\eta}{\varphi}\right)^3 - 1.5\left(1 - \frac{\eta}{\varphi}\right)^2 + 1.5\left(1 - \frac{\eta}{\varphi}\right) \tag{15}$$

Substituting Eq. 15 into Eq. 14 we get :

$$IGLG'I = \left\{\left(1 - \frac{\eta}{\varphi}\right) \cdot \frac{t}{t_c} - \left[\frac{1}{4}\left(1 - \frac{\eta}{\varphi}\right)^4 - \frac{1}{2}\left(1 - \frac{\eta}{\varphi}\right)^3 + \right.\right.$$

$$\left.\left. + \frac{3}{4}\left(1 - \frac{\eta}{\varphi}\right)^2\right]\right\} \cdot Q_r\, t_c \tag{16}$$

The area *IHEE'I* is obtained as :

$$IHEE'I = \left\{1 - \int_0^1 (t/t_c) \cdot d(Q/Q_r) - \left[\frac{\eta}{\varphi} \cdot 1 - \int_0^{\frac{\eta}{\varphi}} (t/t_c)\, d(Q/Q_r)\right]\right\} \cdot Q_r\, t_c$$

$$= \left\{1 - \frac{1}{2} - \frac{\eta}{\varphi} + \left[\frac{1}{4}\left(\frac{\eta}{\varphi}\right)^4 - \frac{1}{2}\left(\frac{\eta}{\varphi}\right)^3 + \frac{3}{4}\left(\frac{\eta}{\varphi}\right)^2\right]\right\} \cdot Q_r\, t_c \tag{17}$$

Hence

$$M_0=\left\{0.5-\frac{\eta}{\varphi}-\left(1-\frac{\eta}{\varphi}\right)\left[\left(1-\frac{\eta}{\varphi}\right)^3-1.5\left(1-\frac{\eta}{\varphi}\right)^2+1.5\left(1-\frac{\eta}{\varphi}\right)\right]+\right.$$

$$+\frac{1}{4}\left[2\left(\frac{\eta}{\varphi}\right)^4-4\left(\frac{\eta}{\varphi}\right)^3+6\left(\frac{\eta}{\varphi}\right)^2+1-4\left(\frac{\eta}{\varphi}\right)\right]-\frac{1}{2}\left[1+3\left(\frac{\eta}{\varphi}\right)^2-3\left(\frac{\eta}{\varphi}\right)\right]$$

$$\left.+\frac{3}{4}\left[2\left(\frac{\eta}{\varphi}\right)^2+1-2\left(\frac{\eta}{\varphi}\right)\right]\right\}\cdot Q_r\,t_c \tag{18}$$

$$M_0=\left[-0.5\left(\frac{\eta}{\varphi}\right)^4+1.5\left(\frac{\eta}{\varphi}\right)^3-1.5\left(\frac{\eta}{\varphi}\right)^2+0.5\left(\frac{\eta}{\varphi}\right)\right]\cdot Q_r\,t_c \tag{19}$$

Hence

$$N_0=\frac{M_0}{Q_0}=\left[-0.5\left(\frac{\eta}{\varphi}\right)^4+1.5\left(\frac{\eta}{\varphi}\right)^3-1.5\left(\frac{\eta}{\varphi}\right)^2+0.5\left(\frac{\eta}{\varphi}\right)\right]\cdot t_c \tag{20}$$

Now let us calculate the expression $-t+\left(\frac{\eta}{\varphi}\right)t$ in Eq. 6

$$-t+\left(\frac{\eta}{\varphi}\right)t=t_c\left[-\left(\frac{\eta}{\varphi}\right)^3+1.5\left(\frac{\eta}{\varphi}\right)^2-1.5\left(\frac{\eta}{\varphi}\right)+\left(\frac{\eta}{\varphi}\right)^4-1.5\left(\frac{\eta}{\varphi}\right)^3+1.5\left(\frac{\eta}{\varphi}\right)^2\right]$$

$$=t_c\left[\left(\frac{\eta}{\varphi}\right)^4-2.5\left(\frac{\eta}{\varphi}\right)^3+3\left(\frac{\eta}{\varphi}\right)^2-1{,}5\left(\frac{\eta}{\varphi}\right)\right] \tag{21}$$

By substituting Eq. 20 and Eq. 21 in Eq. 6, we can write:

$$B=\varphi\left\{t_d\left(1-\frac{\eta}{\varphi}\right)+t_c\left[\left(\frac{\eta}{\varphi}\right)^4-2{,}5\left(\frac{\eta}{\varphi}\right)^3+3\left(\frac{\eta}{\varphi}\right)^2-1{,}5\left(\frac{\eta}{\varphi}\right)-0{,}5\left(\frac{\eta}{\varphi}\right)^4+\right.\right.$$

$$\left.\left.+1{,}5\left(\frac{\eta}{\varphi}\right)^3-1.5\left(\frac{\eta}{\varphi}\right)^2+0.5\left(\frac{\eta}{\varphi}\right)\right]\right\} \tag{22}$$

or

$$B=\varphi\left\{t_d\left(1-\frac{\eta}{\varphi}\right)+t_c\left[0.5\left(\frac{\eta}{\varphi}\right)^4-\left(\frac{\eta}{\varphi}\right)^3+1.5\left(\frac{\eta}{\varphi}\right)^2-\left(\frac{\eta}{\varphi}\right)\right]\right\} \tag{23}$$

For a certain storm duration, B becomes maximum. Therefore we must equalize the first derivative of Eq. 23 to zero:

$$\frac{dB}{dt_d}=\frac{\partial B}{\partial\varphi}\cdot\frac{d\varphi}{dt_d}+\frac{\partial B}{\partial t_d}=0$$

$$\frac{dB}{dt_d}=\left\{t_d-t_c\left[\left(1.5\left(\frac{\eta}{\varphi}\right)^4-2\left(\frac{\eta}{\varphi}\right)^3+1.5\left(\frac{\eta}{\varphi}\right)^2\right]\right\}\frac{d\varphi}{dt_d}+\varphi-\eta=0 \tag{24}$$

Hence we get

$$t_c = \frac{\dfrac{\dfrac{\varphi - \eta}{d\varphi}}{dt_d} + t_d}{\left(\dfrac{\eta}{\varphi}\right)^2 \left[1.5\left(\dfrac{\eta}{\varphi}\right)^2 - 2\left(\dfrac{\eta}{\varphi}\right) + 1.5\right]} \qquad (25)$$

For $\varphi = 24/(t_d + 9)$ we can write

$$t_c = \frac{\dfrac{\eta\,(t_d+9)^2}{24} - 9}{\left(\eta\dfrac{t_d+9}{24}\right)^2 \cdot \left[1.5\left(\eta\dfrac{t_d+9}{24}\right)^2 - 2\left(\eta\dfrac{t_d+9}{24}\right) + 1.5\right]} \qquad (26)$$

or

$$t_c = \frac{24\dfrac{\eta}{\varphi^2} - 9}{\left(\dfrac{\eta}{\varphi}\right)^2 \left[1.5\left(\dfrac{\eta}{\varphi}\right)^2 - 2\left(\dfrac{\eta}{\varphi}\right) + 1.5\right]} \qquad (27)$$

Now we shall consider the special cases explained above. From Eq. 12 and 13 we obtained $IH = IG$. Therefore the point L and E of Fig. 1 coincide in this case. So the special case 1 seen in Fig. 3 does not occur at all. However the special case 2 should be studied for $t_d < t$. For this purpose let us consider Fig. 4 again: Using the following notations:

$$\frac{\eta}{\varphi} = y_0 \; ; \quad \frac{Q_1}{Q_r} = y_1 \; ; \quad \frac{Q_2}{Q_r} = y_2 \qquad (28)$$

$$\frac{t_d}{t_c} = x_0 \; ; \quad \frac{t_1}{t_c} = x_1 \; ; \quad \frac{t_2}{t_c} = x_2 \qquad (29)$$

where $t_1 ; t_2$ and $Q_1 ; Q_2$ are abscissas and ordinates of the points R and S respectively. Since the hatched area shown in Fig. 4 gives the capacity of the retention basin, the coordinates of the intersection points R and S are required. These are the common points of the curves FRS and ORS and satisfy the equation of both the curves. For the curve FRS

$$x - x_0 = (y - y_0)^3 - 1.5\,(y - y_0)^2 + 1.5\,(y - y_0) \qquad (30)$$

can be written. Since R and S are on the curve ORS at the same time

$$x = y^3 - 1.5\,y^2 + 1.5\,y \tag{31}$$

is obtained. By substituting Eq. 31 into Eq. 30

$$y^3 - 1.5\,y^2 + 1.5\,y - x_0 = (y - y_0)^3 - 1.5\,(y - y_0)^2 + 1.5\,(y - y_0)$$

or

$$3\,y_0\,y^2 - y\,(3\,y_0^2 + 3\,y_0) + y_0^3 + 1.5\,y_0^2 + 1.5\,y_0 - x_0 = 0 \tag{32}$$

can be written. The roots of this quadratic equation give the ordinate y_1 and y_2 of the points R and S. According to Fig. 4 the capacity of the basin is:

$$V = (\text{Area } WYSE'R - \text{Area } WYSD'R) \cdot \varphi = (f_1 - f_2) \cdot \varphi \tag{33}$$

For this reason the areas f_1 and f_2 are computed as follows:

$$\frac{f_1}{Q_r\,t_e} = \int_{y_1}^{y_2} y\,dx - y_0\,(x_2 - x_1) \tag{34}$$

where

$$dx = (3\,y^2 - 3\,y + 1.5)\,dy \tag{35}$$

By substituting Eq. 35 into Eq. 34, we get

$$\frac{f_1}{Q_r\,t_e} = \int_{y_1}^{y_2} (3\,y^3 - 3\,y^2 + 1.5\,y)\,dy - y_0(x_2 - x_1) =$$

$$= \left[\frac{3}{4}\,y^4 - y^3 + \frac{3}{4}\,y^2\right]_{y_1}^{y_2} - (x_2 - x_1)\,y_0$$

$$\frac{f_1}{Q_r\,t_e} = \frac{3}{4}\,(y_2^4 - y_1^4) - (y_2^3 - y_1^3) + \frac{3}{4}\,(y_2^2 - y_1^2) - y_0\,(x_2 - x_1) \tag{36}$$

In a similar way

$$\frac{f_2}{Q_r\,t_e} = \frac{3}{4}\,[(y_2 - y_0)^4 - (y_1 - y_0)^4] - [(y_2 - y_0)^3 - (y_1 - y_0)^3] +$$

$$+ \frac{3}{4}\,[(y_2 - y_0)^2 - (y_1 - y_0)^2] \tag{37}$$

is obtained and if necessary calculations are carried out, the following equation can be written:

$$\frac{f_7}{Q_r t_c} = \frac{3}{4}[y_2^2+y_1^2+2y_0(y_0-y_2-y_1)][y_2^2-y_1^2-2y_0(y_2-y_1)]-$$
$$-(y_2-y_1)(y_2^2+y_1^2+3y_0^2-3y_2y_0+y_2y_1-3y_1y_0)+$$
$$+\frac{3}{4}(y_2-y_1)(y_2+y_1-2y_0) \quad (38)$$

Here y_1 and y_2 express the roots of the qadratic equation 32. If an equation of second degree is given in the form

$$ay^2+by+c=0 \quad (39)$$

the following relations can be written between cofficients and the roots:

$$y_1+y_2=-\frac{b}{a}$$
$$y_1 \cdot y_2=\frac{c}{a} \quad (40)$$
$$y_1=\frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$y_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

Using Eqs. 40 the following relations can be written:

$$y_2^2+y_1^2=\frac{b^2}{a^2}-2\frac{c}{a}$$
$$y_2-y_1=\frac{\sqrt{b^2-4ac}}{a}$$
$$y_2^4-y_1^4=\frac{b}{a^2}\sqrt{b^2-4ac}\left(2\frac{c}{a}-\frac{b^2}{a^2}\right) \quad (41)$$
$$y_2^3-y_1^3=\frac{\sqrt{b^2-4ac}}{a}\left(\frac{b^2}{a^2}-\frac{c}{a}\right)$$
$$y_2^2-y_1^2=\frac{-b\sqrt{b^2-4ac}}{a^2}$$

By substituting Eqs. 40 and 41 into Eqs. 36 we get:

$$\frac{f_1}{Q_r t_c} = \frac{3b}{4a^2}\sqrt{b^2-4ac}\left(-1+2\frac{c}{a}-\frac{b^2}{a^2}\right)-\frac{\sqrt{b^2-4ac}}{a}\left(\frac{b^2}{a^2}-\frac{c}{a}\right)-$$
$$-y_0(x_2-x_1) \tag{42}$$

or

$$\frac{f_1}{Q_r t_c} = \frac{\sqrt{b^2-4ac}}{a}\left|\frac{3b}{4a}\left(2\frac{c}{a}-\frac{b^2}{a^2}-1\right)-\frac{b^2}{a^2}+\frac{c}{a}\right|-$$
$$-y_0(x_2-x_1) \tag{43}$$

In a similar way we get:

$$\frac{f_2}{Q_r t_c} = \frac{\sqrt{b^2-4ac}}{a}\left\{\frac{3}{4}\left|\frac{b^2}{a^2}-2\frac{c}{a}+2y_0\left(y_0+\frac{b}{a}\right)\right]\left(-\frac{b}{a}-2y_0\right)-\right.$$
$$\left.-\left(\frac{b^2}{a^2}-2\frac{c}{a}+3y_0^2+3y_0\frac{b}{a}+\frac{c}{a}\right)-\frac{3}{4}\left(\frac{b}{a}+2y_0\right)\right\} \tag{44}$$

Now we can write:

$$\frac{f_1-f_2}{Q_r t_c} = \frac{\sqrt{b^2-4ac}}{a}\left\{\frac{3b}{4a}\left(2\frac{c}{a}-\frac{b^2}{a^2}-1\right)-\frac{b^2}{a^2}+\frac{c}{a}-\right.$$
$$-\frac{3}{4}\left|\frac{b^2}{a^2}-2\frac{c}{a}+2y_0\left(y_0+\frac{b}{a}\right)\right|\left(-\frac{b}{a}+2y_0\right)+$$
$$\left.+\frac{b^2}{a^2}-2\frac{c}{a}+3y_0^2+3y_0\frac{b}{a}+\frac{c}{a}+\frac{3}{4}\left(\frac{b}{a}+2y_0\right)\right\}-$$
$$-y_0(x_2-x_1) \tag{45}$$

or

$$\frac{f_1-f_2}{Q_r t_c} = \frac{\sqrt{b^2-4ac}}{a}\cdot\left[\frac{3b^2}{a^2}y_0-3\frac{c}{a}y_0+(4.5y_0^2+3y_0)\cdot\frac{b}{a}+\right.$$
$$\left.+3y_0^3+3y_0^2+\frac{3}{2}y_0\right|-y_0(x_2-x_1) \tag{46}$$

a, *b*, and *c* are the coefficients of the quadratic Eq. 32:

$$a=3y_0\,,\quad b=-3(y_0^2+y_0)\,,\quad c=y_0^3+1.5y_0^2+1.5y_0-x_0$$

$$\begin{aligned} b^2-4ac &= 9(y_0^2+2y_0^3+y_0^4)-12y_0^4-18y_0^3-18y_0^2+12x_0y_0 \\ &= -3y_0^4-9y_0^2+12x_0y_0 \\ \frac{\sqrt{b^2-4ac}}{a} &= \frac{\sqrt{12x_0y_0-9y_0^2-3y_0^4}}{3y_0} \end{aligned} \tag{47}$$

If we substitute Eqs. 47 into Eq. 46 we obtain:

$$\frac{f_2-f_1}{Q_r t_c} = \sqrt{12x_0y_0-9y_0^2-3y_0^4}\cdot\left(\frac{y_0^2}{6}+\frac{x_0}{3y_0}\right)-y_0(x_2-x_1) \tag{48}$$

We shall determine now (x_2-x_2) · Since x_1 and x_2 are the abscissas of the points R and S we can write:

$$x_2=y_2^3-1.5y_2^2+1.5y_2 \tag{49}$$

$$x_1=y_1^3-1.5y_1^2+1.5y_1 \tag{50}$$

Hence

$$\begin{aligned} x_2-x_1 &= y_2^3-y_1^3-1.5(y_2^2-y_1^2)+1.5(y_2-y_1)= \\ &= (y_2-y_1)\cdot[y_2^2+y_2y_1+y_1^2-1.5(y_2+y_1)+1.5] \end{aligned} \tag{51}$$

Substituting Eqs. 41 into Eq. 51 we get

$$x_2-x_1=\frac{\sqrt{b^2-4ac}}{a}\cdot\left[\frac{b^2}{a^2}-\frac{c}{a}+1.5\frac{b}{a}+1.5\right] \tag{52}$$

By substituting Eqs. 47 into Eq. 52

$$y_0(x_2-x_1)=\frac{\sqrt{12x_0y_0-9y_0^2-3y_0^4}}{3y_0}\cdot\left(\frac{2y_0^3}{3}+\frac{y_0}{2}+\frac{x_0}{3}\right) \tag{53}$$

Now we can write:

$$\frac{f_1-f_2}{Q_r t_c}=\frac{\sqrt{12x_0y_0-9y_0^2-3y_0^4}}{3y_0}\cdot\left(\frac{y_0^3}{2}+x_0-\frac{2y_0^3}{3}-\frac{y_0}{2}-\frac{x_0}{3}\right) \tag{54}$$

$$\frac{f_1-f_2}{Q_r t_c}=\sqrt{12x_0y_0-9y_0^2-3y_0^4}\left(-\frac{y_0^2}{18}-\frac{1}{6}+\frac{2x_0}{9y_0}\right) \tag{55}$$

Hence the value B is obtained for the case $t_d<t$

$$B=\frac{V}{Q_r}=\varphi\,\frac{f_1-f_2}{Q_r}$$

$$B=\varphi\cdot\sqrt{12x_0y_0-9y_0^2-3y_0^4}\cdot\left(-\frac{y_0^2}{18}-\frac{1}{6}+\frac{2x_0}{9y_0}\right)t_c \tag{56}$$

Substituting $x_0=\frac{t_d}{t_c}$, $y_0=\frac{\eta}{\varphi}$ into Eq. 56 we get:

$$B=\varphi\, t_c \sqrt{12\frac{t_d}{t_c}\cdot\frac{\eta}{\varphi}-9\left(\frac{\eta}{\varphi}\right)^2-3\left(\frac{\eta}{\varphi}\right)^4}\left[-\frac{1}{18}\left(\frac{\eta}{\varphi}\right)^2-\frac{1}{6}+\frac{2\,t_d}{9\,t_c}\frac{\varphi}{\eta}\right] \tag{57}$$

We shall differentiate the expression B with respect to t_d in order to find the storm duration which makes B maximum. For this purpose the first derivative should be zero:

$$\frac{dB}{dt_d}=\frac{\partial B}{\partial \varphi}\cdot\frac{d\varphi}{dt_d}+\frac{\partial B}{\partial t_d}=0 \tag{58}$$

Now we must determine the above mentioned derivatives:

$$\frac{\partial B}{\partial \varphi}=\beta\left[\frac{t_c}{18}\left(\frac{\eta}{\varphi}\right)^2+\frac{4}{9}t_d\left(\frac{\varphi}{\eta}\right)-\frac{t_c}{6}\right]+$$
$$+\frac{1}{\beta}\left[\frac{10\,t_d}{6}\left(\frac{\eta}{\varphi}\right)^3+3t_d\left(\frac{\eta}{\varphi}\right)-\frac{12\,t_d^2}{9\,t_c}-\frac{3}{2}t_c\left(\frac{\eta}{\varphi}\right)^4-\right.$$
$$\left.-\frac{3}{2}t_c\left(\frac{\eta}{\varphi}\right)^2-\frac{1}{3}t_c\left(\frac{\eta}{\varphi}\right)^6\right] \tag{59}$$

$$\frac{\partial B}{\partial t_d}=\frac{6\eta}{\beta}\left[-\frac{1}{18}\left(\frac{\eta}{\varphi}\right)^2-\frac{1}{6}+\frac{2}{9}\left(\frac{t_d}{t_c}\right)\left(\frac{\varphi}{\eta}\right)\right]+\frac{2\,\beta\,\varphi^2}{9\,\eta} \tag{60}$$

where

$$\beta=\sqrt{12\left(\frac{t_d}{t_c}\right)\left(\frac{\eta}{\varphi}\right)-9\left(\frac{\eta}{\varphi}\right)^2-3\left(\frac{\eta}{\varphi}\right)^4} \tag{61}$$

Substituting the expressions of $\varphi=24/(t_d+9)$; $d\varphi/dt_d=-24/(t_d+9)^2 = -\varphi^2/24$ and Eqs. 59, 60, 61 into Eq. 58 we get:

$$\frac{dB}{dt_d}=\beta\left[-\frac{t_c}{18}\left(\frac{\eta}{\varphi}\right)^2-\frac{4\,t_d}{9}\left(\frac{\varphi}{\eta}\right)+\frac{t_c}{6}+\frac{48}{9\,\eta}\right]\frac{\varphi^2}{24}+$$
$$+\frac{1}{\beta}\left[-\frac{5t_d}{3}\left(\frac{\eta}{\varphi}\right)^3-3t_d\left(\frac{\eta}{\varphi}\right)+\frac{4\,t_d^2}{3t_c}+\frac{3t_c}{2}\left(\frac{\eta}{\varphi}\right)^4+\right.$$
$$\left.+\frac{3\,t_c}{2}\left(\frac{\eta}{\varphi}\right)^2+\frac{t_c}{3}\left(\frac{\eta}{\varphi}\right)^6-\frac{8}{\varphi}\left(\frac{\eta}{\varphi}\right)^3-\frac{24}{\varphi}\left(\frac{\eta}{\varphi}\right)+\frac{32\,t_d}{\varphi\,t_c}\right]\cdot\frac{\varphi^2}{24}=0 \tag{62}$$

or

$$(t_c)^2\left[1.5\left(\frac{\eta}{\varphi}\right)^4+0.5\left(\frac{\eta}{\varphi}\right)^6\right]+t_c\left[3t_d\left(\frac{\eta}{\varphi}\right)-\frac{48}{\eta}\left(\frac{\eta}{\varphi}\right)^2-\right.$$

$$\left.-t_d\left(\frac{\eta}{\varphi}\right)^3-\frac{16}{\eta}\left(\frac{\eta}{\varphi}\right)^4-\frac{8}{\varphi}\left(\frac{\eta}{\varphi}\right)^3-\frac{24}{\varphi}\left(\frac{\eta}{\varphi}\right)\right]-$$

$$-4\,t_d^2+\frac{64}{\eta}\,t_d\left(\frac{\eta}{\varphi}\right)+\frac{32}{\varphi}\,t_d=0 \quad (63)$$

Eq. 63 is of second degree with rerpect to (t_c). The discriminant of this equation is

$$\Delta=\left[3t_d\left(\frac{\eta}{\varphi}\right)-\frac{48}{\eta}\left(\frac{\eta}{\varphi}\right)^2+3\,t_d\left(\frac{\eta}{\varphi}\right)^3-\frac{16}{\eta}\left(\frac{\eta}{\varphi}\right)^4-\frac{8}{\varphi}\left(\frac{\eta}{\varphi}\right)^3-\right.$$

$$\left.-\frac{24}{\varphi}\left(\frac{\eta}{\varphi}\right)\right]^2 \quad (64)$$

The roots of Eq. 63 are

$$(t_c)_1=\frac{18}{\left(\frac{\eta}{\varphi}\right)^3} \quad (65)$$

$$(t_c)_2=\frac{4\,t_d}{3\left(\frac{\eta}{\varphi}\right)+\left(\frac{\eta}{\varphi}\right)^3} \quad (66)$$

If we substitute Eq. 66 into Eq. 57 we get the following expression for the large bracket in Eq. 57:

$$-\frac{1}{18}\left(\frac{\eta}{\varphi}\right)^2-\frac{1}{6}+\frac{2}{9}\left(\frac{t_d}{t_c}\right)\left(\frac{\varphi}{\eta}\right)=-\frac{1}{18}\left(\frac{\eta}{\varphi}\right)^2-\frac{1}{6}+$$

$$+\frac{2}{4\times 9}\left(\frac{\varphi}{\eta}\right)\left[3\left(\frac{\eta}{\varphi}\right)+\left(\frac{\eta}{\varphi}\right)\right]^3$$

$$=-\frac{1}{18}\left(\frac{\eta}{\varphi}\right)^2-\frac{1}{6}+\frac{1}{6}+\frac{1}{18}\left(\frac{\eta}{\varphi}\right)^2=0 \quad (67)$$

Hence $B=0$ for the second root $(t_c)_2$. Therefore, we conclude that the design storm for the retention basin is to be calculated from Eq. 65 when $t_d < t$.

If we compute t_c from Eq. 26 or Eq. 65 for various values of t_d and substitute into the expression B, we get the function $B=f(\eta, t_c)$ by selecting η as parameter.

$$\text{If } \frac{t_d}{t_c} > \frac{t}{t_c} \text{ Eqs. 23 and 26}$$

$$\text{If } \frac{t_d}{t_c} < \frac{t}{t_c} \text{ Eqs. 57 and 65}$$

will be used. Values of B computed in this way have been shown as a function of concentration time t_c for various values of parameter η in Table 11 of Reference No. 3. The thick horizontal lines indicate the boundary of application of Eq. 26 and Eq. 65. The following inequality is valid above this line:

$$\frac{t_d}{t_c} > \frac{t}{t_c} \tag{68}$$

where Eq. 26 has been applied. Eq. 65 has been used below this line. The function $B = f(\eta, t_c)$ has been plotted in Figure 9.

Determination of Retention Basin Capacity for a Linear S-Curve. Since $M_0 = 0$ and $t = \frac{\eta}{\varphi} \cdot t_c$ we can write:

$$B = \left(t_d - t_c \cdot \frac{\eta}{\varphi}\right)(\varphi - \eta) \tag{69}$$

Here, time of concentration is expressed with the notation t_c. The first derivative of this expression will be equated to zero in order to find the maximum value of B

$$\frac{dB}{dt_d} = \frac{\partial B}{\partial \varphi} \cdot \frac{d\varphi}{dt_d} + \frac{\partial B}{\partial t_d} \tag{70}$$

$$\frac{\partial B}{\partial \varphi} = \frac{\eta}{\varphi^2} \cdot t_c(\varphi - \eta) + t_d - \frac{\eta}{\varphi} \cdot t_c = t_d - t_c \frac{\eta^2}{\varphi^2} \tag{71}$$

$$\frac{\partial B}{\partial t_d} = \varphi - \eta \tag{72}$$

$$\frac{dB}{dt_d} = \left(t_d - \frac{\eta^2}{\varphi^2} \cdot t_c\right) \frac{d\varphi}{dt_d} + \varphi - \eta = 0 \tag{73}$$

By substituting $\varphi = \frac{24}{t_d + 9}$ in Eq. 73 we get:

$$\frac{dB}{dt_d} = -\left(t_d - \frac{\eta^2 (t_d + 9)^2 \cdot t_c}{24^2}\right) \cdot \frac{24}{(t_d + 9)^2} + \frac{24}{t_d + 9} - \eta = 0$$

Hence we obtain:

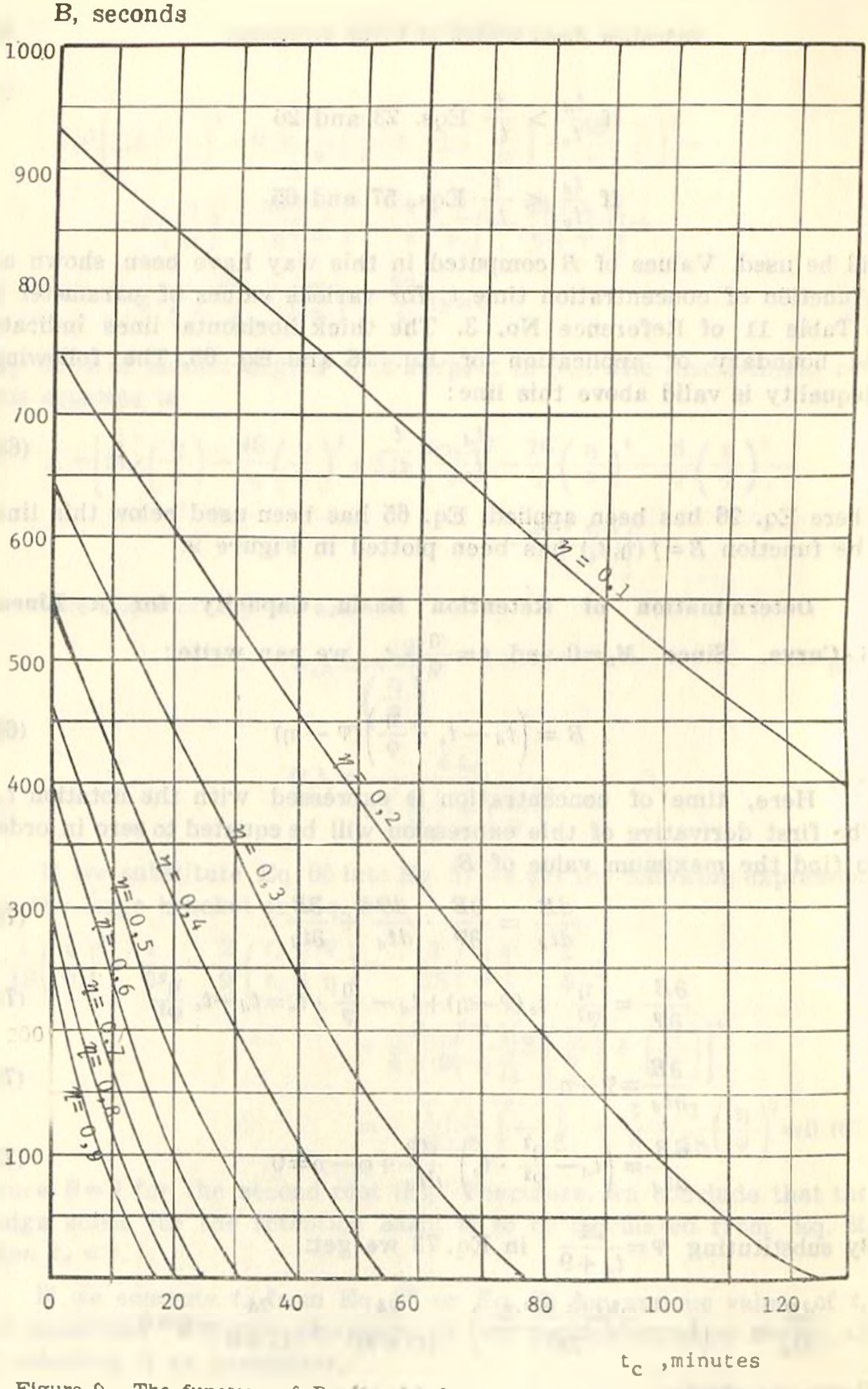

Figure 9. The function of $B = f(\eta, t_c)$ for a dimensionless S - curve with equation of $t/t_c = (Q/Q_r)^3 - 1.5\,(Q/Q_r)^2 + 1.5Q/Q_r$

$$t_d = \frac{24 \times 3}{\sqrt{\eta(24 - \eta t_c)}} - 9 \tag{74}$$

From Eq. 74 we can write:

$$\varphi = \frac{24}{t_d + 9} = \frac{\sqrt{\eta(24 - \eta t_c)}}{3} \tag{75}$$

Substituting Eq. 74 and Eq. 75 in Eq. 69 we obtain :

$$B = \left(\frac{24 \times 3}{\sqrt{\eta(24 - \eta t_c)}} - 9 - \frac{3 \eta t_c}{\sqrt{\eta(24 - \eta t_c)}} \right) \cdot \left(\frac{\sqrt{\eta(24 - \eta t_c)}}{3} - \eta \right) \tag{76}$$

or,

$$B = 24 - \eta(t_c - 9) - 6\sqrt{\eta(24 - \eta t_c)} \tag{77}$$

From Eqs. 69 and 75 it can be easily seen that when

$$\frac{\sqrt{\eta(24 - \eta t_c)}}{3} - \eta = 0 \tag{78}$$

B becomes zero.

From Eq. 78 the following expression can be written for the concentration time which requires no retention basin:

$$t_c = \frac{24}{\eta} - 9 \tag{79}$$

B and t_c computed by Eq. 77 and Eq. 79 for various values of the parameter η have been shown in Figure 10

Summary and Conclusion

To find the required capacity of storm water retention basins it is necessary to determine runoff hydrographs for various storms and compute the storage in each case. The maximum storage gives the required capacity of the basin. However this is a very difficult and time taking procedure. Therefore a new method has been developed to find the storage during various storms by means of a single S - curve plotted for a reference storm r_r, instead of plotting many S - curves for each storm to compute the runoff hydrographs. How the required capacity of the retention ba-

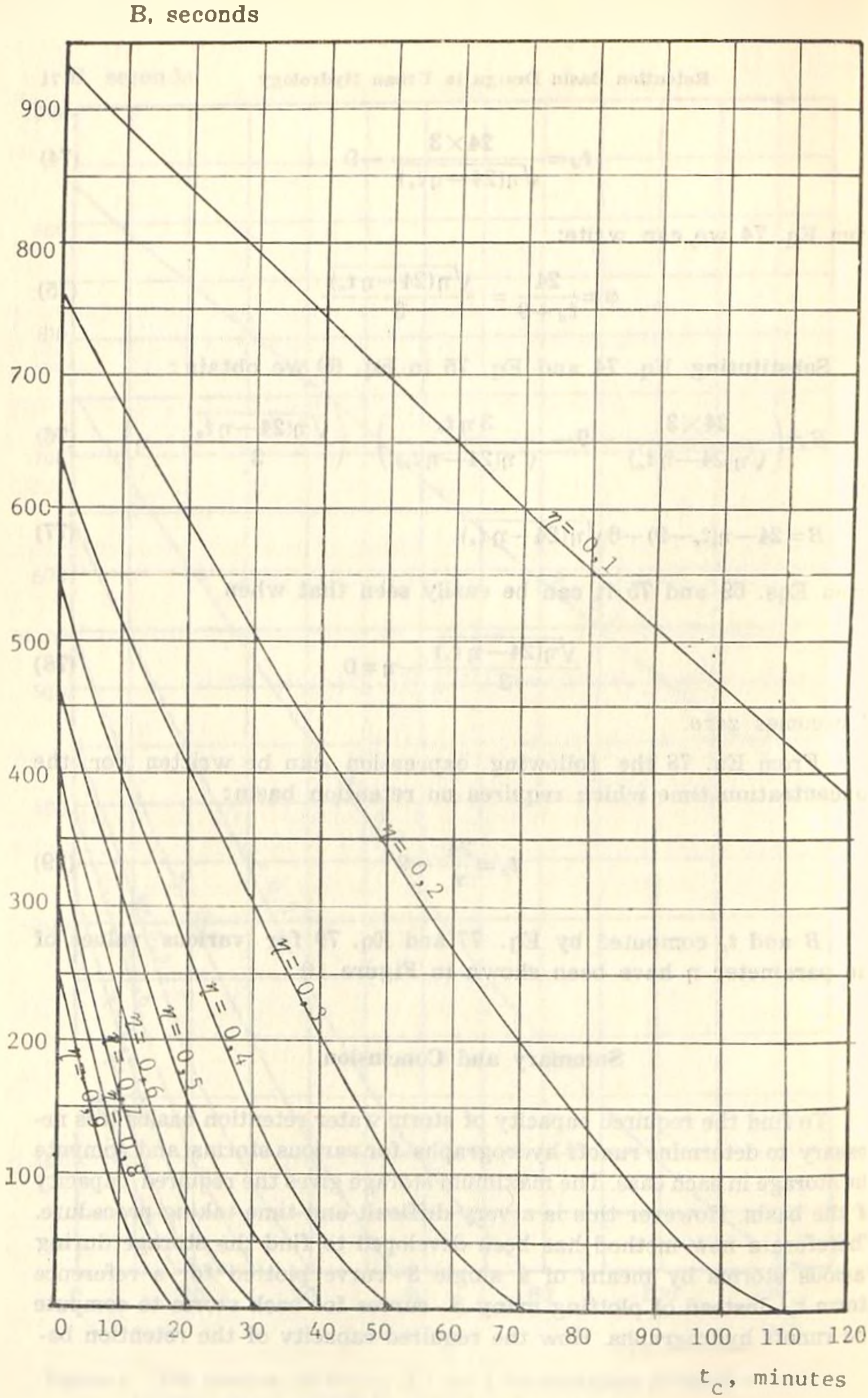

Figure 10. The function of $B = f(\eta, t_c)$ for the linear S - Curve.

sin is determined as a function of the *S* - curve of the water shed has been discussed and its applications on two *S* - curves have been illustrated.

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